A Semi-automatic Alignment Method for Math Educational Standards using the MP (Materialization Pattern) Model

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ABSTRACT

A Semi-automatic Alignment Method for Math Educational Standards using MP (Materialization Pattern) Model

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Educational standards alignment, which matches similar or equivalent concepts of educational standards, is a necessary task for educational resource discovery and retrieval. Automated or semi-automated alignment systems for educational standards have been recently available. However, existing systems frequently result in inconsistency in interpreting a correct alignment or give only a “yes” or no” Boolean decision for alignment. In this research, we present a novel semi-automatic alignment method for math educational standards that goes beyond simple Boolean decision making. Our approach gives seven different degrees of alignments: Strongly Fully-aligned (SFA), Weakly Fully-aligned (WFA), Partially-aligned*** (PA***), Partially-aligned** (PA**), Partially-aligned* (PA*), Poorly-aligned (PR), and Not-aligned (NA). We aim to clarify and extend the notion of alignment for math educational standards, and to broaden categories of resource discovery and retrieval. First, we propose the MP (Materialization Pattern) model for representing the semantics of math educational standards for the purpose of aligning these standards. The MP model captures the semantics of English sentences used in math educational standards based on the Reed-Kellogg sentence diagram. We develop a semi-automatic tool, MPViz, for creating the MP model using the UML notation. The MPViz also converts an MP diagram to two graphs—a verb-phrase
graph and a noun-phrase graph—which facilitate the process of automatic alignments. We align math educational standard statements using graph matching with the Bloom taxonomy, the WordNet, and taxonomies of math concepts. We also develop a semi-automatic tool, MPComp, for aligning math educational standards. This dissertation describes a novel semi-automatic alignment method that utilizes the MP modeling and graph matching. Our experiments show that our alignment method provides the result that is comparable to human judgment. The contributions of our alignment method are as follows: 1) We propose the MP model that can explicitly model the semantics of English sentence structures used in math educational standards; 2) Using the MP model we develop a semi-automatic alignment method that produces seven different degrees of alignments, instead of simple Boolean decisions in existing alignment systems; 3) The multiple degrees of alignments empower education professionals by broadening categories of search or retrieval for educational resources.
CHAPTER 1: INTRODUCTION

The need to search for resources according to educational standards has recently become more vital due to the increasing availability of online K-12 curriculum and the standard-based reform movement for educational systems. Educational resources assigned with one state’s standards can be searched or retrieved by teachers in other states through alignment systems. Automated or semi-automated alignment systems have been recently available (Yilmazel et al. 2007; AlignPro 2006; Sutton and Golder 2008). However, consistent and accurate alignments for educational standards are still challenged due to the lack of uniformity in approach and inconsistency in interpreting a correct alignment (Yilmazel et al. 2007). These systems use “yes or no” Boolean decision making for alignment or an alignment method for making suggestions of relevant standards for human evaluation (Diekema 2006) without clarifying a clear definition of ranking of a correct alignment. This alignment method (Diekma 2006) may also lead to inconsistency in interpreting a correct alignment due to various possible interpretations of correct standard match for alignment of human evaluation.

We present the semi-automated alignment method for math educational standards statements using the MP (Materialization Pattern) model. Our alignment method provides the clear notion of alignment and more detailed level of alignment for math educational standards by giving specifically different degrees of alignments such as strongly Fully-aligned (SFA), Weakly Fully-aligned (WFA), Partially-aligned*** (PA***), Partially-aligned** (PA**), Partially-aligned* (PA*), Poorly-aligned(PR), and Not-aligned(NA). Different degrees of alignments can provide consistency in interpreting a correct alignment and also broaden categories of search or retrieval for educational resources.
assigned with math educational standards for an alignment system. For example, “Strongly Fully-aligned” and “Weakly Fully-aligned” allow alignment for equivalent statements. “Strongly Fully-aligned” is for alignment of identical statements. “Weakly-Fully-aligned” is for more-or-less equivalent statements but one statement includes the meaning of the other one. “Partially-aligned***”, “Partially-aligned**”, and “Partially-aligned*” can allow alignment for two math educational standards statements which have the same math concepts. Degrees of alignments are decreased in order, “Partially-aligned***”, “Partially-aligned**”, and “Partially-aligned*”. “Poorly-aligned” also allow alignment for two math educational standards statements which have different but related math concepts with the same cognitive process of cognitive verbs. “Not-aligned” allows alignment for two statements which have totally different meanings.

In order to develop our alignment method we have a two-stage strategy. The first stage is to develop the MP model for representing the semantics of math educational standards for the purpose of aligning those standards. In order to develop the MP model, we first classify math educational standards statements based on sentence structures into 20 different types for a comprehensive coverage of the possible cases. This is an on-going iterative process, and new types can be added in the future if necessary. We refer to these types as MP statements. Each type can have many math educational standards statements. A math educational standards statement belongs to only one type of MP statements. Each type of MP statement has different relationships between concepts (i.e. classes) when it is converted to the MP model: association, aggregation, dependency, prepositional, transitive verb, and realization relationships. Second, we model different types of math educational standard statements to the MP diagrams with the UML notation using the
semi-automatic tool “MPViz”. Math educational standards (i.e. math standards) express the mathematical understanding, knowledge, and skills that students should obtain from pre-kindergarten through grade 12. Almost all statements are imperative mood sentences; a small portion of them have a subject. “Student” is their subject without exception. Two examples of math educational standards statements are as follows: 1) Add and subtract whole numbers with and without regrouping (Ohio State); 2) Add and subtract decimals using money as a model (Nevada State). The MP model is developed at a sentence level for each statement from typical math educational standards. This MP model can explicitly model the semantics of imperative mood sentence structures used in math standards. Sentence analysis is based on the Reed-Kellogg sentence diagram (Reed & Kellogg 2004). Our MP model captures math concepts and the cognitive process of math concepts from math educational standards statements. Hence, the MP model enables us to compare the level of similarity of two statements from different math standards in terms of math concepts and the cognitive process of math concepts. In other words, the MP model facilitates alignment for math educational standards by capturing math concepts and the cognitive process of math concepts. For example, “whole number” and “decimal” in 1) and 2) are math concepts which students should learn. “Add” and “subtract” are cognitive verbs which stand for cognitive processes of the math concepts “whole number” and “decimal,” respectively. By comparing the two math concepts “whole number” and “decimal”, and cognitive verbs, the two statements can be aligned as “Poorly-aligned” because they have related math concepts and the same cognitive process of cognitive verbs.
In the second stage we utilize graph matching for aligning math educational standards statements as follows: 1) We convert an MP diagram to a verb-phrase graph and a noun-phrase graph using MPViz; 2) We align different math educational standards statements using graph matching with the Bloom taxonomy (Bloom & Krathol 1956) for cognitive verb categorization, the WordNet (Fellbaum 1999) for word similarity, and taxonomies of math concepts for related math concepts. We use a semi-automated alignment tool “MPComp” for aligning the standards.

1.1 Problem Statement

We propose to answer the following research questions during the course of our work:

1. Develop a new semi-automatic alignment method for math educational standards statements.
   (1.a) Develop the MP model for representing the semantics of math educational standards statements.
   (1.b) Develop a graph matching algorithm for a semi-automatic alignment method using the MP model.

2. Evaluate our alignment method against human performance.
   (1) Will our alignment method provide the result that is comparable to human judgment?
1.2 Contributions

The primary contributions from the proposed research are as follows:

1. Development of the MP model for the purpose of aligning math educational standards statements, a semi-automatic tool “MPViz” for modeling semantics of these standards, and a semi-automatic tool “MPComp” for automating the alignment.

2. Development and evaluation of a semi-automatic alignment method which utilizes the MP modeling and graph matching.

3. Our alignment method produces different degrees of alignments that can broaden categories of search or retrieval for educational resources, which in turn will result in additional resources for education professionals.

1.3 Dissertation Organization

This dissertation is organized into five chapters. This first chapter is an introductory chapter and lays out research problems expected to be addressed by the proposed research, and gives an overview of our alignment method, as well as contributions of the work. Chapter 2 details the motivation for our research in an alignment method. Chapter 3 provides a literature review, broken down into three major areas: 1) modeling math educational standards, 2) alignment systems for educational standards, and 3) schema matching. Chapter 4 describes our approach for a semi-automatic alignment method which utilizes the MP modeling and graph matching. It presents semi-automatic tools “MPViz” for creating the MP model with UML (Booch et al. 2005) notation and “MPComp” for alignment. It also provides an evaluation for our
semi-automatic alignment method. Chapter 5 presents a conclusion and suggests future work.
CHAPTER 2: MOTIVATION

Automated or semi-automated alignment systems for educational standards have recently become more available (Yilmazel et al. 2007; AlignPro 2006; Sutton and Golder 2008). Educational resources assigned with one state’s standards can be searched or retrieved by teachers in other states through educational standards alignment. See Figure 2.1 for educational standards alignment.

![Diagram of educational standards alignment between Nevada and Idaho](image)

Figure 2.1: Educational standards alignment: Educational standards between Nevada and Idaho are aligned. A teacher in Nevada can retrieve educational resources tagged with a statement in Idaho which is equivalent or similar to a statement in Nevada.

Alignment is a term used in a variety of contexts within the standard-based reform movement, which currently dominates decisions and actions in schools. The term “alignment” (Nasstrom and Henriksson 2008) is summarized as when two or all three components in a certain education system are consistent (Biggs 1999; Blank et al. 2001), in agreement (Bhola et al. 2003; Webb 1997), matched (La Mara 2001; Olson 2003), work together (Ananda 2003; Roach et al. 2005). In alignment systems for educational standards, Yilmazel et al. (2007) describe standard alignment occurring when “standards describing similar concepts are correlated” and Sutton and Golder (2008) state it as “one statement is more-or-less equivalent to another statement.”
We identify two issues regarding these current alignment systems. First, inconsistency in interpreting a correct alignment exists due to various possible interpretations of a correct standard match for alignment (Yilmazel et al. 2007; Diekema 2006). Second, an alignment method which uses a “yes or no” Boolean decision provides searching or retrieving for resources by only a “yes” category of alignment. These two issues motivate our research for an alignment method.

By giving different degrees of alignments, our alignment method identifies a clear notion of alignment which reflects semantics of math educational standards statements, provides consistency in interpreting a correct alignment for these standards, and broadens categories for search or retrieval for educational resources.

The notion of different degrees of alignments is as follows:

1) **Strongly Fully-aligned (SFA)**: Two math educational standard statements are identical.

2) **Weakly Fully-aligned (WFA)**: The meaning of one math educational standard statement is included in the other one.

3) **Partially-aligned*** (PA***): The statements have the same math concept, same cognitive verbs, and different modifiers of math concepts or cognitive verbs.

4) **Partially-aligned** (PA**): The statements have the same math concept, and the same cognitive process with different cognitive verbs.

5) **Partially-aligned* (PA*)**: The statements have the same math concept, and different cognitive process of cognitive verbs.

6) **Poorly-aligned (PR)**: The statements have related math concepts and the same cognitive process of cognitive verbs.
7) *Not-aligned*(NA): The statements have different math concepts, or related math concepts with different cognitive process of cognitive verbs.
CHAPTER 3: LITERATURE REVIEW

This literature review is divided into three sections. The first area is modeling math educational standards, and the second area is alignment systems for educational standards, and the third area is schema matching.

3.1 Modeling math educational standards

The United Modeling Language (UML) (Booch and et al. 2005) has been established as a standard graphical notation for representing knowledge. Representing natural languages with UML, for instance English, has been an important research issue for various reasons – including the transition of natural language software requirements into modeling (Ilieva & Ormandjieva 2005; Bryant et al. 2003; Takahashi et al. 2008), natural language query sentence processing (Tseng & Chen 2008), and representation of knowledge (Ilieva & Boley 2008) which is extracted from text by an automatic tool. However, little work has been done for modeling imperative mood sentences, the sentence structure of math educational standards statements. Illieva (2007) and Illieva and Boley (2008) divide English sentences into three basic groups, such as the subject, the predicate, and the object in a tabular presentation of sentences and built graphical natural language for UML diagram generation. If the sentences lack a subject, the position of the subject is kept empty in a table and it will be filled by the analyst in an interactive mode. Math educational standards statements, however, have only one subject, “student,” and the subject is omitted because all the statements are imperative mood sentences. Tseng and Chen (2008) briefly mention how to model an imperative mood sentence of English sentences in UML for transforming natural language queries into relational algebra through the UML class diagram notation. Their approach for modeling an imperative
mood sentence of English sentences are as follows: 1) Find out hidden associations between classes, or 2) If the verb does not transfer an action, there is no association at all and the English sentence is modeled as a class hierarchy only without including a verb as an association or a class. In math standards, it’s not easy to find out hidden associations on a sentence level and MP verbs are reified as classes in the MP model. Bryant et al. (2003) describe the method of translating requirements in natural language into UML models and/or executable models of software components. Their method depends on whole requirements in natural language rather than a sentence level. The requirements are refined and processed for creating a knowledge base using natural language processing techniques. And then the knowledge base is converted into TLG (Two-Level Grammar) (Bryant et al. 2002) which is used as an intermediate representation between the informal knowledge base and the formal specification language representation. TLG can be converted into UML at a final step.

We identify that most research in this area has not been focused on an imperative mood sentence. Our MP model is focused on imperative mood sentences for modeling math educational standards.

3.2 Alignment systems for educational standards

Manual alignment methods for educational standards have been implemented (McRel://www.mcrel.org/, Plato Learning, Inc: //www.plato.com/, Align to Achieve://www.aligntoachieve.org/).

In this section, we focus on automatic alignment systems for educational standards. The Standard Alignment tool (SAT) (Yilmazel et al., 2007) and AlignPro (http://www.instron.us/wa/acc_catalog/prod_list.aspx?cid=423&cname=Alignment%20F
ixtures), a product of Instron, are examples of automatic alignment systems for educational standards using natural language processing. AlignPro aligns the standards based on descriptions of content and instructional objectives. These descriptions are used for ranking of documents by concept. The Standard Alignment tool (SAT) uses text categorization for standards alignment. For text categorization, the SAT uses A2A + McREL Compendix which has manual alignments made by experts for educational standards for training a multi-label classifier. The SAT uses three types of text content: benchmark text (McREL), the text of all the levels from the path to the root, and relevant vocabulary assigned by McREL. The SAT also uses the Machine Learning Toolkit for supporting text categorization. The SAT takes a resource and produces all the equivalent educational standards statements with a resource. Alignment method is used in the middle of this process related a resource to educational standards. Label classifier is only trained against A2A + McREL benchmarks for text categorization. McREL vocabulary terms heavily influence for text categorization ability in the system. The SAT may not work correctly against new standards which do not have McREL vocabulary terms. The Achievement Standards Network (ASN) is currently building an alignment system which uses intermediary statements in order to align different state educational standards statements (Sutton & Golder 2008).

3.3 Schema matching

Schema matching has been one major area of database research (Doan et al. 2003A; Batini and Lenzerni 1986; Doan and Halvey 2005; Madhavan et al. 2005; Berstein and Melink 2007; Melink et al. 2007; Nash et al. 2007; Berstein et al. 2006A; Bernstein et al. 2006B; Duchateau et al. 2008; Chai et al. 2008; Giunchiglia et al. 2008). The goal of
Schema matching is to produce a mapping between schema elements of the two input schemas in any data model, where two schemas correspond semantically to each other. Mapping can be directional, bi-directional, or in the form of a query, a set of expressions between elements in each schema. Schema matching can be used for data integration, schema integration, data warehouses, E-commerce, Semantic query processing, database design. Rahm and Bernstein (2001) present a broad survey of approaches to automatic schema matching and also classify schema matching based on matcher criteria which are individual matchers or combining matchers. Individual matchers perform a mapping based on a single matching criterion. Combining matchers use multiple matching criteria by a hybrid matcher or combines multiple match results by a composer matcher. See Fig 27 for their classification of schema matching approaches. We also examine some of schema matching approaches as follows

- **LSD (Learning Source Descriptions) system** (Doan et al. 2003)
  1. It employs a multistrategy learning approach for semi-automatic creation of semantic mappings.
  2. Multistrategy learning approach applies multiple learner modules, where each module exploits a different type of information either in the schemas of the sources or in their data, then combines the predictions of the modules using a meta-learner.
  3. After a small set of data sources have been manually mapped to the mediated schema, a schema-matching system should be able to automatically learn from these mappings to successfully propose mappings for subsequent new data sources.
4. To predict mappings for a new data source, it proposes applying a multitude of learning modules, called base learners, where each base learner exploits well a certain type of information, then combining the base learners’ predictions using a meta-learner. The meta-learner uses the manually mapped sources to learn a set of weights for each base learner. The weights indicate the relative importance of the base learners and can be different for each mediated-schema element, reflecting that different learners may be most appropriate in different cases.

5. Multistrategy learning operates in two phases: training and matching (two phase of LSD). In the training phase they manually mark up the schemas of a few data sources, then use them to train the learners. In the matching phase they apply the trained learner to predict mapping for new sources.

6. LSD exploits domain integrity constraints, user feedback, and nested structures in XML data for improving matching accuracy.

7. The system consists of base learners, meta-learner, prediction combiner, and constraints handler. It operates in two phases: training and matching. In base learners, there are the Name Learner, the Content Learner, the Naïve bayes Learner, the XML Learner. The Name Learner matches an XML element using its tag name by using Whirl, the nearest-neighbor classification model. It stores all training examples of the form (expanded tag-name, Label) that it has been so far. Then given an XML element t, it computes the label for t based on the labels of all examples within a distance delta from t. The similarity distance between any two examples is the TF/IDF distance. The Content Learner uses data
value instead of tag name (using Whirl) (data-value, label). TF/IDF is distance between their data values. It works well on long textual elements. The XML Learner can handle the hierarchical structure of XML data very well. The Naïve Bayes Learner confused instances of classes, contact-info, office-info, agent-info. The meta-learner using stacking combines the predictions of the base learner. The goal of training the meta-learner is to compute for each pair of base learner and label a learner weight which reflects the importance of the learner with respect to predictions regarding that label.

- **SKAT (Semantic Knowledge articulation Tool)** (Mitra et al. 1999)

  SKAT uses a rule-based approach for semi-automatic matching between ontologies (or schema). Rules uses first-order logic to express match and mismatch relationship and methods are defined to derive new match. It supports name matching and simple structural matches based on isa hierarchies. The user’s initial interventions is required for providing application-specific match and mismatch relationships and then accept or reject generated matches. SKAT is used in ONION (Mitra et al. 2000) which is an ontology integration system. ONION is an ontology integration system for heterogeneous ontologies using ontology graphs and articulation ontologies with the purpose of query processing. The main benefits of this system are that its architecture enables support of a scalable framework for ontology integration because it has been designed with strong modularity and it provides interoperability among source (local) ontologies using articulation ontologies. ONION has two kinds of ontologies: source and articulation ontologies. Articulation ontologies contain concepts and relationships. An articulation ontology is created based on articulation rules which specifies mapping between concepts
in source ontologies. Mapping between ontologies is executed by ontology algebra (Mitra and Wiederhold 2001). Ontology algebra consists of three operations: intersection, union, and difference. The intersection determines the part of ontologies which have similar concepts. The union generates consistent unified ontology. The difference is defined as difference between two ontologies and enables local ontologies to be kept different. These articulation ontologies can be organized in a hierarchy. Articulation ontologies can be built based on two other articulation ontologies which integrate different source ontologies (Bruijn et al. 2003). In ONION, queries on the resulting articulation ontology are translated to queries on source ontologies and executed on the underlying ontologies. The returned results are translated to the representation of articulation ontology. Articulation ontologies are organized in a hierarchy as a tree structure. The most general articulation ontology can be considered as a global ontology which provides a virtual view over the underlying source ontologies. Mapping rules between the articulation ontology and local ontologies are specified in an articulation ontology.

- **Similarity flooding** (Melink et al. 2002)

The Similarity Flooding algorithm is a graph matching algorithm based on an iterative fix-point computation which can be applied for various data models and explores its usability for schema matching. The approach converts schema into directed labeled graphs. The algorithm takes two graphs and returns a mapping between corresponding nodes of graphs. Various models to be matched are converted into similarity propagation graphs for an iterative fixed computation to produce mapping between the nodes of input graphs. Mapping represents similarity between corresponding nodes of input graphs. Initial similarity is computed from string comparison of node labels. For computing
similarities, this algorithm uses an idea which is the similarity spreading from similar nodes to the adjacent neighbors. Through iteration similarity are increasing until the fixpoint is reached and the outcome of this step is a refined mapping. Mapping pairs from this algorithm will be filtered for choosing the best match candidates.

- **ARTEMIS** (Castano et al. 2001)

ARTEMIS is a schema integration tool. It matches classes based on name, data type, and structural affinities. Name affinity is based on domain-specific thesauri with synonym, hypernym, or general relationship. Data type affinity is based on a generic table of data type compatibility. Structural affinity of two entities is based on the similarity of relationships. In ARTEMIS, schema matching is done by computing global affinity coefficients among schema elements based on name, data type, and structural affinities. Global affinity is a comprehensive measure of the level of matching of two schema elements. Affinity coefficients are then exploited by a hierarchical clustering algorithm to classify ODLI3 classes based on their level of matching. The output of clustering is an affinity tree. Candidate clusters are interactively selected from the affinity tree using a threshold-based mechanism. The contribution of the ARTEMIS integration proposes affinity-based metrics and clustering procedures for schema matching and integration.

ARTEMIS is used in MOMIS (Mediator environment for Multiple Information Sources). MOMIS creates global virtual view (GVV) of information sources, independent of their location or their data’s heterogeneity. MOMIS (Berneventano et al. 2003) creates a virtual global schema through five phases as follows:

1) Local source schema extraction by wrappers

2) Local source annotation with the WordNet
3) Common thesaurus relationships of inter-schema and intra-schema knowledge about classes and attributes of the source schemas.

4) GVV generation: A global schema and mappings between the global attributes of the global schema and source schema by using the common thesaurus and the local schemas are generated.

5) GVV annotation is generated by exploiting annotated local schemas and mappings between local schemas and a global schema.

MOMIS generates mappings between global attributes of the global schema and source schemas. For each global class in the global virtual view (GVV), a mapping table (MT) stores all generated mappings.
CHAPTER 4: APPROACH

We propose a semi-automatic alignment method for math educational standards. We present terminologies used in the MP model in Section 4.1. In Section 4.2 we present an overview of our alignment method, an analysis of the conversion of different types of MP statements to the MP model, the components of the MP model, heuristics and examples of the MP modeling, and a semi-automatic tool, called “MPViz”, for creating the MP model. In Section 4.3, we provide validation of the MP model, and reasons for using UML for the MP model. In Section 4.4, we detail our alignment method including the overall sketch of an algorithm for aligning math educational standards statements and a graph matching algorithm for our alignment method. We evaluate our alignment method using Cohen’s kappa, precision, recall, and F-measure in Section 4.5.

4.1 Terminology

- **Reed-Kellogg system (Reed and Kellogg 2004):** It is a graphic representation of a sentence structure. It also represents relationships between the elements of sentences and their modifiers. The horizontal main line is for elements such as the subject, the verb, the direct object, and the complement. Modifiers of the subject, the verb, or the object are placed under elements they modify. A simple English sentence in the Reed-Kellogg System is shown as Figure 4.1.1.

```
<table>
<thead>
<tr>
<th>Subject</th>
<th>verb</th>
<th>direct object</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>modifier</td>
<td></td>
<td>modifier</td>
</tr>
</tbody>
</table>
```

*Figure 4.1.1: A simple sentence in the Reed-Kellogg System*
• **Educational standards alignment**: It matches educational standards that describe similar or equivalent concepts.

• **Imperative mood sentence**: It only has a predicate which consists of verbs with verb modifiers, and nouns with noun modifiers. For example, “Write fractions with numerals and number words” is an imperative mood sentence.

• **Math educational standards statement**: Math educational standards express the mathematical understanding, knowledge, and skills that students should obtain from pre-kindergarten through grade 12. In the United States, every state has its own math educational standards. All math educational standards statements have only one subject, the “student”, and the subject is omitted because almost all of the statements are imperative mood sentences. “Write fractions with numerals and number words (Ohio State)” is a math educational standards statement.

• **MP statement**: We classify math educational standard statements based on the sentence structures into 20 different types for a comprehensive coverage of the possible cases. We refer to these 20 different types as *MP statements*. For example, “Write fractions with numerals and number words (Ohio State)” belongs to a Type 3 MP statement. See Figure 4.1.2 for the sentence structure of the math standard statement “Write fractions with numerals and number words (Ohio State)” in the Reed-Kellogg System. “With numerals and words” considers as an MP noun modifier which modifies “fractions”. It is placed under “fractions” using a slanted line and a solid line. A dotted line indicates “and” or “or”. A vertical line can be used between a verb and a direct object, and between a subject and a verb.
Write fractions with numerals and number words

*Figure 4.1.2: A Type 3 MP Statement in the Reed-Kellogg System*

- **Syntactic ambiguity**: It is a property of sentences which may be interpreted in more than one way. For example, “he hit a boy with a stick” can be interpreted: 1) He hit a boy using a stick, 2) He hit a boy who has a stick. “With” causes syntactic ambiguity. A “with” phrase right after a math concept (e.g., with numerals and number words in Figure 4.1.2) is considered as an MP noun modifier in an MP statement for the purpose of math standard alignment.

- **MP verb**: It is a cognitive verb in Figure 4.1.2 at the beginning of an MP statement. For example, “write” is an MP verb.

- **MP noun**: It is a noun in Figure 4.1.2 in an MP statement. For example, “fractions” is an MP noun. An MP noun is a math concept, except Type 2, Type 2A, Type 2B, and Type 2C MP statements.

- **MP verb modifier**: A verb modifier which modifies an MP verb.

- **MP noun modifier**: A noun modifier which modifies an MP noun. For example, “with numerals and words” is an MP noun modifier. If more than one noun in a noun modifier they are connected using connectives “and” or “or”.

- **MP nouns with MP noun modifiers are math concepts and their properties.** They imply what math concepts students are learning.

- **Cognitive process**: The cognitive process has been referred to as the verbs in the educational standards (Williamson and Williams 2010).
• **Cognitive process of a math concept**: It describes how students are learning a math concept.

• **MP diagram**: It is a diagram of an MP statement which is modeled using UML notation.

• **Materialization Pattern (MP)**: It represents an MP class and its verb materialization hierarchy that realizes the behaviors of the MP class. An MP class represents a concept represented by a noun. A materialization hierarchy is a verb hierarchy that models the behaviors of the MP class. The relationship between the MP class and the materialization hierarchy is represented as a realization relationship of UML. See Figure 4.1.3 as an MP diagram for the sentence “Recognize, compare, and classify whole numbers.” A connective “and” in a sentence is simply used for enumeration of classes “Recognize”, “Compare”, and “Classify”. From that sentence we extract an MP class “Whole number” as a math concept, and three verb stereotype classes “Recognize”, “Compare”, and “Classify” as the cognitive process of the MP class “Whole number”. These three verb stereotype classes are subclasses of a class “Realize” which is an abstract class with no instance. A verb materialization hierarchy has verb stereotype classes “Realize”, “Recognize”, “Compare”, and “Classify”. A realization relationship exists between the classes “Whole number” and these verb stereotype classes.
4.2 Developing the MP (Materialization Pattern) for math educational standards

In this section, we present an overview of our alignment method. We also discuss an analysis of the conversion of different types of MP statements to the MP model, the components and examples of the MP model, conversion of 20 types of MP statements to MP diagrams in detail using UML, a semi-automatic tool “MPViz” for creating the MP model, validation of the MP model, and reasons for using UML for the MP model.

4.2.1 An overview of our alignment method

We develop a semi-automatic alignment method as follows:

(1) We propose the MP model that captures the semantics of math educational standards statements based on Reed-Kellogg system.

A) First, we pre-process the standards for modeling and model different types of standards statement as MP diagrams with the UML notation using the semi-automatic tool “MPViz”.

B) Second, we classify statements based on sentence structures into 20 different types for a comprehensive coverage of the possible cases. This is an on-going and iterative process, and new types can be added in the future if necessary.

(2) We align different math educational standards statements.

Figure 4.1.3: An MP diagram for a math standard statement: “Recognize, compare, and classify whole numbers.” A class “Whole number” is an MP class. “Realize” is an abstract verb stereotype class. “Recognize”, “Compare”, and “Classify” are verb stereotype classes.
A) We convert MP diagrams to two graphs using the MPViz: a verb-phrase graph and a noun-phrase graph.

B) We use the semi-automatic alignment tool “MPComp” for aligning math educational standards. Our alignment method uses a graph matching with the Bloom taxonomy for cognitive verb categorization, the WordNet (Fellbaum 1999) for word similarities, and taxonomies of math concepts for related math concepts.

See figure 4.2.1 for the overview of our alignment method.
4.2.2 An analysis of the conversion of different types of MP statements to the MP model

In this section, we pre-process math educational standards statements and analyze how different types of MP statements corresponded to the MP model in terms of classes and relationships between classes. Math educational standards have been classified into 20 different types for a comprehensive coverage of the possible cases for the purpose of representing and modeling these standards. We refer these 20 different types as MP statements. Each type has been classified based on the sentence structure. In these sentences, concepts are represented by nouns or verbs. These concepts are connected by prepositions or verbs. These concepts and connections determine classes and relationships, respectively, in the MP model. Each type of MP statements has different relationships between concepts (i.e. classes) when it is converted to the MP model. These different relationships are association, aggregation, dependency, prepositional, transitive verb, and realization relationships. We also identify a math concept as an MP class or a noun class, and the cognitive process (Bloom & Krathwohl 1956) of a math concept as a verb stereotype class. In the MP model, stereotypes in UML have been used for defining an MP class, a verb stereotype class, a prepositional relationship, and a transitive verb relationship. A distinct feature of the MP model is to extend the granularity of modeling with a verb stereotype class, in which a verb is reified as a class, and thus simplifies modeling of sentences by a Materialization Pattern in a domain class diagram. The MP model is created by a semi-automatic tool “MPViz” using UML from 20 different types of MP statements. We summarize 20 different types of MP statements in a table 4.2.1 and the conversion of these MP statements to the MP model in table 4.2.2 and table 4.2.3.
• Pre-processing math standard statements: It is not needed for most of statements.

➢ When a math educational standard statement starts with “use”, it will become “using” if nouns followed by “use” are not important math concepts in a sentence. For example, “Use concrete objects to illustrate the concepts of addition and subtraction” will be “Illustrate the concepts of addition and subtraction using concrete objects.” If a sentence already has a phrase which starts with “using” it will be replaced by “with”.

➢ When a math concept is too general, it will be omitted. For example, “Find the range of a set of data using whole numbers” will be “Find the range using whole numbers.”

➢ If a math concept which is followed by an infinitive phrase is important, a sentence will start with a verb in an infinitive phrase. For example, “Identify and select appropriate units to measure angles” will be “Measure angles by identifying and selecting appropriate units.”

➢ A math concept will be unified. For example, “Draw solids from different perspectives” will be “Draw solid objects from different perspectives.”

➢ When a math educational standard statement has different MP verbs with different MP nouns, it will become one sentence or two sentences: 1) “Solve systems of two linear equation algebraically and verify solution” will become “Solve and verify solution of systems of two linear equation,” 2) “Interpret data and make prediction using frequency and line plots” will be two sentences: a) “Interpret data,” b) “Make prediction using frequency and line plot.”
Table 4.2.1: An analysis of various types of MP statements

<table>
<thead>
<tr>
<th>Type</th>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
<th>MP verb modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>Recognize, Classify</td>
<td>whole number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognize</td>
<td>repeating pattern</td>
<td></td>
<td>using symbols, objects</td>
<td></td>
</tr>
<tr>
<td>Type 2</td>
<td>Demonstrate</td>
<td>(the) value of irrational number.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 2A</td>
<td>Describe</td>
<td>(a)motion or series of transformation that show two shapes are congruent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 2B</td>
<td>Estimate</td>
<td>(the) result of computations involving whole numbers, fractions, and decimals. using models and words.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 2C</td>
<td>Demonstrate</td>
<td>Fluency in operations with real numbers, vectors, and matrices using mental computation or paper and pencil calculation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 3</td>
<td>Write</td>
<td>Fraction with numeral and number word.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 4</td>
<td>Estimate, use</td>
<td>measuring device with standard and non-standard unit to measure quantity.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 4A</td>
<td>Identify</td>
<td>Functions with graphs that have rotation symmetry or reflection symmetry about the y-axis or x-axis.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 4B</td>
<td>Use</td>
<td>radian measures in the solution problems involving angular velocity and acceleration.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 5</td>
<td>Develop</td>
<td>formula, procedure for determining measurements.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 5A</td>
<td>Create</td>
<td>Plan for collection data for a specific purpose.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 6</td>
<td>Create</td>
<td>two-dimensional design that contains a line of symmetry.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 7</td>
<td>Demonstrate</td>
<td>Skill for using fraction to verify conjectures, confirm computations and explore complex problem-solving situation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 8</td>
<td>Apply</td>
<td>number theory to rename a number quantity.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 8A</td>
<td>Make</td>
<td>geometric shape with concrete models by combining geometric shapes.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 8B</td>
<td>Round</td>
<td>whole number to a given place value.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 8C</td>
<td>Translate</td>
<td>Contextual situation involving area, surface are, volume, and density to mathematical symbols.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 9</td>
<td>Create, solve</td>
<td>word problem involving addition, subtraction, multiplication, and division of a whole number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 10</td>
<td>Interpret data and make prediction using frequency and line plots.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.2.2: An analysis of the conversion of MP statements to the MP model

<table>
<thead>
<tr>
<th>Type</th>
<th>MP verb</th>
<th>MP noun</th>
<th>relationship between an MP noun and nouns in an MP modifier</th>
<th>nouns in an MP modifier</th>
<th>relationship between nouns in an MP modifier</th>
<th>relationship between other noun and other nouns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>Verb stereo type class</td>
<td>MP class (math concept)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 2</td>
<td>Verb stereo type class</td>
<td>Attribute</td>
<td></td>
<td>MP class (math concept)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 2A</td>
<td>Verb stereo type class</td>
<td>Attribute</td>
<td></td>
<td>the 1st noun: MP class (math concept)</td>
<td>association</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>other noun: noun class</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 2B</td>
<td>Verb stereo type class</td>
<td>Attribute</td>
<td></td>
<td>the 1st noun: MP class (math concept)</td>
<td>aggregation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>other noun: noun class</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 2C</td>
<td>Verb stereo type class</td>
<td>Attribute</td>
<td></td>
<td>the 1st noun: MP class (math concept)</td>
<td>prepositional</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>other noun: noun class</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 3</td>
<td>Verb stereo type class</td>
<td>MP class (math concepts)</td>
<td>prepositional</td>
<td>noun class</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 4</td>
<td>Verb stereo type class</td>
<td>MP class (math concepts)</td>
<td>prepositional</td>
<td>noun class (math concepts)</td>
<td>association</td>
<td></td>
</tr>
<tr>
<td>Type 4A</td>
<td>Verb stereo type class</td>
<td>MP class (math concepts)</td>
<td>prepositional</td>
<td>noun class (math concepts)</td>
<td>association</td>
<td>prepositional</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 4B</td>
<td>Verb stereo type class</td>
<td>MP class (math concepts)</td>
<td>prepositional</td>
<td>noun class (math concepts)</td>
<td>aggregation</td>
<td></td>
</tr>
<tr>
<td>Type 5</td>
<td>Verb stereo type class</td>
<td>MP class (math concepts)</td>
<td>association</td>
<td>noun class (math concepts)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 5A</td>
<td>Verb stereo type class</td>
<td>MP class (math concepts)</td>
<td>association</td>
<td>noun class (math concepts)</td>
<td>prepositional</td>
<td></td>
</tr>
<tr>
<td>Type 6</td>
<td>Verb stereo type class</td>
<td>MP class (math concepts)</td>
<td>aggregation</td>
<td>noun class (math concepts)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 7</td>
<td>Verb stereo type class</td>
<td>MP class (math concepts)</td>
<td>association</td>
<td>noun class (math concepts)</td>
<td>association</td>
<td></td>
</tr>
<tr>
<td>Type 8</td>
<td>Verb stereo type class</td>
<td>MP class (math concepts)</td>
<td>association</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 9</td>
<td>Verb stereo type class</td>
<td>MP class(math concepts)</td>
<td>aggregation</td>
<td>MP class (math concepts)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.2.3: An analysis of the conversion of Type 8 MP statements to the MP model

<table>
<thead>
<tr>
<th>Type</th>
<th>MP verb</th>
<th>MP noun</th>
<th>Relationship between an MP noun and nouns in an MP modifier</th>
<th>MP verb modifier</th>
<th>Relationship between MP verb, and nouns or MP verbs in an MP verb modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 8</td>
<td>Verb stereo type class</td>
<td>MP class (math concept)</td>
<td>Verb stereo type class</td>
<td>MP class (math concept)</td>
<td>transitive verb relationship</td>
</tr>
<tr>
<td>Type 8A</td>
<td>Verb stereo type class</td>
<td>MP class (math concept)</td>
<td>Aggregation, association, or prepositional</td>
<td>Verb stereo type class</td>
<td>noun class (math concept)</td>
</tr>
<tr>
<td>Type 8B</td>
<td>Verb stereo type class</td>
<td>MP class (math concept)</td>
<td></td>
<td></td>
<td>noun class (math concept)</td>
</tr>
<tr>
<td>Type 8C</td>
<td>Verb stereo type class</td>
<td>MP class (math concept)</td>
<td>aggregation or association</td>
<td></td>
<td>noun class (math concept)</td>
</tr>
</tbody>
</table>

4.2.3 The component of the MP model

The basic building blocks of the MP model are classes and relationships between classes. These classes consist of an MP class, a noun class, and a verb stereo type class. Predicative, prepositional, transitive verb, realization, generalization, and dependency relationships are used in the MP model. See Table 4.2.4 for a summary of classes and relationships in the MP model.
### Table 4.2.4: A summary of classes and relationships in the MP model

<table>
<thead>
<tr>
<th>class</th>
<th>semantics</th>
<th>notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP class</td>
<td>Abstraction class</td>
<td>&lt;&lt;MP&gt;&gt;</td>
</tr>
<tr>
<td>Verb stereotype class</td>
<td>A verb is reified as a class.</td>
<td>&lt;&lt;Verb&gt;&gt;</td>
</tr>
<tr>
<td>Noun class</td>
<td>Regular UML class</td>
<td>Regular UML notation</td>
</tr>
<tr>
<td>relationship</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Predicative relationship</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Association</td>
<td>It connects two concepts using a verb (Ilieva and Boley 2008).</td>
<td></td>
</tr>
<tr>
<td>Aggregation</td>
<td>It is more specific than association and represents a whole-part relationship.</td>
<td></td>
</tr>
<tr>
<td>Dependency relationship</td>
<td>One class depends on another because it uses it at some points.</td>
<td></td>
</tr>
<tr>
<td>Prepositional relationship</td>
<td>It connects two concepts using a preposition.</td>
<td>&lt;&lt;preposition&gt;&gt;</td>
</tr>
<tr>
<td>Transitive verb relationship</td>
<td>It relates a noun to another noun or a verb.</td>
<td>&lt;&lt;preposition&gt;&gt;</td>
</tr>
<tr>
<td>Realization relationship</td>
<td>The verb stereotype class realizes the behavior that the MP class specifies.</td>
<td></td>
</tr>
<tr>
<td>Generalization relationship</td>
<td>“isa” relationship</td>
<td></td>
</tr>
</tbody>
</table>

### 4.2.4 Heuristics of the MP model

By analyzing math educational standards from 3 different states, we classify statements based on sentence structures into 20 different types of MP statements. Concepts in each type of MP statements are connected differently. Concepts and different connection between concepts in MP statements are modeled as classes and relationships.
in the MP model, respectively. We now present heuristics for modeling each different type of MP statements as an MP diagram.

2. Heuristic to determine classes

   1) All MP verbs are converted to verb stereotype classes, which represent the cognitive process of math concepts.

   2) All MP nouns except the Type 2, Type 2A, Type 2B, and Type 2C are converted to MP classes. These MP classes are math concepts. A superclass is created as an MP class if a superclass exists when more than one MP class exist.

   3) All nouns in an MP noun modifier or an MP verb modifier are converted to noun classes. A superclass is created if a superclass exists when more than one noun class in an MP modifier or an MP verb modifier exist.

3. Heuristic to determine relationships

   We identify relationships as follows:

   - between an MP class and noun classes in an MP noun modifier,
   - between noun classes in an MP modifier or an MP verb modifier, or
   - between an MP verb class and noun classes in an MP verb modifier.

   1) There is always a realization relationship between an MP class and a verb stereotype type class.

   2) There is a predicative relationship (association or aggregation) when two concepts are connected using a verb.

   3) There is a prepositional relationship when two concepts are connected using a preposition.
4) There is a transitive verb relationship when a transitive verb relates a concept to another concept (Hartman and Link 2007) or a verb stereotype class.

5) There is a dependency relationship when an MP verb modifier or an MP noun modifier starts with “using.”

6) There is a generalization relationship if a superclass exists.

4.2.5 Development and examples of the MP model

In order to develop an MP diagram, we take four different steps for each type of MP statements as follows:

1. Step 1: Write an MP statement which is a math educational standard statement.

2. Step 2: Create the table format of the MP statement.

3. Step 3: Draw a diagram of the MP statement based on the Reed-Kellogg system.

4. Step 4: Develop an MP diagram using the UML notation from the Reed-Kellogg diagram of the MP statement.

1. Type 1 MP Statement

1) Step 1: Recognize, classify, order, and compare whole numbers.

2) Step 2: Create the table format of the Type 1 MP statement.

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognize, classify,</td>
<td>whole number</td>
</tr>
<tr>
<td>compare</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2.5: A Type 1 MP Statement

3) Step 3: Draw the Reed-Kellogg diagram of a Type 1 MP Statement

“Recognize, classify, and compare whole numbers.
4) Step 4: Verbs (e.g., recognize, classify, and compare) and nouns (e.g., whole number) in the horizontal line of the Reed-Kellogg System (except Types 2, 2A, 2B, 2C) are converted to classes in the MP model. A dotted line implies “and”. A connective “and” in a sentence is simply used for enumeration of classes in the MP model.

Figure 4.2.3: An MP diagram of a Type 1 MP statement: A class “Whole number” is an MP class which is a math concept. “Realize“ is an abstract verb stereotype class. Verb stereotype classes “Recognize”, “Classify”, and “Compare” are the cognitive process of the class “Whole number” that is a math concept.

2. Type 2 MP Statement

1) Step 1: A math standard statement: Estimate the value of irrational numbers. See Table 4.2.7 and Figure 4.2.4 for Step 2 and Step 3, respectively.

2) Step 2: Create the table format of a Type 2 MP statement.
Table 4.2.6: A Type 2 MP Statement

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>value</td>
<td>of irrational numbers</td>
</tr>
</tbody>
</table>

3) Step 3: Draw the Reed-Kellogg diagram of a Type 2 MP statement “Estimate the value of irrational numbers.”

![Reed-Kellogg diagram](image)

Figure 4.2.4: A Type 2 MP Statement in the Reed-Kellogg system

4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a complement in the form of a prepositional phrase such as “of irrational numbers.”
- Model a noun (e.g., irrational number) in an MP noun modifier as an MP class, which is a *math concept* and an MP noun(s) (e.g., value) as an *attribute* of the MP class. See Figure 4.2.5. A class “Irrational number” is an MP class which is a math concept. “Value” is an attribute of the class “Irrational number”. “Estimate” is a verb stereotype class which represents the cognitive process of the MP class “Irrational number“.
Figure 4.2.5 An MP diagram of a Type 2 MP Statement: A class “Irrational number” is an MP class which has an attribute “value”. The class “Irrational number” is a math concept. A verb stereotype class “Estimate” implies the cognitive process of the MP class “Irrational number”.

3. Type 2A MP Statement:

1) Step 1: A math standard statement: Describe a motion or series of transformations that show two shapes are congruent.

2) Step 2:

Table 4.2.7: A Type 2A MP Statement

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>motion, series of transformations that show congruency of two shapes</td>
<td></td>
</tr>
</tbody>
</table>

3) Step 3:

Figure 4.2.6: A Type 2B MP Statement in the Reed-Kellogg system
4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a complement in the form of a relative noun clause such as “of transformations that show two shapes are congruent.”

- Model a noun (for example, transformation) in an MP noun modifier as an MP class, which is a *math concept* and an MP noun(s) (for example, motion, and series) as an *attribute* of the MP class. See Figure 4.2.7. A class “Transformation” is an MP class which is a math concept. “Motion” and “series” are attributes of the class “Transformation”. “Describe” is a verb stereotype class which represents the cognitive process of the MP class “Transformation”.

- A noun clause “that show two shapes are congruent” is pre-processed as “the congruency of two shapes.” Model a noun “two shapes” as a noun class and “congruency” as an attribute of a class “Two shapes”. An association relationship exists between an MP class “Transformation” and a noun class “Two shapes”.

![Figure 4.2.7: An MP diagram of a Type 2A MP Statement: A class “Transformation” is an MP class which have attributes “motion” and “series”. The class “Transformation” is a math concept. A verb stereotype class “Describe” implies the cognitive process of the MP class “Transformation”.](image)
4. Type 2B MP Statement:

1. Step 1: A math standard statement: Estimate the result of computations involving whole numbers, fractions, and decimals.

2. Step 2:

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>result</td>
<td>of computations involving whole numbers, fractions, and decimals</td>
</tr>
</tbody>
</table>

3. Step 3:

Figure 4.2.8: A Type 2B MP Statement in the Reed-Kellogg system

4. Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a prepositional phrase such as “of computations involving whole numbers, fractions, and decimals.”

- Model a noun (e.g., computation) in an MP noun modifier as an MP class, which is a math concept and an MP noun(s) (e.g., result) as an attribute of the MP class. See Figure 8. A class “Computation” is an MP class which is a math
concept. “Result” is an attribute of the class “Computation”. “Estimate” is a verb stereotype class which represents the cognitive process of the MP class “Computation”.

- Model nouns “Whole number”, “Fraction”, and “Decimal” as noun classes and An aggregation relationship exists between an MP class “Computation” and noun classes “Whole number”, “Fraction”, and “Decimal”.

![Diagram](image)

*Figure 4.2.9 An MP diagram of a Type 2B MP Statement: A class “Computation” is an MP class which has an attribute “result”. The class “Computation” is a math concept. A verb stereotype class “Estimate” implies the cognitive process of the MP class “Computation”.*

5. Type 2C MP Statement:

1) Step 1: A math standard statement: Demonstrate fluency in operations with real numbers, vectors, and matrices.

2) Step 2:

*Table 4.2.9: A Type 2C MP Statement*

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate</td>
<td>fluency</td>
<td>in operations with real numbers, vectors, and matrices</td>
</tr>
</tbody>
</table>
3) Step 3:

Demonstrate fluency in operation with real numbers, vectors, and matrices.

Figure 4.2.10: A Type 2C MP Statement in the Reed-Kellogg system

4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a prepositional phrase such as “in operations with real numbers, vectors, and matrices.”

- Model a noun (e.g., operation) in an MP noun modifier as an MP class, which is a math concept and an MP noun(s) (e.g., fluency) as an attribute of the MP class. See Figure 4.2.11. A class “Operation” is an MP class which is a math concept. “Fluency” is an attribute of the class “Operation”. “Demonstrate” is a verb stereotype class which represents the cognitive process of the MP class “Operation”.

- Model nouns “Real number”, “Vector”, and “Matrix” as noun classes and a prepositional relationship exists between an MP class “Operation” and noun classes “Real number”, “Vector”, and “Matrix”.
Figure 4.2.11 An MP diagram of a Type 2C MP Statement: A class “Operation” is an MP class which has an attribute “fluency”. The class “Operation” is a math concept. A verb stereotype class “Demonstrate” implies the cognitive process of the MP class “Operation”.

6. Type 3 MP Statement:

1) Step 1: Write fractions with numerals and number words.

2) Step 2:

Table 4.2.10: A Type 3 MP Statement

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write</td>
<td>fractions</td>
<td>with numerals and number words.</td>
</tr>
</tbody>
</table>

3) Step 3:

Write fractions with numerals and number words.

Figure 4.2.12: A Type 3 MP Statement in the Reed-Kellogg system
4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a prepositional phrase such as “with numerals and number words.”
- There is a prepositional relationship (e.g., <<with>>) between an MP noun class (e.g., Fraction) and noun classes (e.g., Numeral and Number word) in an MP noun modifier.

![An MP diagram using UML notation](image)

*Figure 4.2.13 An MP diagram of a Type 3 MP Statement: A class “Fraction” is an MP class. The class “Fraction” is a math concept. A verb stereotype class “Write” implies the cognitive process of the MP class “Fraction”.*

7. Type 4 MP Statement:

1) Step 1: Estimate and use measuring devices with standard units and non-standard units to measure length, weight, and volume.

2) Step 2:

*Table 4.2.11: A Type 4 MP Statement*

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate, Use</td>
<td>measuring device</td>
<td>with standard units and non-standard units to measure length, weight, and volume.</td>
</tr>
</tbody>
</table>
3) Step 3:

Estimate and device use measuring with standard unit and non-standard unit measure length and height volume

Figure 4.2.14: A Type 4 MP Statement in the Reed-Kellogg system

4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a prepositional phrase such as “with standard unit and non-standard unit to measure length, height, volume.”
- There is a prepositional relationship (e.g., <<with>>) an MP noun class (for example, Measuring device) and noun classes (e.g., Standard unit and Non-standard unit) in an MP noun modifier.
- A superclass “Quantity” of classes “Length”, “Weight”, and “Volume” has been created.
- There is an association relationship (e.g., measure) between noun classes (e.g., “Standard unit” and “Quantity”, and “Non-standard unit”, and “Quantity”).

Figure 4.2.15 An MP diagram of a Type 4 MP Statement: A class “Measuring device” is an MP class. The class “Measuring device” is a math concept. A verb stereotype class “Estimate” implies the cognitive process of the MP class “Measuring device”.
8. Type 4A MP Statement:

1) Step 1: Identify functions with a graph that has rotation symmetry and reflection symmetry about x-axis and y-axis.

2) Step 2:

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify</td>
<td>functions</td>
<td>with a graph that has a rotation symmetry and a reflection symmetry about x-axis and y-axis.</td>
</tr>
</tbody>
</table>

3) Step 3:

4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a prepositional phrase such as “with a graph that has a rotation symmetry and reflection symmetry about x-axis and y-axis.”
- There is a prepositional relationship (e.g., <<with>>) between an MP noun class (e.g., Function) and noun classes (e.g., Graph) in an MP noun modifier.
• There is an association relationship (e.g., has) between noun classes (e.g., rotation symmetry and reflection symmetry).
• There is a prepositional relation (e.g., about) between noun classes (e.g., Rotation symmetry, Reflection symmetry and X-axis, Y-axis).

*Figure 4.2.17: An MP diagram of Type 4A MP Statement*

9. Type 4B MP Statement:
   1) Step 1: Use radian measures in solution problems involving angular velocity and accelerations.
   2) Step 2:

*Table 4.2.13: A Type 4B MP Statement*

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use</td>
<td>radian measure</td>
<td>in solution problems involving angular velocity and accelerations.</td>
</tr>
</tbody>
</table>
3) Step 3:

```plaintext
Use radian measures in solution problem involving angular velocity involving ; and accelerations
```

*Figure 4.2.18: A Type 4B MP Statement in the Reed-Kellogg system*

4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a prepositional phrase such as “in solution problems involving angular velocity and accelerations.”
- There is a prepositional relationship (e.g., <<in>>) between an MP class (e.g., “Radian measure”) and a noun class (e.g., Solution problem).
- There is an aggregation relationship between noun classes (e.g., Solution problem and Angular Velocity, and Solution problem and Acceleration).

*Figure 4.2.19: An MP diagram of Type 4B MP Statement*
10. Type 5 MP Statement

1) Step 1: Develop formulas and procedures that determine measurement.

2) Step 2:

Table 4.2.14: A Type 5 MP Statement

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Develop</td>
<td>formulas and</td>
<td>that determine measurement.</td>
</tr>
<tr>
<td></td>
<td>procedures</td>
<td></td>
</tr>
</tbody>
</table>

3) Step 3:

Figure 4.2.20: A Type 5 MP Statement in the Reed-Kellogg system

4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a relative noun clause such as “that determine measurement.”
- There is an association relationship between MP classes “Formula“, “Procedure”, and a noun class “Measurement”. 
Figure 4.2.21 An MP diagram of a Type 5 MP Statement: Classes “Formula” and “Procedure” are MP classes and math concepts. A verb stereotype class “Develop” implies the cognitive process of the MP class “Formula” and “Procedure”.

11. Type 5A MP Statement

1) Step 1: Create a plan that collects data for a purpose.

2) Step 2:

Table 4.2.15: A Type 5A MP Statement

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create</td>
<td>plan</td>
<td>that collects data for a purpose.</td>
</tr>
</tbody>
</table>

3) Step 3:

Figure 4.2.22: A Type 5A MP Statement in the Reed-Kellogg system
4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a relative noun clause such as “that collects data for a purpose.”
- There is an association relationship between MP classes “Plan” and a noun class “Data”.
- There is a prepositional relationship between noun classes “Data” and “Purpose”.

![MP Diagram]

Figure 4.2.23 An MP diagram of a Type 5A MP Statement: A class “Plan” is an MP class and a math concept. A verb stereotype class “Create” implies the cognitive process of the MP class “Plan”.

12. Type 6 MP Statement

1) Step 1: Create a two-dimensional design that contains line of symmetry.

2) Step 2:

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create</td>
<td>two-dimensional design</td>
<td>that contains line of symmetry.</td>
</tr>
</tbody>
</table>
3) Step 3:

```
Create | two-dimensional design
\a that \contains \line of symmetry
```

Figure 4.2.24: A Type 6 MP Statement in the Reed-Kellogg system

4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is a relative noun clause such as “that contains line of symmetry.”
- There is an aggregation relationship between an MP class “Two-dimensional design” and a noun class “Line of symmetry.”

Figure 4.2.25: An MP diagram of Type 6 MP Statement

13. Type 7 MP Statement:

1) Step 1: Demonstrate skills using fractions to confirm computation, to verify conjecture, and to explore complex problem-solving situation.
2) Step 2:

Table 4.2.17: A Type 7 MP Statement

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate</td>
<td>skills</td>
<td>using fraction to confirm computation, to verify conjecture, and to explore complex problem-solving situation.</td>
</tr>
</tbody>
</table>

3) Step 3:

Figure 4.2.26: A Type 7 MP Statement in the Reed-Kellogg system

4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is “using fractions to confirm computation, to verify conjecture, and to explore complex problem-solving situation.”
- There is an independency relationship between an MP class “skill” and a noun class “Fraction”. There is also an association relationship between class “Fraction” and classes “Computation”, and “Complex problem-solving situation.”
14. Type 8 MP Statement:

1) Step 1: Apply number theory to rename number quantity.

2) Step 2:

\[ \text{Apply number theory to rename number quantity.} \]

3) Step 3:

\[ \text{Apply } \text{number theory to rename number quantity.} \]
4) Step 4: An MP diagram using UML notation is as follows:

- An MP verb modifier is “to rename number quantity”, and has an MP verb (e.g., rename) and an MP noun (e.g., number quantity).
- There is a transitive verb relationship between MP verbs “Apply” and “Rename”.

![Figure 4.2.29: An MP diagram of Type 8 MP Statement](image)

15. Type 8A MP Statement:

1) Step 1: Make new geometric shapes with concrete models by combining geometric shapes.

2) Step 2:

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
<th>MP verb modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make</td>
<td>geometric</td>
<td>with concrete</td>
<td>by combining</td>
</tr>
<tr>
<td>shape</td>
<td>models</td>
<td>models</td>
<td>geometric shapes</td>
</tr>
</tbody>
</table>
3) Step 3:

![Diagram of MP Statement]

*Figure 4.2.30: A Type 8A MP Statement in the Reed-Kellogg system*

4) Step 4: An MP diagram using UML notation is as follows:

- An MP verb modifier is “to by combining geometric shapes,” has an MP verb (e.g., combine) and an MP noun (e.g., geometric shape).
- There is a transitive verb relationship between MP verbs “Make” and “Combine”.
- There is a prepositional relationship between an MP noun (e.g., geometric shape) and a noun (e.g., concrete model) in an MP noun modifier.

![Diagram of MP Statement]

*Figure 4.2.31: An MP diagram of Type 8A MP Statement*

16. Type 8B MP Statement:

1) Step 1: Relate addition and subtraction as an inverse operation.

2) Step 2:
Table 4.2.20: A Type 8B MP Statement

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP verb modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apply</td>
<td>addition, subtraction</td>
<td>as an inverse operation.</td>
</tr>
</tbody>
</table>

3) Step 3:

Figure 4.2.32: A Type 8B MP Statement in the Reed-Kellogg system

4) Step 4: An MP diagram using UML notation is as follows:

- An MP verb modifier is “as an inverse operation”.
- There is a transitive verb relationship between an MP verbs “Apply” and a noun class “Inverse operation.”

Figure 4.2.33: An MP diagram of Type 8B MP Statement

17. Type 8C MP Statement:

1) Step 1: Translate contextual situation involving area, surface area, volume, and density to Mathematical symbols.

2) Step 2:
3) Step 3:

![Diagram of Translate contextual situation to symbols involving area, surface area, volume, and density]

*Figure 4.2.34: A Type 8C MP Statement in the Reed-Kellogg system*

4) Step 4: An MP diagram using UML notation is as follows:

- There is an aggregation relationship an MP class “Contextual situation”, and noun classes “Area”, “Surface area”, “Volume“, and “Density”.
- There is a transitive verb relationship between an MP verb “Translate” and a noun class “Mathematical symbol” in an MP verb modifier.

![Diagram of MP classes and relationships]

*Figure 4.2.35: An MP diagram of Type 8C MP Statement*
18. Type 8D MP Statement:

1) Step 1: Apply combination as a method which creates coefficient of binomial theorem.

2) Step 2:

   Table 4.2.22: A Type 8D MP Statement

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP verb modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apply</td>
<td>combination</td>
<td>as a method which creates coefficient of binomials</td>
</tr>
</tbody>
</table>

3) Step 3:

   Figure 4.2.36: A Type 8D MP Statement in the Reed-Kellogg system

4) Step 4: An MP diagram using UML notation is as follows:

   - There is an association relationship between noun classes “Mathematical method”, and “Bionomial theorem”.
   - There is a transitive verb relationship between MP verb “Apply” and a noun class “Method” in an MP verb modifier.
Realize

Combination

Apply

Realize

Figure 4.2.37: An MP diagram of Type 8D MP Statement

19. Type 9 MP Statement:

1) Step 1: Create and solve word problems involving addition, subtraction, multiplication, and division of a whole number.

2) Step 2:

Table 4.2.23: A Type 9 MP Statement

<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
<th>MP noun modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create,</td>
<td>word problem</td>
<td>involving addition, subtraction, multiplication, and division of a whole number.</td>
</tr>
<tr>
<td>solve</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3) Step 3:

Figure 4.2.38: A Type 9 MP Statement in the Reed-Kellogg system
4) Step 4: An MP diagram using UML notation is as follows:

- An MP noun modifier is “involving addition, subtraction, multiplication, and division of a whole number.” It has an MP noun “Whole number”, and MP verbs “Add”, “Subtract”, “Multiply”, and “Divide”.
- There is an aggregation relationship between two MP classes “Word problem” and “Whole number”.

![MP diagram](image)

*Figure 4.2.39: An MP diagram of a Type 9MP Statement*

20. Type 10 MP Statement:

It can be any combination of Type 1 MP Statements through Type 9 MP Statements.

- Interpret data and make predictions using frequency and line plots.

This MP statement has two Type 1 MP statements: 1) Interpret data, 2) Make predictions using frequency and line plots.

### 4.2.6 A semi-automatic tool “MPViz” for creating the MP model

We develop the MPViz for creating the MP model. The MPViz visualizes the MP model. The MPViz cooperates with Graphviz Dotty ([http://www.graphviz.org/](http://www.graphviz.org/)). The MPViz can create 20 different types of MP diagrams. A Type 2B MP statement and a Type 9 MP statement are converted to MP diagrams as follows:
Figure 4.2.40 A GUI screen of the MPViz: Data for a Type 2B Statement “Estimate the results of computation involving whole numbers, fractions, and decimals” was entered into the MPViz.

Figure 4.2.41: A GUI screen of the MPViz for creating the MP model from a Type 2B Statement “Estimate the results of computation involving whole numbers, fractions, and decimals.”
Fig. 4.2.42 A text file which was created by entering a Type 2B Statement: “Estimate computation involving whole numbers, fractions, and decimals.”

Fig. 4.2.43 An MP diagram of a Type 2B statement by the MPViz: “Estimate involving whole numbers, fractions and decimals.”

Figure 4.2.44: A GUI screen of the MPViz for creating the MP model from a Type 9 Statement: “Create and solve word problems involving addition, subtractions, multiplication, and division of a whole number.”
4.3 Validation and UML for the MP model

4.3.1 Why UML for the MP model

The different degrees of alignments are measured in terms of math concepts and the cognitive process of math concepts in our alignment method. Capturing math concepts and the cognitive process of math concepts from math educational standards statements is a necessary and important task. In the MP model, an MP class represents a math concept represented by a noun and a verb materialization hierarchy realizes the behaviour of an MP class. A verb materialization hierarchy represents the cognitive process of math concepts. Therefore, UML has been chosen for the MP model for capturing math concepts and the cognitive process of math concepts for the following
reasons: 1) UML has a realization relationship which semantically represents a relationship between an MP class and a verb materialization hierarchy; 2) UML has a stereotype which extends current feature, and a stereotype has been used for defining an MP class for math concepts and a verb stereotype class for the cognitive process of math concepts.

We use taxonomies of math concepts for related math concepts and WordNet for word similarity for our alignment method. Taxonomies of math concepts can be considered a light-weight ontology. However, ontology has not been used for representing the semantics of math educational standards because the concept of the MP (Materialization Pattern) model which includes a realization relationship, an MP class, and a verb stereotype class, cannot be expressed well with OWL as an ontology. In general, a verb describes a relation between resources as a property in RDF or RDFS, and a relationship between entities as an entity relationship in ER model. A verb also relates an object to other objects as an object property in OWL. Math educational standards statements have only one subject, “student”, and the subject is omitted because they are imperative mood sentences. For example, “Write fractions with numerals and number words (Ohio State).” is a math educational standards statement. If we model a verb as a relationship between a “student” and math concepts using OWL, a concept “student” will be repeatedly shown as a domain in an object property because math educational standard statement has only one subject “student”. The concept “student” is insignificant because these standards state what kinds of math concepts should be learned and how they can be learned. Instead, a verb is significant as the cognitive process of a math concept and is modelled as a verb stereotype class. Therefore, we think the MP model using the UML
notation is a right choice for capturing math concepts and the cognitive process of math concepts for aligning standards.

### 4.3.2 Validation of the MP model

Model validation is based on the purpose of a model and its intended use. Validation guarantees that the model satisfies its intended use in terms of the methods employed and the results produced (Macal 2005). The MP model has been proposed for representing the semantics of imperative mood sentences used in math educational standards using UML notation for the purpose of aligning math educational standards.

Our semi-automatic modeling tool “MPViz” has been embedded in our semi-automatic alignment tool “MPComp”. The MPComp has been used as a formal validation tool for the MP model. The validation of the MP model has been clearly shown when we evaluated our alignment tool for the intended use of the MP model. As a validation (evaluation) method we set up a domain expert judgment as the gold standard because a domain expert judgment is the best for interpreting and aligning math educational standards. We wanted to prove whether or not our result from the alignment tool MPComp is comparable to a domain expert judgment. We compared results from the alignment tool to a domain expert’s results using the Cohen’s kappa test. We extracted and tested 122 pairs of math educational standards from Idaho and Nevada, and 80 pairs from Ohio and Texas. It clearly showed that results from our alignment tool “MPComp” are comparable to results from a domain expert. See Section 4.5 for validation (evaluation) in detail.
4.4 Graph matching for aligning math educational standards

Our alignment method also utilizes graph matching which is one method of schema matching (Doan et al. 2003A; Batini and Lenzerni 1986; Doan and Halvey 2005; Madhavan et al. 2005; Berstein and Melink 2007; Melink et al. 2007; Nash et al. 2007; Berstein et al. 2006A; Bernstein et al. 2006B; Duchateau et al. 2008; Chai et al 2008; Giunchiglia et al. 2008) for aligning math educational standards statements. A semi-automatic tool MPViz creates an MP diagram with UML notation from different types of MP statements.

4.4.1 Terminology

Each MP diagram is converted to a verb-phrase graph and a noun-phrase graph using a semi-automatic tool MPViz. In this section, we present a simple example of conversion of MP diagram to a verb phrase graph and a noun phrase graph, overall sketch of an algorithm for our alignment method, and the graph matching algorithm using Bloom’s taxonomy, the WordNet, and the math ontology.

- MP statements, MP verb, MP noun, MP verb modifier, MP noun modifier, cognitive process, and cognitive process of math concepts are already explained in Section 3.1.
- Generalization set (Ambler, 2010): In UML a taxonomic classification creates generalization hierarchy. UML 2.0 uses generalization set concept, an inheritance arrowhead with a label representing the name of the set. It is used for different taxonomic classification about the same class. There are three generalization sets for person: Role, Age, and Gender in Figure 4.4.1. But we omitted the name of the generalization set in taxonomies of math concepts.
A graph (Skavarcius & Robinson 1986) is a pair $G = (V, E)$ where $V$ is a finite set of vertices, and $E$ is an irreflexive and symmetric relation on $V$. The ordered pairs in $E$ are called the edges of the graph. The irreflexivity of $E$ implies that there are no edges from a vertex to itself. The symmetry of $E$ implies that $(u, v) \in E$ if and only if $(v, u) \in E$.

“A graph $G = (V, E)$ is called a tree if $G$ is connected and acyclic (Skavariuc & Robinson 1986).”

A list is a data structure that implements an ordered collection of values, where the same value may occur more than once

Bloom’s Taxonomy (Bloom and Krathwohl 1956): In 1956, a committee of colleges, headed by Benjamin Bloom recognizes three domain of educational learning such as cognitive domain, affective domain, and psychomotor domain. The cognitive domain involves mental skill. The affective domain deals with growth in feelings or emotional areas. The Psychomotor domain includes manual
or physical skills. These three domains often are referred as KSA (Knowledge, Skills, and Attitude).

- Bloom’s cognitive domain (Bloom and Krathwohl 1956): The cognitive domain involves knowledge and the development of intellectual ability and skills. Bloom identifies six categories within the cognitive domain from the lowest level, through increasingly more abstract mental levels to the most complex level. The categories can be considered as degrees of difficulties. These categories are: knowledge, comprehension, application, analysis, synthesis, and evaluation. These categories are referred to as follows: 1) Knowledge: Recalling data or information learned, 2) Comprehension: Demonstrating understanding of information in one’s own words or interpreting it, 3) Application: Using information in a new situation, 4) Analysis: Breaking down material or concepts into component parts, 5) Synthesis: Creating a new meaning or structure with previous learning, and 6) Evaluation: Make a judgment or decision about the value of ideas or materials.

4.4.2 Conversion of an MP diagram to a verb phrase graph and a noun phrase graph

An MP statement has an MP verb(s) with an MP verb modifier(s) and an MP noun(s) with an MP noun modifier(s). See Figure 4.4.2 for an MP statement in the Reed-Kellogg system.

```
<table>
<thead>
<tr>
<th>MP verb</th>
<th>MP noun</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP verb modifiers</td>
<td>MP noun modifiers</td>
</tr>
</tbody>
</table>
```

*Figure 4.4.2: An MP statement in the Reed-Kellogg system*
This MP statement is modeled to a MP diagram. The MP diagram is converted to a verb-phrase graph and a noun-phrase graph for matching. Therefore, a verb-phrase graph is created from an MP verb(s) with an MP verb modifier(s) and a noun-phrase graph is created from a MP noun(s) with a MP noun modifier(s). A simple example of conversion of an MP diagram to two graphs is as follows using an MP statement “Demonstrate the value of irrational number.”

![Figure 4.4.3: An MP diagram of a Type 2 statement by Rational Rose](image)

![Figure 4.4.4: An MP diagram of a Type 2 statement by the MPViz](image)

![Figure 4.4.5: A verb phrase graph and a noun phrase graph of Figure 4.4.4 by the MPViz](image)
4.4.3 Graph matching (Melink et al. 2002) using Bloom taxonomy, the WordNet, and the math ontology

Graph matching is used for aligning two math educational standards statements on the sentence level with the Bloom taxonomy (Bloom & Krathol 1956) for cognitive verb categorization, the WordNet (Fellbaum 1999) for word similarity, and a math ontology for related math concepts.

Two problems for sentence alignment are as follows:

1) Sentences have the same information but little similarity on the surface (Brazilay and Elhadad 2003).

2) Sentences don’t convey the same information but have overlapping vocabularies (Yilmazel et al. 2007).

These two problems can be mostly ignored for alignment of math educational standards because these statements are imperative mood sentences and very well defined. Math concepts (e.g., Irrational number in Figure 4.4.3) and the cognitive process of math concepts (for example, Demonstrate in Figure 4.4.3) are well-defined terms. Alignment can be performed by comparing math concepts and the cognitive process of math concepts. Math concepts and the cognitive process of math concepts in MP diagrams are converted to verb phrase graphs and noun phrase graphs for alignment. Therefore, in order to align two math educational standards statements, each verb-phrase graph from two statements, and each noun-phrase graph from two statements can be matched, respectively.
4.4.3.1 Bloom’s Cognitive Taxonomy

The cognitive process has been referred as the verbs in the educational standards (Williamson and Williams 2010). Verbs on math educational standards statements are referred as MP verbs in MP Statements. MP verbs are cognitive process verbs. For alignment of math educational standards, MP verbs are categorized based on Bloom’s Cognitive Taxonomy. The Bloom’s Cognitive Taxonomy has six categories as follows: 1) knowledge, 2) comprehension, 3) application, 4) analysis, 5) synthesis, and 6) evaluation. When two MP verbs belong to the same category of the cognitive domain of Bloom’s taxonomy, it is recognized that they belong to the same cognitive process. See Table 4.4.1 for categorization of cognitive verbs based on Bloom’s cognitive taxonomy. This table has cognitive verbs from two different sources. One was collected at various conferences by Dr. Cia Verschelden; it was originally posted on the Office of Assessment web site (www.k-state.edu/assessment) in 2003. The other is from NWEA (Northwest Evaluation Association 2008).
### Table 4.4.1: Category of cognitive verbs based on Bloom’s cognitive taxonomy

<table>
<thead>
<tr>
<th>Knowledge</th>
<th>Comprehension</th>
<th>Application</th>
<th>Analysis</th>
<th>Synthesis</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
<td>Answer</td>
<td>Add</td>
<td>Analyze</td>
<td>Categorize*</td>
<td>Appraise</td>
</tr>
<tr>
<td>Count on</td>
<td>Associate</td>
<td>Apply</td>
<td>Arrange</td>
<td>Combine</td>
<td>Approximate</td>
</tr>
<tr>
<td>Display</td>
<td>Classify*</td>
<td>Build</td>
<td>Breakdown</td>
<td>Compile</td>
<td>Assess</td>
</tr>
<tr>
<td>Define</td>
<td>Compare*</td>
<td>Calculate</td>
<td>Categorize*</td>
<td>Compose</td>
<td>Assess</td>
</tr>
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<td>Change</td>
<td>Choose</td>
<td>Create</td>
<td>Assess</td>
</tr>
<tr>
<td>Draw</td>
<td>Defend</td>
<td>Classify*</td>
<td>Classify*</td>
<td>Develop</td>
<td>Assess</td>
</tr>
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<td>Discuss</td>
<td>Copy</td>
<td>Combine</td>
<td>Design</td>
<td>Consider</td>
</tr>
<tr>
<td>Labels</td>
<td>Distinguish</td>
<td>Complete</td>
<td>Compare*</td>
<td>Drive</td>
<td>Consider</td>
</tr>
<tr>
<td>List</td>
<td>Estimate</td>
<td>Compute</td>
<td>Compose</td>
<td>Devise</td>
<td>Consider</td>
</tr>
<tr>
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<td>Exemplify</td>
<td>Collect</td>
<td>Construct</td>
<td>Device</td>
<td>Critique</td>
</tr>
<tr>
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<td>Explain</td>
<td>Conduct</td>
<td>Decompose</td>
<td>Explain*</td>
<td>Critique</td>
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<td>Design</td>
<td>Express</td>
<td>Determine</td>
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<td>Outlines</td>
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<td>Demonstrate</td>
<td>Detect</td>
<td>Estimate</td>
<td>Determine</td>
</tr>
<tr>
<td>Point</td>
<td>Find</td>
<td>Determine</td>
<td>Develop</td>
<td>Evaluate</td>
<td>Determine</td>
</tr>
<tr>
<td>Quote</td>
<td>Generalize</td>
<td>Discover</td>
<td>Diagram</td>
<td>Evaluate</td>
<td>Determine</td>
</tr>
<tr>
<td>Read</td>
<td>Give examples</td>
<td>Divide</td>
<td>Differentiate</td>
<td>Group</td>
<td>Evaluate</td>
</tr>
<tr>
<td>Recall</td>
<td>Illustrate*</td>
<td>Draw</td>
<td>Discriminate</td>
<td>Judge</td>
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</tr>
<tr>
<td>Recite</td>
<td>Interpret</td>
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<td>Distinguish</td>
<td>Justify</td>
<td>Evaluate</td>
</tr>
<tr>
<td>Recognize</td>
<td>Infer</td>
<td>Examine</td>
<td>Illustrate*</td>
<td>Measure</td>
<td>Evaluate</td>
</tr>
<tr>
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<td>Locate</td>
<td>Extend</td>
<td>Infer</td>
<td>Order</td>
<td>Evaluate</td>
</tr>
<tr>
<td>Repeat</td>
<td>Match*</td>
<td>Graph</td>
<td>Outline</td>
<td>Plan</td>
<td>Evaluate</td>
</tr>
<tr>
<td>Replicate</td>
<td>Order</td>
<td>Gather</td>
<td>Partition</td>
<td>Prevent</td>
<td>Evaluate</td>
</tr>
<tr>
<td>Reproduce</td>
<td>Read</td>
<td>Interpolate</td>
<td>Point out</td>
<td>Prevent</td>
<td>Evaluate</td>
</tr>
<tr>
<td>Specify</td>
<td>Represent</td>
<td>Manipulate</td>
<td>Relate</td>
<td>Prevent</td>
<td>Evaluate</td>
</tr>
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<td>State</td>
<td>Paraphrase</td>
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<td>Reorganize</td>
<td>Evaluate</td>
</tr>
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<td>Predict</td>
<td>Measure</td>
<td>Separate</td>
<td>Review</td>
<td>Evaluate</td>
</tr>
<tr>
<td>Write</td>
<td>Provide examples</td>
<td>Model</td>
<td>Solve*</td>
<td>Rewrite</td>
<td>Evaluate</td>
</tr>
<tr>
<td></td>
<td>Rewrite</td>
<td>Operate</td>
<td>Subdivide</td>
<td>Summarize*</td>
<td>Evaluate</td>
</tr>
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<td>Sort</td>
<td>Perform</td>
<td>Utilize</td>
<td>Transform</td>
<td>Evaluate</td>
</tr>
<tr>
<td></td>
<td>Summarize</td>
<td>Prepare</td>
<td>Specify</td>
<td>Specify</td>
<td>Evaluate</td>
</tr>
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<td>Understand</td>
<td>Produce</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
<td>Round</td>
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<td>Show</td>
<td></td>
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<tr>
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<td>Simplify</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td></td>
<td>Sketch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Solve*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Subtract</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
<td>Translate</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Use</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 4.4.3.2 Taxonomies of math concepts

We create taxonomies of K-12 math concepts based on McREL’s (Mid-continent Research for Education and Learning) standards. This is also an on-going process. Generalization (i.e. isa relationship) and aggregation (i.e. whole-part relationship) relationships in UML have been used for taxonomies of math concepts. It is used for
finding out related math concepts for alignment. Related math concepts are relationships in sibling, parent, or children in a tree where we consider each generalization set as a tree. See definitions 4.4.3, 4.4.4, and 4.4.9 for related concepts.

Figure 4.4.6: A taxonomy of a math concept “number”: The math concept “number” has five different generalization sets.

See Figure 4.4.6 for finding out related concepts. For examples, related concepts of a math concept “rational number” are real number, irrational number, fraction, decimal, integer, whole number, or natural number. Related concepts of a math concept “positive integer” are integer or negative integer. They are relationships in parent, children, or sibling in a tree where each generalization set in a taxonomy of math concepts is considered as a tree. The easiest way of finding out related concepts is to make a list of nodes in each subtree with depth 1 which has a math concept for related concepts. For
example, we want to find out related concepts for a math concept “rational number”.

Lists of subtrees will be as follows:

1. Real number, Irrational number, Rational number, and

2. Rational number, Fraction, Integer, Decimal, Whole number, Natural number.

Related concepts of a math concept “rational number” are real number, irrational number, fraction, integer, decimal, whole number, or natural number. They are a union of nodes in a tree with depth one which has “rational number.”

4.4.3.3 WordNet and a threshold value for word similarity

WordNet (Princeton University 2006) is an online lexical database for the English language. It provides synonyms called synsets, and semantic relations between these synonym sets. Synsets provide general definitions of words.

- Function for word similarities based on the WordNet

We used a predefined function (Simpson and Dao 2009) for word similarities between nouns, verbs, math concepts, or attributes of MP statements (math educational standard statements). This predefined function calculates the semantic similarity between two sentences based on the similarity of the pairs of words. It computes the similarity between two words based on the WordNet dictionary. They use a revised version of Wu & Palmer’s method (1994) for word similarity such as

\[
\text{Sim}_{\text{ad}}(w_1, w_2) = 2 \times \frac{\text{depth}(\text{LCS})}{\text{depth}(w_1) + \text{depth}(w_2)};
\]

where \( w_1 \) and \( w_2 \) are two words for comparison,

\( \text{Depth}(w_1) \) & \( \text{depth}(w_2) \) are depth of nodes \( w_1 \) & \( w_2 \) respectively in WordNet taxonomies,

\( \text{LCS} \) is the Least Common Submer of \( w_1 \) & \( w_2 \) in WordNet taxonomies.
This implies that $0 < \text{Sim}_{\text{id}} \leq 1$. Sim can never be zero because the depth of LCS cannot be zero (The depth of the root node of a taxonomy is 1). A value of $\text{Sim}_{\text{id}}$ is set up as 1 when two words are the same. We referred this predefined function which we use as $\text{Sim}_{\text{id}}$.

- Setting up a *threshold value* for a similarity value:

  In order to define the equivalent meaning of a math concept (MP noun), an attribute of a math concept, a cognitive verb (MP verb), a noun in an MP noun modifier or an MP verb modifier, we set up our own threshold for a similarity value as 0.95. See Definition 4.4.8 in Section 4.4.9. For setting up our own threshold we had two sets of testing as follows:

  1) We tested data with $\text{Sim}_{\text{id}}$, and compared a result against existing data (Resnik 1999). This test result shows us that a pair of words has equivalent meaning when a similarity value by $\text{Sim}_{\text{id}}$ is over 0.95. Therefore, a threshold has been set up as 0.95. See table 4.4.2 for a test result. Three computational similarity measures such as $\text{wsim}$, $\text{wsim}_{\text{edge}}$, and $\text{wsim}_{\text{p(c)}}$ in table N were used by Resnik (1999). Their formulas are as follows:

  1. $\text{wsim}(w_1, w_2) = \max_{c_1, c_2} \left[ \text{sim}(c_1, c_2) \right] \quad (4.1)$
     where $c_1$ ranges over $s(w_1)$ and $c_2$ ranges over $s(w_2)$

  2. $\text{wsim}_{\text{p(c)}}(w_1, w_2) = \max_{c_1, c_2} \left[ \text{simp}_{\text{p(c)}}(c_1, c_2) \right] \quad (4.2)$
     where $c_1$ ranges over $s(w_1)$ and $c_2$ ranges over $s(w_2)$ with probability $p(c)$ for any given concept $c$. $s(w)$ indicates the set of concepts in the taxonomy that represent sense of word $w$. 

3. \( w_{sim\_edge} (w1, w2) = (2 \times \text{max}) - \text{min}\{\text{len}(c_1, c_2)\} \) \hspace{1cm} (4.3)

where \( c_1 \) ranges over \( s(w1) \), \( c_2 \) ranges over \( s(w2) \), \text{max} is the maximum depth of taxonomy, and \( \text{len}(c_1, c_2) \) is the length of the shortest path from \( c_1 \) to \( c_2 \).

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Word Pair} & \text{wsim} & \text{wsim\_edge} & \text{wsim\_p(c)} & \text{Sim\_ad} \\
\hline
\text{car} & \text{automobile} & 8.0411 & 30 & 0.9962 & 1 \\
\text{gem} & \text{jewel} & 14.9286 & 30 & 1.0000 & 1 \\
\text{journey} & \text{voyage} & 6.7537 & 29 & 0.9907 & 0.95 \\
\text{boy} & \text{lad} & 8.4240 & 29 & 0.9971 & 0.92 \\
\text{coast} & \text{shore} & 10.8076 & 29 & 0.9994 & 0.91 \\
\text{asylum} & \text{madhouse} & 15.6656 & 29 & 1.0000 & 0.95 \\
\text{magician} & \text{wizard} & 13.6656 & 30 & 0.9999 & 1 \\
\text{midday} & \text{noon} & 12.3925 & 30 & 0.9998 & 1 \\
\text{furance} & \text{stove} & 1.7135 & 23 & 0.6951 & 0.54 \\
\text{food} & \text{fruit} & 5.0076 & 27 & 0.9689 & 0.4 \\
\text{bird} & \text{cock} & 9.3139 & 29 & 0.9984 & 0.95 \\
\text{bird} & \text{crane} & 9.3139 & 27 & 0.9984 & 0.87 \\
\text{tool} & \text{implement} & 6.0787 & 29 & 0.9852 & 0.93 \\
\text{bother} & \text{monk} & 2.9683 & 24 & 0.8722 & 0.93 \\
\text{crane} & \text{implement} & 2.9683 & 24 & 0.8722 & 0.75 \\
\text{lad} & \text{brother} & 2.9355 & 26 & 0.8693 & 0.67 \\
\text{journey} & \text{car} & 0 & 0 & 0 & 0.14 \\
\text{monk} & \text{oracle} & 2.9683 & 24 & 0.8722 & 0.53 \\
\text{food} & \text{rooster} & 1.0101 & 18 & 0.5306 & 0.29 \\
\text{coast} & \text{hill} & 6.2344 & 26 & 0.9867 & 0.67 \\
\text{forest} & \text{graveyard} & 0 & 0 & 0 & 0.43 \\
\text{monk} & \text{slave} & 2.9683 & 27 & 0.8722 & 0.67 \\
\text{coast} & \text{forest} & 0.0000 & 0.0000 & 0.0000 & 0.55 \\
\text{lad} & \text{wizard} & 2.9683 & 26 & 0.8722 & 0.67 \\
\text{chord} & \text{smile} & 2.3544 & 20 & 0.8044 & 0.38 \\
\text{glass} & \text{magician} & 1.0105 & 22 & 0.5036 & 0.36 \\
\text{noon} & \text{string} & 0.0000 & 0.0000 & 0.0000 & 0.27 \\
\text{rooster} & \text{voyage} & 0.0000 & 0.0000 & 0.0000 & 0.27 \\
\hline
\end{array}
\]

2) We also tested 33 pairs of math educational standards statements from two different states (Nevada, Idaho) using our alignment tool “MPComp” with
different threshold values 0.99, 0.95, 0.93 and 0.90 for word similarities.

The MPComp uses $\text{Sim}_{sd}$ for computing a similarity of a pair of words.

We compare our results from our alignment method to a domain expert’s judgment using Cohen’s kappa (Cohen 1960; Cohen et al. 1968). Precision, recall, and F-measure also have been measured as evaluation metrics for measuring correctness of different degrees of alignment. When threshold values are 0.99, 0.95, 0.93, and 0.90, testing results showed that kappa values are 0.699, 0.699, 0.664, and 0.410, respectively. Fleiss’ (1981) guidelines characterize kappa over .75 excellent, .40 to .75 as fair to good, and below .40 as poor. Altman (1990) also interprets kappa 0.80 to 1 as very good, 0.60 to 0.80 as good, 0.40 to 0.60 as moderate, 0.20 to 0.40 as fair, and less than 0.20 as poor. We have a value of kappa as 0.699 which falls into the categories “fair to good” and “good” with threshold values 0.99 and 0.95 according to Fleiss and Altman, respectively. Therefore our alignment method is comparable to a domain expert’s judgment when we set up a threshold value for word similarity as 0.95. See Tables 4.4.3 and 4.4.4.
We noticed that kappa equals to 0.699 with p = .000 < 0.001. This result shows that Cohen’s kappa coefficient, kappa = 0.699 and p value which measures statistical significance is .000. We have a value of kappa as 0.699 which falls into the categories “fair to good” and “good” based on Fleiss’ and Altman’s guidelines, respectively. See Table 4.4.5 for precision, recall, and F-measure.

### Table 4.4.3: Threshold0.99 • DomainExpert Cross Tabulation

<table>
<thead>
<tr>
<th>Frequency</th>
<th>DomainExpert</th>
<th>NA</th>
<th>PA*</th>
<th>PA**</th>
<th>PA***</th>
<th>PR</th>
<th>SFA</th>
<th>WFA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold0.99</td>
<td>NA</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>PA*</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>PA**</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>PA***</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>PR</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>SFA</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>WFA</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>5</td>
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<td>3</td>
<td>4</td>
<td>10</td>
<td>2</td>
<td>8</td>
<td>33</td>
</tr>
</tbody>
</table>

### Table 4.4.4: Symmetric Measures

<table>
<thead>
<tr>
<th>Measure of Agreement</th>
<th>Value</th>
<th>Asymp Std. Errora</th>
<th>Approx. Tb</th>
<th>Approx. Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kappa</td>
<td>.699</td>
<td>.092</td>
<td>8.562</td>
<td>.000</td>
</tr>
<tr>
<td>N of valid cases</td>
<td>33</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Not assuming the null hypothesis
b. Using the asymptotic standard error assuming the null hypothesis

We run SPSS using Cohen’s kappa’s option with a threshold value as 0.99.
We have extremely high precision in “Strongly Fully-aligned” and “Partially-aligned*”, high precision in “Poorly-aligned”, and low precision in “Partially-aligned***”.

- We run SPSS using Cohen’s kappa’s option with a threshold value as 0.95

See tables 4.4.6 & 4.4.7 for a result.

### Table 4.4.6: Threshold0.95 • DomainExpert Cross Tabulation

<table>
<thead>
<tr>
<th>Frequency</th>
<th>DomainExpert</th>
<th>NA</th>
<th>PA*</th>
<th>PA**</th>
<th>PA***</th>
<th>PR</th>
<th>SFA</th>
<th>WFA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold0.95</td>
<td>NA</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>PA*</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>PA**</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>PA***</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>PR</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>SFA</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>WFA</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>2</td>
<td>8</td>
<td>33</td>
<td></td>
</tr>
</tbody>
</table>
This result shows that Cohen’s kappa coefficient, \( \kappa = 0.699 \) with
\( p = .000 < 0.001 \). This kappa value, 0.699 falls into the categories “fair to good” by
Fleiss’ guidelines and “good” by Altman’s interpretation. See Table 4.4.8 for precision,
recall, and F-measure.

Table 4.4.8: Precision, recall, and F-measure for different degrees of alignments with
a threshold value as 0.95

<table>
<thead>
<tr>
<th>Alignments</th>
<th>Precision</th>
<th>Recall</th>
<th>F-measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strongly Fully-aligned (SFA)</td>
<td>100</td>
<td>50</td>
<td>66.67</td>
</tr>
<tr>
<td>Weakly Fully-aligned (WFA)</td>
<td>77.78</td>
<td>87.5</td>
<td>82.35</td>
</tr>
<tr>
<td>Partially-aligned*** (PA***)</td>
<td>75</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>Partially-aligned**(PA**)</td>
<td>40</td>
<td>66.67</td>
<td>50</td>
</tr>
<tr>
<td>Partially-aligned* (PA*)</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Poorly aligned(PR)</td>
<td>87.50</td>
<td>70</td>
<td>77.78</td>
</tr>
<tr>
<td>Not aligned(NA)</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
</tbody>
</table>

We have extremely high precision in “Strongly Fully-aligned” and “Partially-
aligned*", high precision in “Poorly-aligned”, and low precision in “Partially-
aligned**".
We run SPSS using Cohen’s kappa’s option with a threshold value as 0.93

See tables 4.4.9 & 4.4.10 for a result.

### Table 4.4.9: Threshold0.93 • DomainExpert Cross Tabulation

<table>
<thead>
<tr>
<th>DomainExpert</th>
<th>NA</th>
<th>PA*</th>
<th>PA**</th>
<th>PA***</th>
<th>PR</th>
<th>SFA</th>
<th>WFA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold0.93</td>
<td>NA</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>PA*</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>PA**</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>PA***</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>PR</td>
<td>1</td>
<td>0</td>
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<td>0</td>
<td>7</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>SFA</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>WFA</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>2</td>
<td>33</td>
</tr>
</tbody>
</table>

### Table 4.4.10: Symmetric Measures

<table>
<thead>
<tr>
<th>Measure of Agreement</th>
<th>Value</th>
<th>Asymp Std. Error</th>
<th>Approx. T</th>
<th>Approx. Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kappa</td>
<td>.664</td>
<td>.093</td>
<td>8.341</td>
<td>.000</td>
</tr>
<tr>
<td>N of valid cases</td>
<td>33</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Not assuming the null hypothesis  
b. Using the asymptotic standard error assuming the null hypothesis

This result shows that Cohen’s kappa coefficient, $\kappa = 0.664$ with $p = .000$. Kappa $= 0.664$ falls into the “fair to good” category based on Fleiss’ guidelines.

See Table 4.4.11 for precision, recall, and F-measure.
Table 4.4.11: Precision, recall, and F-measure for different degrees of alignments with a threshold value as 0.93

<table>
<thead>
<tr>
<th>Precision</th>
<th>Recall</th>
<th>F-measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strongly Fully-aligned (SFA)</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Weakly Fully-aligned (WFA)</td>
<td>46.67</td>
<td>87.5</td>
</tr>
<tr>
<td>Partially-aligned*** (PA***)</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>Partially-aligned**(PA**)</td>
<td>33.33</td>
<td>66.67</td>
</tr>
<tr>
<td>Partially-aligned*(PA*)</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>Poorly aligned(PR)</td>
<td>87.5</td>
<td>70</td>
</tr>
<tr>
<td>Not-aligned(NA)</td>
<td>100</td>
<td>60</td>
</tr>
</tbody>
</table>

We have extremely high precision in “Strongly Fully-aligned” and “Not-aligned”, high precision in “Poorly-aligned”, and low precision in “Partially-aligned***” and “Partially-aligned**”.

We run SPSS using Cohen’s kappa’s option with a threshold value as 0.90

See tables 4.4.12 & 4.4.13 for a result.

Table 4.4.12: Threshold0.90 • DomainExpert Cross Tabulation

<table>
<thead>
<tr>
<th>DomainExpert</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NA</td>
</tr>
<tr>
<td>Threshold0.90</td>
<td>3</td>
</tr>
<tr>
<td>NA</td>
<td>1</td>
</tr>
<tr>
<td>PA*</td>
<td>0</td>
</tr>
<tr>
<td>PA**</td>
<td>0</td>
</tr>
<tr>
<td>PA***</td>
<td>1</td>
</tr>
<tr>
<td>PR</td>
<td>0</td>
</tr>
<tr>
<td>SFA</td>
<td>0</td>
</tr>
<tr>
<td>WFA</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
</tr>
</tbody>
</table>
This result shows that Cohen’s kappa coefficient, kappa = 0.410 with p = .001. It falls into the “fair to good” category according to Fleiss’ guidelines and “moderate” category based on Altman. See Table 4.4.14 for precision, recall, and F-measure.

Table 4.4.13: Symmetric Measures

<table>
<thead>
<tr>
<th>Measure of Agreement Kappa</th>
<th>Value</th>
<th>Asymp Errora</th>
<th>Std. Errorb</th>
<th>Approx. T</th>
<th>Approx. Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>N of valid cases</td>
<td>33</td>
<td>.100</td>
<td>5.728</td>
<td>.000</td>
<td></td>
</tr>
</tbody>
</table>

a. Not assuming the null hypothesis
b. Using the asymptotic standard error assuming the null hypothesis

Table 4.4.14: Precision, recall, and F-measure for different degrees of alignments with a threshold value as 0.90

<table>
<thead>
<tr>
<th>Alignment Type</th>
<th>Precision</th>
<th>Recall</th>
<th>F-measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strongly Fully-aligned (SFA)</td>
<td>100</td>
<td>50</td>
<td>66.67</td>
</tr>
<tr>
<td>Weakly Fully-aligned (WFA)</td>
<td>38.89</td>
<td>87.5</td>
<td>53.85</td>
</tr>
<tr>
<td>Partially-aligned*** (PA***)</td>
<td>66.67</td>
<td>50</td>
<td>57.14</td>
</tr>
<tr>
<td>Partially-aligned**(PA**)</td>
<td>33.33</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>Partially-aligned* (PA*)</td>
<td>50</td>
<td>100</td>
<td>66.67</td>
</tr>
<tr>
<td>Poorly-aligned (PR)</td>
<td>50</td>
<td>10</td>
<td>16.67</td>
</tr>
<tr>
<td>Not-aligned (NA)</td>
<td>75</td>
<td>60</td>
<td>66.67</td>
</tr>
</tbody>
</table>

We have extremely high precision in “Strongly Fully-aligned”, low precision in “Weakly Fully-aligned”, “Partially-aligned***”, “Partially-aligned**”, and “Partially-aligned*”, “Poorly-aligned”. 
4.4.4 A semi-automatic tool “MPComp” for alignment

The MPComp is developed for alignment of math educational standards. The MPViz is embedded in the MPComp. The MPComp aligns two math educational standards statement using graph matching. Verb phrase graphs and noun graphs which were already created by the MPViz are inputs for the MPComp. The MPComp produces output such as “Strongly Fully-aligned (SFA)”, “Weakly Fully-aligned (WFA)”, “Partially-aligned*** (PA***)”, “Partially-aligned** (PA**)”, “Partially-aligned* (PA*)”, “Poorly-aligned (PR)”, or “Not-Aligned (NA)”.

![Figure 4.4.7: A GUI screen of an alignment tool MPComp.](image)

4.4.5 Overall sketch of an algorithm for aligning math educational standards statements

An overview of our alignment method (See Figure 4.2.1.) is as follows:

1) Classify math educational standards statement into MP statements.

2) Create the MP model in UML diagrams from MP statements using the MPViz.

3) Transform UML diagrams to graph forms using the MPViz.
4) Match two graphs for comparing two MP models using the MPComp.

5) Get alignment result.

Steps 4 and 5 are related to alignments of two MP statements (math educational standard statements). Figure 4.4.8 is an overall sketch of a matching algorithm for our alignment method.

![Flowchart for a graph matching algorithm](image)

Figure 4.4.8: A flowchart for a graph matching algorithm

4.4.6 Formal notations and examples of different degrees of alignments

In this section we present these different degrees of alignments for math educational standard statements with formal notation and examples.

4.4.6.1 The notion of different degrees of alignments with examples

1. **Strongly Fully-aligned**: Two math standard statements have the exact same meaning. They have the same math concepts, the same properties (attributes or modifiers) in verb phrase and noun phrase graphs, and the same cognitive verbs.
Recall basic addition and subtraction facts through 18.
Recall basic addition facts (sums to 18) and corresponding subtraction facts immediately.

**Figure 4.4.9: An example of “Strongly Fully-aligned”**

**Figure 4.4.10: An output of alignment (Strongly Fully-aligned) of above two MP statements (math educational standard statements) by the MPComp**

2. *Weakly Fully-aligned*: The meaning of one math standard statement has been included in the other statement. They have the same math concepts and the same cognitive verbs with different numbers of math concepts or cognitive verbs, or with no modifier or attribute of an MP noun(s) of one statement. See Figure 4.4.11 and Figure 4.4.12 for examples and outputs of examples by MPComp for Weakly Fully-aligned.
Identify, label, draw, and describe points, line segments, rays, and angles.

Identify and label points, line segments, rays, and angles.

Read, write, compare, and order whole numbers.

Read, write, compare, and order whole numbers to one million.

Figure 4.4.11: An example of “Weakly Fully-aligned”

Figure 4.4.12: An output of alignment (Weakly Fully-aligned) of above MP statements (math educational standard statements) by the MPComp

3. Partially-aligned***: Two math standard statements have the same math concepts with different attributes or modifiers, and the same cognitive verbs. See Figure 4.4.13 and Figure 4.4.14 for examples and outputs of examples by MPComp for Partially-aligned***.
Read, write, compare, and order commonly used fractions with pictorial representation.

Read and write unit fractions with numbers and words

- **same cognitive verb**
- **same math concept**
- **different modifiers of a math concept**

*Figure 4.4.13: An example of “Partially-aligned***”*

*Figure 4.4.14: An output of alignment (Partially-aligned*** ) of above MP statements (math educational standard statements) by the MPComp*

4. **Partially-aligned**: Two math standard statements have the same math concepts and the same cognitive process with different cognitive verbs. See Figure 4.4.15 and Figure 4.4.16 for examples and outputs of examples by MPComp for Partially-aligned**.
Identify and model basic addition facts (sums to 18) and the corresponding subtraction facts.
Recall basic addition and subtraction facts thru 18.

**Figure 4.4.15: An example of “Partially-aligned**”

5. **Partially-aligned**: Two math standard statements have the same math concepts and the different cognitive process of verbs. See Figure 4.4.17 and Figure 4.4.18 for examples and outputs of examples by MPComp for Partially-aligned*.
Evaluate algebraic expressions and formula for a given integer value.

Simplify algebraic expressions.

different cognitive process

same math concepts

A modifier of a math concepts

Figure 4.4.17: An example of “Partially-aligned”

Figure 4.4.18: An output of alignment (Partially-aligned) of above MP statements (math educational standard statements) by the MPComp

6. Poorly-aligned: Two math standard statements have different but related math concepts and the same cognitive process of verbs. See Figure 4.4.19 and Figure 4.4.20 for examples and outputs of examples by MPComp for Poorly-aligned.
Add and subtract fractions with unlike denominators.

Add, subtract, multiply, and divide rational numbers.

Figure 4.4.19: An example of “Poorly-aligned”

Figure 4.4.20: An output of alignment (Poorly-aligned) of above MP statements (math educational standard statements) by the MPComp

7. **Not-aligned**: Two math standard statements have different math concepts, or different but related math concepts with different cognitive process of verbs. See Figure 4.4.21 and Figure 4.4.22 for examples and outputs of examples by MPComp for Not aligned.
Collect and organize classification of data using concrete materials.

Represent and use numbers in the equivalent form.

An attribute of a math concept

different math concepts

different modifiers of a math concept

different cognitive process

Figure 4.4.21: An example of “Not-aligned”

Figure 4.4.22: An output of alignment (Not-aligned) of above MP statements (math educational standard statements) by the MPComp

See Table 4.4.15 for a summary of different degrees of alignments.
Table 4.4.15: A summary of different degrees of alignments

<table>
<thead>
<tr>
<th>Different degree of alignment for two math educational standards statements</th>
<th>Notion</th>
<th>Meaning</th>
<th>Example</th>
<th>Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strongly Fully-aligned (SFA)</strong></td>
<td>The same math concepts, the same cognitive verbs, &amp; the same modifiers of math concepts</td>
<td>Exactly same meaning</td>
<td>Recall basic addition and subtraction fact through 18. Immediately recall basic facts (sums to 18) and corresponding subtraction facts.</td>
<td>It allows alignment for two identical statements.</td>
</tr>
<tr>
<td><strong>Weakly Fully-aligned (WFA)</strong></td>
<td>The same math concepts, the same cognitive verbs, &amp; One of statements has no modifiers of math concepts</td>
<td>The meaning of one math standards statement is included in the other statement.</td>
<td>Read, write, compare, and order whole numbers. Read, write, compare, and order whole numbers to one million.</td>
<td>It allows alignment for more-or-less equivalent statements.</td>
</tr>
<tr>
<td><strong>Partially-aligned</strong>* (PA***))</td>
<td>The same math concept, the same cognitive verbs &amp; different modifiers of math concepts</td>
<td>The meaning of two math standards statement is more-or-less similar.</td>
<td>Read, write, compare, and order commonly used fraction with pictorial representation. Read and write unit fraction with numbers and words.</td>
<td>It can be useful for teachers to know how to teach the same math concepts through different methods.</td>
</tr>
<tr>
<td><strong>Partially-aligned</strong> (PA**)</td>
<td>The same math concept and the same cognitive process of different cognitive verbs</td>
<td>The meaning of two math standards statement is more-or-less similar.</td>
<td>Identify and model basic addition facts (sums to 18) and the corresponding subtraction facts. Recall basic addition and subtraction facts thru 18.</td>
<td>It can be useful for teachers to know how to teach the same math concepts through different methods.</td>
</tr>
<tr>
<td><em><em>Partially-aligned</em> (PA</em>)**</td>
<td>The same math concept and different cognitive process of cognitive verbs</td>
<td>Two statements have the same math concept.</td>
<td>Evaluate algebraic expression and formula for given integer values. Simplify algebraic expressions.</td>
<td>It can be useful for teachers to know how to teach the same concepts through different methods.</td>
</tr>
<tr>
<td><strong>Poorly-aligned (PR)</strong></td>
<td>Different but related math concepts and same cognitive process of cognitive verbs</td>
<td>Two statements related math concepts and the same cognitive process of math concepts.</td>
<td>Identify and generate equivalent forms of whole numbers. Identify and generate equivalent forms of fractions and decimals.</td>
<td>It can be useful for teachers to know related concepts.</td>
</tr>
<tr>
<td><strong>Not-aligned (NA)</strong></td>
<td>Different math concepts or related math concepts with different cognitive process of cognitive verbs</td>
<td>Two statements have totally different meaning.</td>
<td>1. Add and subtract simple fraction. Add and subtract measurement. 2. Use model to translate among fraction, decimals, and percents. Locate the position of rational numbers on number line</td>
<td></td>
</tr>
</tbody>
</table>
4.4.6.2 Formal notations of different degrees of alignments

**Definition 4.4.1.** Let w1 and w2 be two math concepts (MP nouns or MP class), two cognitive verbs (MP verbs or verb stereotype classes), or two attributes of math concepts.

\[
same(w1, w2) = \begin{cases} 
true, & \text{if } \text{wordNetSimilarity}(w1, w2) \geq 0.95 \\
false, & \text{otherwise}
\end{cases}
\]

where \( \text{wordNetSimilarity}(w1, w2) \) is similarity by WordNet,

See Section 4.4.3.3 for setting a value of threshold as 0.95.

**Definition 4.4.2.** Let w1 and w2 be two cognitive verbs (MP verbs or verb stereotype classes).

\[
sameCP(w1, w2) = \begin{cases} 
true, & \text{if } w1, w2 \text{ belong to the same cognitive process} \\
false, & \text{otherwise}
\end{cases}
\]

\text{of Bloom’s taxonomy}

**Definition 4.4.3.** Let w1 and w2 be two math concepts (MP nouns or MP classes).

\[
related(w1, w2) = \begin{cases} 
true, & \text{if } w1, w2 \text{ are relationships in sibling, parent, or children} \\
false, & \text{otherwise}
\end{cases}
\]

\text{in a tree.}

where each generalization set in a taxonomy of a math concept is considered as a tree.

**Definition 4.4.4.** Let \( W[] \) be non-empty lists of words (math concepts, attributes of math concepts, or cognitive verbs).

\(|W|\) is the number of words (math concepts, attributes of math concepts, or cognitive verbs) in \( W \).

**Definition 4.4.5.** Let \( W1[] \) and \( W2[] \) be non-empty lists of words (math concepts, attributes of math concepts, or cognitive verbs).
1) We define $\text{ListMatched}(1, W_1, W_2)$ is true iff for all $i$, there exists $j$ such that $\text{same}(W_1[i], W_2[j])$ is true.

2) We define $\text{ListMatched}(2, W_1, W_2)$ is true iff for all $i$, there exists $j$ such that $\text{sameCP}(W_1[i], W_2[j])$ is true.

3) We define $\text{ListMatched}(3, W_1, W_2)$ is true iff for all $i$, there exists $j$ such that $\text{related}(W_1[i], W_2[j])$ is true.

4) We define $\text{MultiWordMatched}(W_1, W_2)$ as follows: for $i = 1, 2, 3$

   $$\text{MultiWordMatched}(i, W_1, W_2) = \begin{cases} 
   \text{ListMatched}(i, W_1, W_2), & \text{if } |W_1| < |W_2| \\
   \text{ListMatched}(i, W_2, W_1), & \text{if } |W_1| > |W_2| \\
   \text{ListMatched}(i, W_1, W_2) \text{ or } \text{ListMatched}(i, W_2, W_1), & \text{if } |W_1| = |W_2|
   \end{cases}$$

**Definition 4.4.6.** $CV_1$ & $CV_2$ are the same cognitive verbs (MP verbs or verb stereotype classes). Let $CV_1[]$ and $CV_2[]$ be non-empty lists of cognitive verbs (MP verbs or verb stereotype classes).

$CV_1 == CV_2$ if and only if $\text{MultiWordMatched}(1, CV_1, CV_2)$

**Definition 4.4.7.** $\text{MathCP}_1$ & $\text{MathCP}_2$ are the same math concept (MP nouns or MP classes). Let $\text{MathCP}_1[]$ and $\text{MathCP}_2[]$ be non-empty lists of math concepts (MP nouns or MP classes).

$\text{MathCP}_1 == \text{MathCP}_2$ if and only if $\text{MultiWordMatched}(1, \text{MathCP}_1, \text{MathCP}_2)$
Definition 4.4.8. Attr1 & Attr2 are the same attribute of math concepts. Let Attr1[] and Attr2[] be non-empty lists of attributes.

\[ \text{Attr1} == \text{Attr2} \text{ if and only if } \text{MultiWordMatched(1, Attr1, Attr2)} \]

Definition 4.4.9. CV1 & CV2 are the same cognitive process. Let CV1[] and CV2[] be non-empty lists of cognitive verbs (MP verbs or verb stereotype classes).

\[ \text{CV1} \approx \text{CV2} \text{ if only if } \text{MultiWordMatched(2, CV1, CV2)} \]

Definition 4.4.10. MathCP1 & MathCP2 are related math concepts (MP nouns or MP classes). Let MathCP1[] and MathCP2[] be lists of math concepts (MP nouns or MP classes).

\[ \text{MathCP1} \equiv\text{MathCP2} \text{ if and only if } \text{MultiWordMatched(3, MathCP1, MathCP2)} \]

Definition 4.4.11. Modifiers of cognitive verbs (MP verb modifiers) from different math standard statements (MP statements) are the same. Let VG1 & VG2 be verb phrase graphs of each MP statements (math educational standard statements). Modifiers of cognitive verbs CVMOD1 & CVMOD2 are subgraphs of VG1 & VG2, respectively. Let MP1 & MP2 be math educational standard statements (MP statements).

\[ \text{CVMOD1} \cong \text{CVMOD2} \text{ if and only if } \text{CVMOD1, CVMOD2 are isomorphic and all nodes, edges of CVMOD1, CVMOD2 are the same.} \]

When either MP1 or MP2, but not both, has a modifier of cognitive verbs, we say that OneMPverbModifier (MP1, MP2) is true.
**Definition 4.4.12.** Modifiers of math concepts from different math educational standards (MP statements) are the same or only one math educational standard statement (MP statements) has a modifier.

Let MP1 & MP2 be math educational standard statements (MP statements).

Let NG1 & NG2 be noun phrase graphs of each math educational standard statements (MP statements). Let modifiers of math concepts MathMOD1 & MathMOD2 be subgraphs of NG1 & NG2, respectively.

MathMOD1 $\approx$ MathMOD2 if and only if MathMOD1, MathMOD2 are isomorphic and all nodes, edges of MathMOD1, MathMOD2 are the same.

When either MP1 or MP2, but not both, has a modifier of math concepts, we say that

OneMathConceptModifier(MP1, MP2) is true.

The different degrees of alignment can be denoted as follows:

- **Strongly Fully-aligned:**
  
  \[(CV1 == CV2) \land (\text{MathCP1} == \text{MathCP2}) \land (\text{MathMOD1} \approx \text{MathMOD2}) \land (\text{Attr1} == \text{Attr2}) \land (\text{CVMOD1} \approx \text{CVMOD2})\]

  where $(|\text{MathCP1}| = |\text{MathCP2}|), (|\text{Attr1}| = |\text{Attr2}|)$, and $(|CV1| = |CV2|)$

**Figure 4.4.23: Variables in an algorithm**

1) **Strongly Fully-aligned:**

\[(CV1 == CV2) \land (\text{MathCP1} == \text{MathCP2}) \land (\text{MathMOD1} \approx \text{MathMOD2}) \land (\text{Attr1} == \text{Attr2}) \land (\text{CVMOD1} \approx \text{CVMOD2})\]
2) **Weakly Fully-aligned:**

A. \((CV1 == CV2)\) and \((MathCP1 == MathCP2)\) and \((MathMOD1 \approx MathMOD2)\)

B. \((CV1 == CV2)\) and \((MathCP1 == MathCP2)\) and \((OneMathConceptModifier(MP1, MP2)\) or \((OneMPverbModifier(MP1, MP2))\))

3) **Partially-aligned***:

\((CV1 == CV2)\) and \((MathCP1 == MathCP2)\) and \(not (MathMOD1 \approx MathMOD2)\)

4) **Partially-aligned**

\((CV1 \approx CV2)\) and \((MathCP1 == MathCP2)\)

5) **Partially-aligned**

\(not (CV1 \approx CV2)\) and \((MathCP1 == MathCP2)\)

6) **Poorly-aligned**

\((CV1 \approx CV2)\) and \((MathCP1 \approx\approx MathCP2)\)

7) **Not-aligned**

\(not (MathCP1 == MathCP2)\) or \((not (CV1 \approx CV2)\) and \((MathCP1 \approx\approx MathCP2))\)

### 4.4.7 Graph matching algorithm

This section presents algorithms for a graph matching for alignment of two MP statements (math educational standards statements) in detail.

An algorithm for alignment is as follows:

**Algorithm** \(\) AlignTwoMP(VG1, VG2, NG1, NG2)

**Input**: VG1, VG2: Verb phrase graphs of two MP Statements, NG1, NG2: noun phrase graphs of two MP Statements

**Output**: Strongly Fully-aligned (SFA), Weakly Fully-aligned (WFA), Partially-aligned*** (PA***), Partially-aligned** (PA**), Partially-aligned* (PA*), Poorly-aligned (PR), or Not-aligned (NA)

**Steps**:

// Let MathCP1, MathCP2 be a single math concept or multiple math concepts in graphs NG1, NG2, respectively.
// Let CV1, CV2 be a single cognitive verb or multiple cognitive verbs in graphs VG1, VG2, respectively.
Let MathMOD1, MathMOD2 be modifiers of math concepts in graphs NG1, NG2, respectively.
Let CVMOD1, CVMOD2 be modifiers of cognitive verbs in graphs VG1, VG2, respectively.
Let Attr1, Attr2 be a single attribute or multiple attributes of math concepts MPNC1, MPNC2, respectively.
Let |MathCP1| and |MathCP2| be no. of math concepts where MathCP1[] & MathCP2[] are non-empty lists of math concepts.
Let |CV1|, |CV2| be no. of cognitive verbs where CV1[] & CV2[] are non-empty lists of cognitive verbs.

1. Check whether or not math concepts MathCP1 and MathCP2 are the same.
   If MathCP1 and MathCP2 are the same
   (a) Check whether or not CV1 and CV2 are the same cognitive verbs.
      if CV1 and CV2 are the same cognitive verb
      i. Check whether or not Attr1 and Attr2 are the same, and MathMOD1 and MathMOD2 are the same.
      if Attr1 and Attr2 are the same, and MathMOD1 and MathMOD2 are the same
         ① Check whether or not CVMOD1 and CVMOD2 are the same.
            If CVMOD1 and CVMOD2 are the same
               ①  If (|MathCP1| = |MathCP2|) and (|CVMOD1| = |CVMOD2|)
                  "Strongly Fully-aligned (SFA)"
               ② else
                  "Weakly Fully-aligned (WFA)"
            ② else
                  "Weakly Fully-aligned (WFA)"
      ii. else if one math standard statement has no modifiers of math concepts
           "Weakly Fully-aligned (WFA)"
      iii. else "Partially-aligned*** (PA***)
   ② else
        "Partially-aligned (PA*)"
2. Check whether or not MathCP1 and MathCP2 are related concepts.
   else if MathCP1 and MathCP2 are related concepts
   (a) Check whether or not CV1 and CV2 belong to the same cognitive process
      if CV1 and CV2 belong to the same cognitive process
         "Poorly-aligned (PR)"
      (b) else
         "Not-aligned (NA)"
   3. else "Not-aligned (NA)"
Algorithm AlignTwoMP(VG1, VG2, NG1, NG2) {
Input: VG1, VG2: Verb phrase graphs of two MP Statements, NG1, NG2: noun phrase graphs of two MP Statements
Output: strongly Fully-aligned (SFA), weakly Fully-aligned (WFA), Partially-aligned*** (PA***), Partially-aligned** (PA**), Partially-aligned* (PA*), Poorly-aligned (PR), or Not-aligned (NA)
If (CheckMathConcept(1, MPNC1, MPNC2) == “same”) // MPNC1, MPNC2 are math concepts.
    If (CheckVerbPhrase(1, VP1, VP2) == “same cognitive verb”) // VP1, VP2 are cognitive verbs.
        If (CheckAttributes(1, Attr1, Attr2) == “same” && CheckMathCPMod(M1, M2) == “same”)
            { // Attr1, Attr2 are attributes of math concepts MPNC1, MPNC2, respectively.
                // M1, M2 are modifiers of math concepts MPNC1, MPNC2, respectively.
                If (CheckVerbModifier(VM1, VM2) == “same”) // VM1, VM2: modifiers of cognitive verbs.
                    If (|MPNC1| == |MPNC2|) and (|VP1| == |VP2|) // no. of math concepts, no. of cognitive verbs
                        “Strongly Fully-aligned”
                    else “Weakly Fully-aligned”;
            } else “Weakly Fully-aligned”;
        else if (CheckMathCPMod(M1, M2) == “different” and “only one math standard has a modifier of math concepts”)  
            “Weakly Fully-aligned***”;
    else if (CheckMathConcept(3, MPNC1, MPNC2) == “related”)
        if (CheckVerbPhrase(2, VP1, VP2) == “same cognitive process”)  
            “Partially-aligned***”
        else “Partially-aligned**”;
    else if (CheckMathConcept(3, MPNC1, MPNC2) == “related”)  
        if (CheckVerbPhrase(2, VP1, VP2) == “same cognitive process”)  
            “Poorly-aligned”
        else “Not-aligned”;
}

Figure 4.4.24: An algorithm for alignment of two math educational standards statements

Function CheckMathConcept( type, MP1, MP2)
Steps:
// This function is to check multiple math concepts are the same, related, or different . ;
1. if (type = 1 and MultiWordMatched(1, MP1, MP2))
    Return “same” // MP1 and MP2 are the same math concepts.
2. else if (type = 2 and MultiWordMatched(2, MP1, MP2))
    Return “related” // MP1 and MP2 are related math concepts.
3. else Return “different”

Function CheckVerbPhrase (type, VP1, VP2)
// This function is to check whether cognitive verbs are the same, in the same cognitive process, or different
// cognitive process.
Steps:
1. if (type = 1 and MultiWordMatched(1, VP1, VP2))
   Return “same cognitive verb”
2. else if (type = 2 and MultiWordMatched(2, VP1, VP2))
   Return “same cognitive process” // VP1 & VP2 belong to the same cognitive process.
3. else Return “different cognitive process”; // VP1 & VP2 belong to the different cognitive process

Function CheckAttributes(Attr1, Attr2)
// Attr1 & Attr2 are attributes of math concepts (MP nouns)
// This function is to check attributes are the same or different.
   if (type = 1 and MultiMatched(1, Attr1, Attr2))
      return “same” // Attr1 & Attr2 are the same.
   else “different”

Function CheckMathCPMod (MPMOD1, MPMOD2)
// MPMOD1 & MPMOD2 are modifiers of math concepts in each MP statement (math educational standards // statement. MPMOD1 & MPMOD2 are subgraphs of noun phrase graphs NG1, NG2
Steps:
1. if (MPMOD1 and MPMOD2 are not isomorphic) // Their length are not the same.
   Return “different”
2. for each (node n1 in MPMOD1)
   {
       n2 = the node of MPMOD2 which is matched to n1 according to the isomorphism
       if (n1 and n2 have difference labels each other)
           Return “different”
   }
3. for each (edge e1 in MPMOD1)
   {
       e2 = the edge of MPMOD2 which is matched to e1 according to the isomorphism
       if (e1 and e2 have difference labels each other)
           return “different”
   }
4. return “same”

Function CheckVerbModifier(VM1, VM2) {
// VG1 & VG2 are verb phrases graphs of each MP statement.
// Modifiers of cognitive verbs VM1, VM2 are subgraphs of VG1, VG2
// This function is to check modifiers of cognitive verbs are the same.
Step:
1. if (VM1 and VM2 are not isomorphic) // Their length are not the same.
   return “different”
2. for each (node n1 in VM1)
   {
       n2 = the node of VM2 which is matched to n1 according to the isomorphism
       if (n1 and n2 have difference labels each other)
   }
return "different"
}
3. for each (edge e1 in VM1)
{
    e2 = the edge of VM2 which is matched to e1 according to the isomorphism
    if (e1 and e2 have difference labels each other)
    return "different"
}
4. return "same"

4.5 Experimental Evaluation

Although there is no specific unified way to evaluate computational measures of alignments for math educational standards, one reasonable way to evaluate it would be agreement with a human judgment. This can be assessed by using computational measures of alignments to compare with a domain expert’s judgment, and looking at how well computational measures of alignments are comparable to a domain expert’s judgment.

4.5.1 Experimental Settings

Using our alignment tool “MPComp” we align 33 pairs of math educational standard statements from Nevada and Idaho, respectively with threshold values 0.99, 0.95, 0.93, and 0.90 for setting up a threshold value for an equivalent meaning of two words. For an evaluation of our alignment method, we extract 122 pairs from Nevada and Idaho, respectively, and 80 pairs from Ohio and Texas, respectively. Their math educational standards have five subcategories such as 1) Numbers, Number Sense, and Computation, 2) Patterns, Functions, and Algebra, 3) Measurement, 4) Geometry, and 5) Data Analysis. Each pair has been extracted from the same subcategory for alignment. A research question, “Will our alignment method provide results that are comparable to human judgment?” has been tested. We have chosen a domain expert’s judgment as a
gold standard. In a math educational standards domain, a math teacher can be considered as a domain expert. A person who has been a math teacher more than 20 years in middle or high schools with a master’s degree in mathematics has been chosen as a domain expert. We compare our results from our alignment method to a domain expert’s judgment. Data sets for these testing have been attached as Appendix 1 and Appendix 2.

4.5.2 Evaluation Methodology

We compare our results from our alignment method to a domain expert’s judgment using Cohen’s kappa (Cohen 1960; Cohen et al. 1968). Precision, recall, and F-measure also have been measured as evaluation metrics for measuring correctness of different degrees of alignment.

4.5.2.1 Cohen’s kappa

The Cohen’s kappa coefficient is used to assess a statistical measure of inter-rater agreement for categorical (qualitative) variables. Cohen’s kappa measures the agreement between two raters. Each rater classifies items into categories which are mutually exclusive. The formula for the Cohen’s kappa coefficient ($\kappa$) is:

$$\kappa = \frac{Pr(a) - Pr(e)}{1 - Pr(e)} , \quad (4.5.1)$$

Where $Pr(a)$ is the observed percentage agreement among raters,

$Pr(e)$ is the probability of random agreement among raters,

$\kappa < = 1$ (A value of 1 implies perfect agreement).

Fleiss’s guidelines (Fleiss 1981) characterize kappa as follows:

- Excellent agreement = over .75
• Fair to good agreement = .40 to .75
• Poor agreement = below .40

Altman (1990) also interprets kappa as follows:
• Poor agreement: less than 0.20
• Fair agreement: 0.20 to 0.40
• Moderate agreement: 0.40 to 0.60
• Good agreement: 0.60 to 0.80
• Very good agreement: 0.80 to 1.

4.5.2.2 Precision, Recall, and F-measure

As evaluation metrics for measuring correctness of each category, precision, recall, and F-measure have been measured based on the following formulas:

1. Precision = no. of answers that are correctly labeled in each category by our alignment method / no. of answers labeled in each category by our alignment method.

2. Recall = no. of answers that are correctly labeled in each category by our alignment method / no. of answers that should be labeled in each category

3. F-measure = 2*Precision*Recall / (Precision + Recall)

4.5.3 Evaluation results and discussions

In order to examine agreements between results from our algorithm and results from a domain expert, SPSS with Cohen’s kappa option has been used. We also measure precision, recall, and F-measure for measuring correctness of different degrees of alignments.
4.5.3.1 A threshold value for word similarity

A threshold value for equivalent meaning of words (math concepts, attributes of math concepts, or cognitive verbs) has been set up as 0.95 through testing. We tested 33 pairs of math educational standards statements from Nevada and Idaho, respectively with threshold values 0.99, 0.95, 0.93, and 0.90. See Section 4.4.3.3 for test results in detail.

4.5.3.2 Testing results for alignments

Our different degrees of alignments are Strongly Fully-aligned (SFA), Weakly Fully-aligned (WFA), Partially-aligned*** (PA***), Partially-aligned** (PA**), Partially-aligned* (PA*), Poorly aligned(PR), and Not aligned(NA).

1) Cohen’s kappa:

- Testing for 122 pairs of math educational standards statement from Nevada and Idaho, respectively. See Tables 4.5.1, 4.5.2, and 4.5.3 for results.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Valid</th>
<th>Missing</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>Percent</td>
<td>N</td>
</tr>
<tr>
<td>MyAlgorithm *</td>
<td>122</td>
<td>100.0%</td>
<td>0</td>
</tr>
<tr>
<td>DomainExpert</td>
<td></td>
<td></td>
<td>122</td>
</tr>
</tbody>
</table>
From our above output the result of interrater analysis is $\kappa = 0.671$ with $p = .000 < 0.001$. The result shows that Cohen’s kappa coefficient, $\kappa = 0.671$ and p value which measures statistical significance is .000. According to p value, this measure of agreement is statistically significant but it is reported that in general statistical significance for kappa is not a useful guide. We have a value of kappa as 0.671 which falls into the categories “fair to good” and “good” based on Fleiss and Altman,
respectively. Therefore our alignment method is comparable to a domain expert’s judgment.

- Testing for 80 pairs of math educational standards statement from Ohio and Texas, respectively. See Tables 4.5.4, 4.5.5, and 4.5.6 for results.

**Table 4.5.4: Case Processing Summary Test 2**

<table>
<thead>
<tr>
<th>Cases</th>
<th>Valid</th>
<th>Missing</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>Percent</td>
<td>N</td>
</tr>
<tr>
<td>MyAlgorithm * DomainExpert</td>
<td>80</td>
<td>100.0%</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 4.5.5: MyAlgorithm * DomainExpert Cross Tabulation Test 2**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>DomainExpert</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NA</td>
<td>PA*</td>
</tr>
<tr>
<td>MyAlgorithm</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>PA*</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>PA**</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>PA***</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PR</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SFA</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>WFA</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>
From a Table 4.5.6, the result of interrater analysis is kappa = 0.711 with p = .000 < 0.001. This kappa value 0.711 also falls into the categories “fair to good” and “good” according to Fleiss’ (1981) and Altman’s (1990) guidelines, respectively. Therefore our alignment method is comparable to a domain expert’s judgment.

2) Precision, Recall, and F-measure:

- See Table 4.5.7 for precision, recall, and F-measure for 122 pairs of math educational standard statements from Nevada and Idaho, respectively.

<table>
<thead>
<tr>
<th>Measure of Agreement kappa</th>
<th>Value</th>
<th>Asymp Std. Errora</th>
<th>Approx. Tb</th>
<th>Approx. Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N of valid cases</td>
<td>.711</td>
<td>.057</td>
<td>14.160</td>
<td>.000</td>
</tr>
</tbody>
</table>

a. Not assuming the null hypothesis
b. Using the asymptotic standard error assuming the null hypothesis

Table 4.5.7: Precision, recall, and F-measure for different degrees of alignments for test 1

<table>
<thead>
<tr>
<th>Degree of Alignment</th>
<th>Precision</th>
<th>Recall</th>
<th>F-measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strongly Fully-aligned (SFA)</td>
<td>100</td>
<td>54.55</td>
<td>70.59</td>
</tr>
<tr>
<td>Weakly Fully-aligned (WFA)</td>
<td>84.21</td>
<td>61.54</td>
<td>71.53</td>
</tr>
<tr>
<td>Partially-aligned*** (PA*** )</td>
<td>55</td>
<td>91.67</td>
<td>68.75</td>
</tr>
<tr>
<td>Partially-aligned** (PA**)</td>
<td>77.78</td>
<td>77.78</td>
<td>77.78</td>
</tr>
<tr>
<td>Partially-aligned* (PA*)</td>
<td>73.33</td>
<td>78.57</td>
<td>75.86</td>
</tr>
<tr>
<td>Poorly aligned (PR)</td>
<td>84</td>
<td>80.77</td>
<td>82.35</td>
</tr>
<tr>
<td>Not-aligned (NA)</td>
<td>47.37</td>
<td>60</td>
<td>52.94</td>
</tr>
</tbody>
</table>
We have extremely high precision in “Strongly Fully-aligned”, high precision in “Poorly-aligned” and “Weakly-Fully-aligned”, and low precision in “Not-aligned”.

- See Table 4.5.8 for precision, recall, and F-measure for 80 pairs of math educational standard statements from Ohio and Texas, respectively.

**Table 4.5.8: Precision, recall, and F-measure for different degrees of alignments for test 2**

<table>
<thead>
<tr>
<th>Alignment Type</th>
<th>Precision</th>
<th>Recall</th>
<th>F-measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strongly Fully-aligned (SFA)</td>
<td>50</td>
<td>16.67</td>
<td>25</td>
</tr>
<tr>
<td>Weakly Fully-aligned (WFA)</td>
<td>88.89</td>
<td>66.67</td>
<td>76.19</td>
</tr>
<tr>
<td>Partially-aligned*** (PA***)</td>
<td>75</td>
<td>92.31</td>
<td>82.76</td>
</tr>
<tr>
<td>Partially-aligned**(PA**)</td>
<td>58.33</td>
<td>87.5</td>
<td>70</td>
</tr>
<tr>
<td>Partially-aligned* (PA*)</td>
<td>92.85</td>
<td>81.25</td>
<td>86.66</td>
</tr>
<tr>
<td>Poorly-aligned (PR)</td>
<td>71.43</td>
<td>90.91</td>
<td>80</td>
</tr>
<tr>
<td>Not-aligned (NA)</td>
<td>50</td>
<td>100</td>
<td>66.67</td>
</tr>
</tbody>
</table>

We have high precision in “Weakly Fully-aligned” and “Partially-aligned*”, and low precision in “Not-aligned”.

3) Discussions:

No similar alignment methods are available, neither are alignment tools for a math educational standard domain. Therefore, we have compared results from our alignment method to a domain expert’s judgment. Our experiments have shown good levels of agreement, where kappa values are .696, .671, and .711 with a threshold value 0.95 for word similarity. All of these results fall into the “good” category based on Altman’s guidelines (1990). Therefore, our alignment method produces results that are comparable to a domain expert’s judgment. We have found that our alignment method was unable to
measure alignment of math standard statements correctly when different concepts have been used for the same meaning in a context or when concepts of words should be interpreted in a context. The following are examples.

1) Use money notation to add and subtract given monetary amounts.
   Add and subtract decimals using money.

2) Read time to the nearest hour.
   Tell time to the hour.

3) Identify and use place value positions of whole numbers and decimals to hundreds.
   Identify and apply place value positions of whole numbers and decimals to thousandths.

In the first example, a result of a domain expert’s judgment is Strongly Fully-aligned but our alignment method produced Not-aligned as an output because “monetary amount” and “decimal” are different concepts but a domain expert interprets them as the same meaning in a sentence. In the second example, from a domain expert a result is “Strongly Fully-aligned” and from our alignment method a result is “Partially-aligned***” because “read” and “tell” are different concept but a domain expert interprets them as the same meaning in a sentence.

In the third example a domain expert’s judgment is “Weakly Fully-aligned” but an output of our alignment method is “Partially-aligned***”. A domain expert interprets that the meaning of hundredth has been included in the meaning of thousandth in a sentence. Our alignment method interprets hundredth and thousandth with different meanings. In these three examples, the limitation of our
alignment method was unable to measure semantic similarities between words in math educational standard statements. This limitation has resulted from a syntactic-based method for measuring word similarity in math educational standards. Therefore, for our future work we need to develop a semantic-based method for measuring word similarity in math educational standards. We should utilize it with a syntactic-based method for measuring word similarity in math educational standards for our alignment method.
CHAPTER 5: CONCLUSION and FUTURE WORK

We have presented a semi-automatic alignment method for math educational standards. Our alignment method utilized the MP modeling, and graph matching with Bloom taxonomy (Bloom & Krathol 1956) for cognitive verb categorization, the WordNet (Fellbaum 1999) for word similarity, and taxonomies of math concepts for related math concepts. This alignment method has extended the notion of alignment for math educational standards by giving different degrees of alignments such as Strongly Fully-aligned or Weakly Fully-aligned, Partially-aligned*, Partially-aligned**, Partially-aligned***, Poorly-aligned, and Not-aligned. Different degrees of alignments have provided consistency in interpreting a correct alignment and also empowered educational professionals by broadening categories of search or retrieval for educational resources assigned with math educational standards.

We also have proposed the MP model for modeling math educational standards statements. The MP model has been developed at a sentence level for each statement from typical math educational standards. This MP model can explicitly model the semantics of imperative mood sentence structures used in math standards. Our sentence analysis is based on the Reed-Kellogg sentence diagram (Reed & Kellogg 2004). Our MP model has facilitated alignments of math educational standards by capturing math concepts and the cognitive process of math concepts from math educational standards statements. With modification, our approach can be utilized for modeling other educational standards which have imperative mood sentence structure.
We also developed a semi-automatic tool “MPViz” for creating the MP model. And we developed a semi-automatic tool “MPComp” for alignment of math educational standards. The MPViz is embedded in the MPComp.

We compared the results from our alignment method to a domain expert’s judgment using Cohen’s kappa (Cohen 1960; Cohen et al. 1968). Precision, recall, and F-measure also have been measured as evaluation metrics for measuring correctness of different degrees of alignment. Our two sets of experiments have been performed for 122 pairs of math educational standard statements from Nevada and Idaho, and 80 pairs from Ohio and Texas. They have shown good levels of agreement, where kappa values are .671 and .711. Therefore, our experiments showed that our alignment method has provided results that are comparable to a domain expert’s judgment.

In the future, we plan to develop an automatic alignment method for math educational standards. In order to develop an automatic alignment method, we will develop a parsing algorithm for capturing math concepts (MP noun), modifiers of math concepts (MP noun modifiers), cognitive verbs (MP verbs), and modifiers of cognitive verbs (MP verb modifiers) automatically based on the Reed-Kellogg sentence diagram from math educational standard statements. We also plan to develop a web-based alignment system which uses our alignment method and extends our alignment method to other educational standards with modification. For future work we also need to develop a semantic-based method for measuring word similarity in math educational standards and utilize it with a current method for measuring word similarity for our alignment method.

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SPSS Tutorials for Statistical Data Analysis


Appendix A – Test data A

- Test data A: 35 pairs of math educational standards statements from Nevada and Idaho states, respectively are extracted and tested for different thresholds (0.95, 0.90, 0.85, and 0.80) for word similarities.

1) Identify, use, and model place value positions of 1’s, 10’s, 100’s, and 1000’s.
   Identify place value through 9,999.

2) Identify and use place value positions of whole numbers to one million.
   Identify and apply place value in whole numbers.

3) Read and write unit fractions with numbers and words.
   Read, write, compare, and order commonly used fractions with pictorial representations.

4) Identify fractions and compare fractions with like denominators using models, drawings, and numbers.
   Compare and order commonly used fractions and their equivalents.

5) Add and subtract fractions with like denominators using models, drawings, and numbers.
   Add and subtract fractions with like denominators without simplification.

6) Add and subtract fractions with unlike denominators.
   Add, subtract, multiply, and divide rational numbers.

7) Compare fractions with unlike denominators using models and by finding common denominators.
Compare and order commonly used fractions and their equivalents

8) Write a number word.
   Write a number sentence from addition or subtraction problem-solving solution.

9) Compare and order real numbers, including powers of whole numbers in
   mathematical and practical situations.
   Compare magnitudes and relative magnitudes of rational numbers, including
   Integer, fractions, and decimals.

10) Add and subtract decimals using money as a model.
    Add and subtract whole numbers.

11) Add and subtract multi-digit numbers.
    Add and subtract whole numbers.

12) Generate and solve one-step addition and subtraction problems based on practical
    situations.
    Choose addition or subtraction to solve word problems and explain the choice.

13) Add and subtract decimals.
    Add and subtract whole numbers with and without regrouping through 999.

14) Identify, describe, and represent patterns and relationships in the number system,
    including arithmetic and geometric sequences.
    Translate a repeating pattern from one representation to another.

15) Solve and graphically represent equations and simple inequalities in one variable.
    Solve one- and two-step equations and inequalities.

16) Evaluate formulas and algebraic expressions for given integer values.
    Simplify algebraic expressions.
17) Simplify algebraic expressions by combining like terms.

   Simplify algebraic expressions.

18) Model and solve equations using concrete and visual representations.

   Solve one-step equations.

19) Solve systems of two linear equations algebraically and verify solutions.

   Use appropriate procedures to solve linear systems of equations involving two
   variables.

20) Compare, order, and describe objects by size.

   Compare the lengths or sizes of objects.

21) Identify and sort pennies, nickels, and dimes.

   Identify a penny as a value of money.

22) Use decimals to show money amounts.

   Use decimal numbers with money.

23) Use a calendar to identify days, weeks, months, and a year.

   Name the day of the week and the day’s date using a calendar.

24) Name, sort, and sketch two-dimensional shapes (circles, triangles, rectangles
   including squares) regardless of orientation.

   Recognize, name, build, draw, and sort two- and three-dimensional shapes (triangle,
   rectangle, square, circle, cone, cube, cylinder).

25) Identify congruent and similar shapes (circles, triangles, and rectangles including
   square).

   Identify shapes as congruent, similar, or symmetrical.

26) Identify and copy two-dimensional designs that contain a line of symmetry.
Identify multiple lines of symmetry in two-dimensional shapes.

27) Construct geometric figures using a variety of tools.

   Recognize congruency and similarity of two-dimensional figures.

28) Collect, organize, and record data in response to questions pose by teacher and/or students.

   Collect, organize, and display data in tables, charts, or bar graphs in order to answer a question.

29) Collect, record, and classify data in response to questions pose by teacher and/or students.

   Collect, organize, and display data in tables, charts, or bar graphs in order to answer a question.

30) Organize, display, and read data using the appropriate graphical representations
    (with and without technology)

   Collect, organize, and display the data with appropriate notation in tables, charts, bar graphs, and line graphs.

31) Recite in order the months of the year.

   Recite the months of the year, in order.

32) Use concrete objects to model simple addition and subtraction.

   Add three one-digit addends.

33) Compare and measure length and weight using non-standard measurement.

   Estimate measurement using non-standard.
Appendix B – Test data B

- Test data 1: 122 pairs of math educational standards statements from Nevada and Idaho states, respectively are extracted and tested.

1) Identify, use, and model place value positions of 1’s, 10’s, 100’s, and 1000’s.
   Identify place value through 9,999.

2) Identify and use place value positions of whole numbers to one million.
   Identify and apply place value in whole numbers.

3) Read and write unit fractions with numbers and words.
   Read, write, compare, and order commonly used fractions with pictorial representations.

4) Identify fractions and compare fractions with like denominators using models, drawings, and numbers.
   Compare and order commonly used fractions and their equivalents.

5) Identify and use place value positions of whole numbers and decimals to hundredths.
   Identify and apply place value in whole numbers and decimal numbers to thousandths.

6) Identify and use place value positions to thousandths.
   Identify and apply place value in whole numbers and decimal numbers to thousandths.

7) Add and subtract fractions with like denominators using models, drawings, and numbers.
   Add and subtract decimal numbers through thousandths.
8) Add and subtract fractions with like denominators using models, drawings, and numbers.
Add and subtract fractions with like denominators without simplification.

9) Add and subtract fractions with unlike denominators.
Add, subtract, multiply, and divide rational numbers.

10) Compare fractions with unlike denominators using models and by finding common denominators.
Compare and order commonly used fractions and their equivalents.

11) Use models to translate among fractions, decimals, and percents.
Locate the position of rational numbers on a number line.

12) Translate among fractions, decimals, and percents, including fractional percents.
Convert between decimals and fractions.

13) Explain and use the relationship among equivalent representations of rational numbers in mathematical and practical situations.
Explain the interrelationship of fractions, decimals, and percents.

14) Read, write, compare, and order numbers from 0 – 100.
Read, write, compare, and order whole numbers to 100.

15) Read, write, compare, and order numbers from 0 - 999.
Read, write, compare, and order whole numbers to 1,000.

16) Read, write, compare, and order whole numbers.
Read, write, compare, and order whole numbers to 1,000.

17) Write a number word.
Write a number sentence from addition or subtraction problem-solving solution.
18) Count by multiples of a given number.
    Find the Least Common Multiple and the Greatest Common Divisor.

19) Compare and order a combination of rational numbers including fractions, decimals, percents, and integers in mathematical and practical situations.
    Compare magnitudes and relative magnitudes of rational numbers, including integers, fractions, and decimals.

20) Compare and order real numbers, including powers of whole numbers in mathematical and practical situations.
    Compare magnitudes and relative magnitudes of rational numbers, including integers, fractions, and decimals.

21) Use concrete objects to model simple addition and subtraction.
    Use concrete objects to illustrate the concepts of addition and subtraction.

22) Use concrete objects to model simple addition and subtraction.
    Add three one-digit addends.

23) Identify and model basic addition facts (sums to 18) and the corresponding subtraction facts.
    Use strategies for addition and subtraction combinations through 18.

24) Immediately recall basic addition facts (sums to 18) and the corresponding subtractions facts.
    Recall basic addition and subtraction facts through 18.

25) Immediately recall addition and subtractions facts
    Recall basic multiplication and division facts up to 10’s.

26) Immediately recall multiplication facts (products to 81).
Recall multiplication facts through 10 x 10.

27) Immediately recall and use multiplication and corresponding division facts (products to 144).
   Recall basic multiplication and division facts up to 10’s.

28) Estimate the number of objects in a set using various techniques.
   Use estimation to identify a number of objects.

29) Identify equivalent expressions between and among fractions, decimals, and percents.
   Recall the common equivalent fractions, decimals, and percents of halves, thirds, fourths, fifths, and tenths.

30) Identify absolute values of integers.
   Describe the use of integers in real-world situations.

31) Determine an approximate value of radical and exponential expressions using a variety of methods.
   Evaluate numerical expressions with rational numbers using the order of operations.

32) Add and subtract one- and two-digit numbers without regrouping.
   Add and subtract whole numbers with and without regrouping through 999.

33) Add and subtract two- and three-digit numbers without regrouping.
   Add and subtract whole numbers with and without regrouping through 999.

34) Add and subtract decimals using money as a model.
   Add and subtract decimals using money.

35) Add and subtract decimals using money as a model.
Add and subtract whole numbers.

36) Add and subtract multi-digit numbers.
Add and subtract whole numbers.

37) Use mathematical vocabulary and symbols to describe addition, subtraction, and equality.
Use concrete objects to illustrate the concepts of addition and subtraction.

38) Generate and solve one-step addition and subtraction problems based on practical situations.
Choose addition or subtraction to solve word problems and explain the choice.

39) Generate and solve addition, subtraction, multiplication, and division problems using whole numbers in practical situations.
Use the order of operations and perform operations with rational number.

40) Add and subtract decimals.
Add and subtract whole numbers with and without regrouping through 999.

41) Add and subtract decimals
Add and subtract decimal numbers through thousandths.

42) Use the concepts of number theory, including prime and composite numbers, factors, multiples, and the rules of divisibility to solve problems.
Apply the number theory concepts of primes, composites, and prime factorization.

43) Recognize, describe, label, extend, and create simple repeating patterns using symbols, objects, and manipulative.
Replicate and extend simple repeating patterns.

44) Recognize, describe, label, extend, and create simple repeating patterns using
symbols, objects, and manipulative.

Describe and extend a repeating pattern

45) Recognize, describe, extend, and create repeating and increasing patterns using symbols, objects, and manipulative.

Describe and extend patterns by using manipulative and pictorial representations.

46) Use patterns and their extensions to solve problems.

Use patterns to represent problems.

47) Recognize, describe, and create patterns using objects and numbers found in tables, number chars, and charts.

Describe and extend patterns by using manipulative and pictorial representations.

48) Identify, describe, and represent patterns and relationships in the number system, including arithmetic and geometric sequences.

Translate a repeating pattern from one representation to another.

49) Use variables and open sentences to express relationships.

Use symbols “<,” “>,” “≤,” “≥,” “≠,” “=” and “≥” to express relationships.

50) Evaluate formulas and algebraic expressions using whole number values.

Evaluate simple algebraic expressions using substitution.

51) Solve and graphically represent equations and simple inequalities in one variable.

Solve one- and two-step equations and inequalities.

52) Evaluate formulas and algebraic expressions for given integer values.

Simplify algebraic expressions.

53) Create, compare, describe sets of objects as greater than, less than, or equal to.

Compare numbers to 99 using vocabulary (less than, greater than, equal to, more,
less, same, fewer).

54) Complete number sentences with the appropriate words and symbols (+, -, >, <, =).
   Write a number sentence using simple geometric shapes as symbols to represent an unknown number.

55) Simplify algebraic expressions by combining like terms.
   Simplify algebraic expressions.

56) Identify, model, describe, and evaluate functions.
   Given a function, identify domain and range.

57) Solve linear equations and represent the solution graphically.
   Match graphical representations with simple linear equations.

58) Model and solve equations using concrete and visual representations.
   Solve one-step equations.

59) Solve systems of two linear equations algebraically and verify solutions.
   Use appropriate procedures to solve linear systems of equations involving two variables.

60) Compare, order, and describe objects by size.
   Compare the lengths or sizes of objects.

61) Compare, order, describe, and represent objects by length and weight.
   Compare the lengths or sizes of objects.

62) Estimate and convert units of measure for length, area, and weight within the same measurement system (customary and metric).
   Convert units of measurement within each system in one-step problems.
63) Estimate and convert units of measure for length, area, and weight within the same measurement system (customary and metric).

Convert units of measurement within each system.

64) Compare and measure length and weight using non-standard measurement.

Estimate measurement using non-standard units.

65) Compare and measure length and weight using non-standard measurement.

Use non-standard tools and units for measuring length, volume (capacity), and weight.

66) Select and use appropriate units of measure.

Determine and use appropriate units.

67) Select and use appropriate units of measure.

Select and use appropriate units and tools to make formal measurements of length and temperature in both systems.

68) Convert and estimate units of measure for mass and capacity within the same measurement system (customary and metric).

Convert units of length within each system.

69) Convert and estimate between customary and metric system.

Convert units of measurement within each system.

70) Measure volume and weight to a required degree of accuracy and metric system.

Identify relationships of length and time within the U.S. customary system and within the metric system.

71) Given a measurement, identify the greatest possible error.

Approximate error in measurement situations.
72) Define and determine the perimeter of polygons and the area of rectangles, including squares.

   Calculate the perimeter of polygons and the area of rectangles and squares.

73) Identify and sort pennies, nickels, and dimes.

   Identify each and state the value of pennies, nickels, and dimes.

74) Identify and sort pennies, nickels, and dimes.

   Identify a penny as a value of money.

75) Determine the value of any set of pennies, nickels, and dimes.

   Identify each and state the value of pennies, nickels, and dimes.

76) Determine the value of any given set of coins.

   Count the value of a collection of pennies, nickels, dimes, and quarters up to $1.00.

77) Use decimals to show money amounts.

   Use decimal numbers with money.

78) Determine possible combinations of coins and bills to equal given amounts.

   Count the value of a collection of bills and coins up to $100.00.

79) Read, write, and use money notation.

   Count the value of a collection of bills and coins up to $100.00.

80) Use money notation to add and subtract given monetary amounts.

   Add and subtract decimals using money.

81) Calculate percents in monetary problems.

   Use decimal numbers with money.

82) Recite in order the days of the week.
Name the day of the week and the day’s date using a calendar.

83) Recite in order the months of the year.
Recite the months of the year, in order.

84) Use a calendar to identify days, weeks, months, and a year.
Name the day of the week and the day’s date using a calendar.

85) Read time to the nearest hour.
Tell time to the hour.

86) Tell time to the nearest minute, using analog and digital clocks.
Tell time to the nearest minute using digital and analog clocks.

87) Write and apply ratios in mathematical and practical problems involving measurement and monetary conversions.
Identify and write ratios and scales.

88) Identify two-dimensional shapes (circles, triangles, rectangles including squares) regardless of orientation.
Identify shapes as congruent, similar, or symmetrical.

89) Name, sort, and sketch two-dimensional shapes (circles, triangles, rectangles including squares) regardless of orientation.
Recognize, name, build, draw, and sort two- and three-dimensional shapes (triangle, rectangle, square, circle, cone, cube, cylinder).

90) Demonstrate an understanding of relative position words, including before/after, far/near, and over/under, to place objects.
Describe the location of an object relative to another (e.g., next to, under, over, behind).
91) Identify congruent and similar shapes (circles, triangles, and rectangles including square).
   Recognize congruency and similarity of two-dimensional figures.

92) Identify congruent and similar shapes (circles, triangles, and rectangles including square).
   Identify shapes as congruent, similar, or symmetrical.

93) Identify, draw, and classify angles, including straight, right, obtuse, and acute.
   Classify angles without formal measures as acute, right, obtuse, and/or straight.

94) Demonstrate and describe the transformational motions of geometric figures (translation/slide, reflection/flip, and rotation/turn).
   Discuss and predict the results of sliding, flipping, and turning two-dimensional shapes.

95) Demonstrate and describe the transformational motions of geometric figures (translation/slide, reflection/flip, and rotation/turn).
   Describe and classify relationships among types of one-, two-, and three-dimensional geometric figures, using their defining properties.

96) Identify, classify, compare, and draw triangles and quadrilaterals based on their properties.
   Discuss perimeters of polygons, and areas and perimeters of rectangles and squares, using concrete objects.

97) Identify, classify, compare, and draw regular and irregular quadrilaterals.
Discuss perimeters of polygons, and areas and perimeters of rectangles and squares, using concrete objects.

98) Identify, classify, compare, and draw regular and irregular polygons. Recognize congruency and similarity of two-dimensional figures.

99) Identify two dimensional figures as they appear in the environment. Recognize congruency and similarity of two-dimensional figures.

100) Identify and copy two-dimensional designs that contain a line of symmetry. Identify multiple lines of symmetry in two-dimensional shapes.

101) Demonstrate translation, reflection, and rotation using coordinate geometry and models. Describe reflections, translations, and rotations on various shapes.

102) Demonstrate dilation using coordinate geometry and models. Describe reflections, translations, and rotations on various shapes.

103) Identify, draw, label, and describe points, line segments, rays, and angles. Identify and label points, lines, line segments, rays, and angles.

104) Construct geometric figures using a variety of tools. Recognize congruency and similarity of two-dimensional figures.

105) Collect, organize, and record data in response to questions pose by teacher and/or students. Collect, organize, and display data in tables, charts, or bar graphs in order to answer a question.

106) Use tally marks to represent data. Use tally marks to represent data.
107) Collect, record, and classify data in response to questions posed by teacher and/or students.

Collect, organize, and display data in tables, charts, or bar graphs in order to answer a question.

108) Use tables, pictographs, and bar graphs to represent data.

Collect, organize, and display the data with appropriate notation in tables, charts, bar graphs, and line graphs.

109) Model and compute range.

Find the range of a set of data using whole numbers.

110) Model the measures of central tendency for mode and median.

Find measures of central tendency - median and mode - with simple sets of data using whole numbers.

111) Organize, display, and read data using the appropriate graphical representations (with and without technology)

Collect, organize, and display the data with appropriate notation in tables, charts, bar graphs, and line graphs.

112) Organize statistical data through the use of tables, graphs, and matrices.

Collect, organize, and display data in tables, charts, and graphs.

113) Compute range.

Calculate the range of a set of data.

114) Model and compute the measures of central tendency for mean, median, and mode.

Find measures of central tendency – mean, median, and mode – with simple sets
of data.

115) Model and compute the measures of central tendency for mean, median, and model.
    Determine the measures of central tendency – mean, median and mode – with sets of data.

116) Select and apply the measures of central tendency to describe data.
    Choose and calculate the appropriate measure of central tendency – mean, median, and mode.

117) Conduct simple probability experiments using concrete materials.
    Predict the results of simple probability experiments using coins or spinners.

118) Represent the results of simple probability experiments as fractions to make predictions about future events.
    Predict, perform, and record results of simple probability experiments using fraction notation.

119) Conduct simple probability experiments using concrete materials.
    Predict the results of simple probability experiments using coins or spinners.

120) Represent the results of simple probability experiments as decimals to make predictions about future events.
    Model situations of probability using simulations.

121) Find experimental probability using concrete materials.
    Predict, perform, and record results of simple probability experiments using fraction notation.
122) Represent the results of simple probability experiments as fractions, decimals, percents, and ratios to make predictions about future events. Predict, perform, and record results of simple probability experiments using fraction notation.

- Test data 2: 80 pairs of math educational standards statements from Ohio and Texas states, respectively are extracted and tested.

1. Recognize, classify, compare and order whole numbers. (Ohio)
   Compare and order whole numbers up to 99 using sets of concrete objects and pictorial models. (Texas)

2. Identify and state the value of a penny, nickel, and dime. (Ohio)
   Use words and numbers to describe the values of individual coins such as penny, nickel, dime, and quarter and their relationships. (Texas)

3. Determine the value of a collection of coins and dollar bills. (Ohio)
   Determine the value of collection of coins less than one dollar. (Texas)

4. Determine the value of a collection of coins and dollar bills. (Ohio)
   Determine the value of a collection of coins and bills. (Texas)

5. Use place value structure of the base-ten number system to read, write, represent and compare whole number and decimals. (Ohio)
   Use place value to read, write (in symbols and words), and describe the value of whole numbers through 999,999. (Texas)

6. Use place value concepts to represent, compare, and order whole numbers using numerals, words and physical models. (Ohio)
Use place value to compare and order whole numbers through 9,999. (Texas)

7. Compare and order whole number up to 10. (Ohio)
   Compare and order whole numbers using place value. (Texas)

8. Represent fractions using words, numeral, and physical models. (Ohio)
   Construct concrete models of fractions. (Texas)

9. Identify and illustrate parts of a whole and parts of sets of objects. (Ohio)
   Compare fractional parts of whole objects or sets of objects in a problem situation using concrete models. (Texas)

10. Model and represent addition as combining sets and counting on, and subtraction as take-away and comparison. (Ohio)
    Model addition and subtraction using pictures, words, and numbers. (Texas)

11. Demonstrate fluency in multiplication facts through 10 and corresponding division facts. (Ohio)
    Learn and apply multiplication facts through the tens using concrete models. (Texas)

12. Round whole numbers to a given place value. (Ohio)
    Round two-digit numbers to the nearest ten and three-digit numbers to the nearest hundred. (Texas)

13. Develop strategies for basic addition facts. (Ohio)
    Learn and apply basic addition facts (sums to 18) using concrete models. (Texas)

14. Compare, order, and convert among fractions, decimals, and percents. (Ohio)
    Compare and order common fractions and decimals. (Texas)

15. Recognize and classify numbers as prime or composite and list factors. (Ohio)
Identify prime and composite numbers using concrete models and patterns in factor pairs. (Texas)

16. Add and subtract whole numbers with and without regrouping. (Ohio)

Add and subtract decimal to the hundredths place using concrete and pictorial models. (Texas)

17. Use place value structure of the base-ten number system to read, write, represent and compare whole numbers through millions and decimals through thousandths. (Ohio)

Use place value to read, write, compare, and order whole numbers through the millions place. (Texas)

18. Round whole numbers to a given place value. (Ohio)

Round whole numbers and decimals through tenths to approximate reasonable results in problem situation. (Texas)

19. Compare and order whole number up to 10. (Ohio)

Compare and order fractions using concrete and pictorial models. (Texas)

20. Demonstrate fluency in multiplication facts through 10 and corresponding division facts. (Ohio)

Recall and apply multiplication facts through 12X12. (Texas)

21. Compare, order, and covert among fractions, decimals, and percents. (Ohio)

Convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator. (Texas)

22. Compare, order, and determine equivalent forms of real numbers. (Ohio)
Generate equivalent forms of rational numbers including whole numbers, fractions, and decimals. (Texas)

23. Use the prime factorization to recognize the least common multiple. (Ohio)

   Identify factors and multiples including common factors and common multiples. (Texas)

24. Recognize and identify perfect squares and their roots. (Ohio)

   Represent squares and square roots using geometric models. (Texas)

25. Write prime factorizations using exponents. (Ohio)

   Apply prime factorization to solve problems and explain solution. (Texas)

26. Represent multiplication and division situations involving fraction and decimals with models and visual representations. (Ohio)

   Represent multiplication and division situations in picture, word, and number form. (Texas)

27. Use physical models, points of reference, and equivalent forms to add and subtract commonly used fractions with like and unlike denominators and decimals. (Ohio)

   Model and Record addition and subtraction of fractions with like denominators in problem-solving situations. (Texas)

28. Add and subtract whole numbers with and without regrouping. (Ohio)

   Add, subtract, multiply, and divide rational numbers in problem situations. (Texas)

29. Compare, order, and determine equivalent forms of real numbers. (Ohio)

   Compare and order rational numbers in various forms including integers, percents, and positive and negative fraction and decimal. (Texas)
30. Add and subtract whole numbers with and without regrouping. (Ohio)

Add, subtract, multiply, and divide rational numbers in problem situations. (Texas)

31. Model, represent, and explain multiplication. (Ohio)

Model factors and products using arrays and area models. (Texas)

32. Tell time to the hour and half hour on digital and analog timepieces. (Ohio)

Describe time on a clock using hours and half hours. (Texas)

33. Estimate and measure weight using non-standard unit. (Ohio)

Estimate and measure length, capacity, and weight of objects using nonstandard unit. (Texas)

34. Estimate and measure lengths using non-standard and standard units. (Ohio)

Measure length, capacity, and weight using concrete models that approximate standard units. (Texas)

35. Tell time to the hour and half hour on digital and analog timepieces. (Ohio)

Tell and write time shown on traditional and digital clocks. (Texas)

36. Read thermometers in both Fahrenheit and Celsius scales. (Ohio)

Use a thermometer to measure temperature. (Texas)

37. Estimate and measure weight using non-standard unit. (Ohio)

Estimate and measure weight using standard unit including ounces, pounds, grams and kilograms. (Texas)

38. Measure length, and capacity using uniform objects in the environment. (Ohio)

Estimate and measure capacity using standard units including milliliters, liters, cups, pints, quarts and gallons. (Texas)

39. Estimate and measure length, weight, capacity, using metric and customary unit.
Estimate and measure weight using standard unit including ounces, pounds, grams and kilograms. (Texas)

40. Identify and select appropriate units to measure angles. (Ohio)

Measure angles. (Texas)

41. Convert units of length, area, volume, mass, and time within the same measurement system. (Ohio)

Convert measures within the same measurement system (customary and metric) based on relationships between units. (Texas)

42. Identify and select appropriate units for measuring length, capacity, weight, and temperature. (Ohio)

Select and use appropriate units, tools, or formulas to measure and to solve problems involving length, area, time temperature, capacity, and weight. (Texas)

43. Describe and create plane figures: circle, rectangle, square, triangle, hexagon, trapezoid, parallelogram and rhombus, and identify them in the environment. (Ohio)

Describe, identify, and compare circles, triangles, and rectangles including squares. (Texas)

44. Create new shapes by combining or cutting apart existing shapes. (Ohio)

Combine geometric shapes to make new geometric shapes using concrete models. (Texas)

45. Identify, explain, and model the concept of shapes being congruent and similar. (Ohio)
Use attributes to describe how two shapes or two solids are alike or different.

(Texas)

46. Identify and draw figures with line of symmetry. (Ohio)

Identify lines of symmetry in shapes. (Texas)

47. Create and identify two-dimensional figures with line of symmetry. (Ohio)

Create shapes with lines of symmetry using concrete models and technology.

48. Describe, identify, and model reflections, rotations, and translations, using physical materials. (Ohio)

Sketch the results of translations, rotations, and reflections. (Texas).

49. Describe a motion or series of transformations that show two shapes are congruent. (Ohio)

Describe the transformation that generates one figure from the other when given two congruent figures. (Texas)

50. Classify, identify, and draw right, acute, obtuse and straight angles. (Ohio)

Use angle measurements to classify angles as acute, obtuse, or right. (Texas)

51. Identify and define triangles based on angle measures and side lengths. (Ohio)

Identify relationships involving angles in triangles and quadrilaterals. (Texas)

52. Prove the Pythagorean Theorem. (Ohio)

Use pictures or models to demonstrate the Pythagorean Theorem. (Texas)

53. Draw circles, and identify and determine the relationship among the radius, diameter, center and circumference. (Ohio)

Describe the relationship between radius, diameter, and circumference of a circle. (Texas)
54. Draw the results of translations, reflections, rotations, and dilations of objects in the coordinate plane. (Ohio)
   Graph dilations, reflections, and translations on a coordinate plane. (Texas)

55. Make and test conjectures about characteristics and properties (e.g., sides, angles, symmetry) of two dimensional figures and three-dimensional objects. (Ohio)
   (The students) Makes and verifies conjectures about angles, lines, polygons, circles, and three-dimensional figures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic. (Texas)

56. Describe, classify, compare, and model two- and three-dimensional objects using their attributes. (Ohio)
   (The student) Describes, and draws cross sections and other slices of three-dimensional objects. (Texas)

57. Identify, describe, and model intersecting, parallel, and perpendicular lines and line segments. (Ohio)
   (The student) Uses one- and two-dimensional coordinate systems to represent points, lines, line segments, and figures. (Texas)

58. Use Pythagorean Theorem to solve problems involving triangles. (Ohio)
   (The student) Develops, extends, and uses the Pythagorean Theorem. (Texas)

59. Draw representation of three-dimensional geometric objects from different view.
   (Ohio)
   Draw solids from different perspectives. (Texas)

60. Identify similarities and differences of quadrilaterals (e.g. squares, rectangle, parallelograms, and trapezoids). (Ohio)
Use properties to classify shapes including triangles, quadrilaterals, pentagons, and circles. (Texas)

61. Gather and sort data in response to questions posed by teacher and students. (Ohio)

Collect and sort data. (Texas)

62. Read, interpret, and construct bar graphs with intervals greater than one. (Ohio)

Use organized data to construct real object graphs, picture graphs, and bar-type graphs. (Texas)

63. Draw lines of symmetry to verify symmetrical two-dimensional shapes. (Ohio)

Identify lines of symmetry in shapes. (Texas)

64. Identify and draw figures with line symmetry. (Ohio)

Identify lines of symmetry in shapes. (Texas)

65. Describe the likelihood of simple events as possible/impossible and more likely/less likely. (Ohio)

Use data to describe events as more likely, less likely, or equally likely. (Texas)

66. Conduct a simple probability experiment and draw conclusions about the likelihood of possible outcomes. (Ohio)

Use fractions to describe the results of a probability experiment. (Texas)

67. Describe data using mode, median, and range. (Ohio)

Use median, mode, and range to describe data. (Texas)

68. Understand the different information provided by measures of center (mean, mode, and median) and measures of range. (Ohio)
Select the appropriate measure of central tendency to describe a set of data for a particular purpose. (Texas)

69. Compute probabilities of compound events, independent events, and simple dependent events. (Ohio)

Find the probabilities of compound events (dependent and independent). (Texas)

70. Make predictions based on theoretical probabilities and experimental results. (Ohio)

Evaluate predictions and conclusions based on data analysis. (Texas)

71. Describe sampling methods and analyze the effects of method chosen on how well the resulting sample represents the population. (Ohio)

Evaluate methods of sampling to determine validity of an inference made from a set of data. (Texas)

72. Develop strategies for basic addition fact. (Ohio)

Use patterns to develop strategies to remember basic addition facts. (Texas)

73. Continue repeating and growing patterns with materials, pictures and geometric items. (Ohio)

Identify, describe, and extend patterns to make predictions and solve problems. (Texas)

74. Describe orally the basic unit or general plan of a repeating or growing pattern. (Ohio)

Identify, describe, and extend patterns to make predictions and solve problems. (Texas)

75. Describe and compare qualitative and quantitative change. (Ohio)
Describe and interpret rates of change from graphical and numerical data. (Texas)

76. Solve and graph linear equations and inequalities. (Ohio)

   Formulate linear equations or inequalities to solve problems. (Texas)

77. Use the quadratic formula to solve quadratic equations that have complex roots.

   (Ohio)

   Solve quadratic equations using the quadratic formula. (Texas)

78. Solve quadratic equations with real roots by graphing, formula and factoring.

   (Ohio)

   (The student) Determine a quadratic function from its roots or a graph. (Texas)

79. Solve linear equations and inequalities graphically, symbolically and using technology. (Ohio)

   (The student) Solve systems of linear equations using concrete models, graphs, tables, and algebraic methods. (Texas)

80. Compare, order, and convert among fractions, decimals, and percents. (Ohio)

   Compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals. (Texas)
Appendix C – Taxonomies of Math Concepts

These taxonomies have hierarchies of K-12 math concepts. Generalization (i.e. isa relationship) and aggregation (i.e. whole-part relationship) relationships in UML have been used for hierarchies of math concepts in this ontology.

Figure A.1: A taxonomy of a math concept “number”
Figure A.2: A taxonomy of a math concept “numeral”

Figure A.3: A whole-part lexical relation of math concepts “equivalent form”, percent”, “fraction”, and “decimal”

Figure A.4: A taxonomy of a math concept “place value”

Figure A.5: A whole-part lexical relation of math concepts “exponentiation”, “base”, and “exponent”

Figure A.6: A taxonomy of a math concept “root”
Figure A.7: A taxonomy of a math concept “mathematical expression”

Figure A.8: A taxonomy of a math concept “numeration system”

Figure A.9: A taxonomy of a math concept “fact”
Property of operation

Distributive property
Commutative property
Associative property
Inverse property
Identify property

Graph

Table graph
Picture graph
Bar graph

Measure of central tendency

Mean
Media
Mode

Figure A.10: A taxonomy of a math concept “mathematical operation” and a whole-part lexical relation in a math concept “mathematical operation”

Figure A.11: A taxonomy of a math concept “property of operation”

Figure A.12: A taxonomy of a math concept “graph”

Figure A.13: A Whole-part lexical relation in math concepts “means”, “media”, “mode”, and “measure of central tendency”
Figure A.14: A taxonomy of a math concept

Figure A.15: A taxonomy of a math concept “sentence”

Figure A.16: A taxonomy of a math concept “sequence”
Symmetry
  Line symmetry  Rotation symmetry  Reflection symmetry  Point symmetry

Figure A.17: A taxonomy of a math concept “symmetry”

Coordinate system
  Polar coordinate system  Cartesian coordinate system

Figure A.18: A taxonomy of a math concept “coordinates system”

Function
  X-intercept  Y-intercept  Slope  Domain  Variable  Range
    Dependent variable  Independent variable

Figure A.19: A whole-part lexical relations in a math concept “function” and a taxonomy of a math concept “variable”
Figure A.20: A taxonomy of a math concept “equation”

Figure A.21: A taxonomy of a math concept “Change”

Figure A.22: A taxonomy of a math concept “variable”
Inequality

Quadratic inequality
Power inequality
Logarithm inequality

Figure A.23: A taxonomy of a math concept “inequality”

Quantity

Length

Height
Width
Temperature
Mass
Angle
Volume
Capacity
Circumference

Figure A.24: A taxonomy of a math concept “quantity”

Unit of measure

Unit of length
Unit of time
Unit of width
Unit of height
Unit of temperature
Unit of rate

Figure A.25: A taxonomy of a math concept “unit of measure”
Figure A.26: A taxonomy of a math concept “unit”

Figure A.27: A taxonomy of a math concept “measuring system”

Figure A.28: A taxonomy of a math concept “money”

Figure A.29: A taxonomy of a math concept “word”
Figure A.30: A taxonomy of a math concept “geometric figure”

Figure A.31: A taxonomy of a math concept “shape”

Figure A.32: A taxonomy of a math concept “angle”
Figure A.33: A taxonomy of a math concept “line”

Figure A.34: A taxonomy of a math concept “transformation”

Figure A.35: A taxonomy of a math concept “object”
VITA

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