Realizable Constraint Driven Capacitor Placement and Control Sequences for Voltage Spread Reduction in Distribution Systems

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DEDICATIONS

To those who seek knowledge and are insistent on lifelong learning.
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ABSTRACT

Realizable Constraint Driven Capacitor Placement and Control Sequences for Voltage Spread Reduction in Distribution Systems
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There is a continued focus on advancing and diversifying the US electric energy sector. Some federal and state initiatives have been imposed to: reduce system peak load, increase the amount of renewable generators, and increase the number of demand response participants. In distribution systems, various combinations of network devices (e.g. capacitors, distributed generators, loads) are used to achieve a reduction in peak load. Historically, capacitors have been installed and employed by system operators for reactive power compensation, voltage regulation, power factor correction and energy loss reduction.

In this thesis, capacitor placement and control sequences for voltage spread reduction in distribution systems is developed and delivered. Additionally, direct load control participants and photovoltaic generators are included in the control problem in order to investigate impacts of federal and state peak load reduction goals.

Existing capacitor placement and control problem formulations do not address bulk transmission system requirements and whether an optimal solution is physically attainable by system operators. Here, two new constraints are included in the problem formulation. A substation reactive power constraint is included in order to comply with the transmission system operating requirements. A voltage rise constraint is included so that bus voltage magnitudes between pre and post device switch actions are held to an
acceptable change (rise/drop) in bus voltage. Subsequently, heuristic based greedy algorithms were developed to find a solution.

The results show that constraint-driven methodologies are needed to generate control sequences, which can realize the objectives. The order in which capacitors actions are taken throughout a day is significant and should be guided by the voltage rise constraint. A set of feasible non-inferior solutions which are attained via a search of feasible switching sequences was found. Also, the transmission system reactive power requirements significantly impact the placement and control results. DLC and PhV results showed that a tradeoff exists between a reduction in real power and increase in the total reactive power in the circuit.
1. INTRODUCTION

This dissertation proposes constraint driven capacitor placement and control sequences for select network devices within a distribution system. It will be demonstrated that in order to achieve the desired objectives it is necessary to provide distribution system operators with a feasible sequence of control actions that transitions the system’s control devices along given load settings. In this thesis, the problem of capacitor placement and control for voltage spread reduction and real power loss reduction objectives is addressed.

The current/present-day capacitor placement and control literature's problem formulations do not typically address whether the optimal solution is attainable by system operators. Here, practical control sequences for multiple load settings are determined by including a voltage rise constraint and a substation reactive power constraint in the problem formulation. Subsequently, heuristic based greedy algorithms were developed that implement the constraints to find a solution. Lastly, direct load control participants and photovoltaic generators are included in the control problem in order to include emerging systems and reflect federal and state guidelines in load reduction and renewable generation standards for distribution systems.

In this chapter, distribution system characteristics are given, then some background and motivation for the problem is presented, followed by the objectives and a summary of contributions. Subsequently, an overview of the thesis’s layout is provided.
1.1. **Distribution System Characteristics**

Power systems are generally separated into three areas of study, i.e. generation, transmission and distribution systems. Historically, power has been supplied unidirectional from the generators (source) in transmission systems to the loads (customers) in distribution systems. As a result, transmission system operating requirements (e.g. North American Reliability Corporation (NERC) VAR-001-3 (voltage and reactive control standard) [1], NERC TOP-001-3 (transmission operating standard) [2] and American National Standards Institute (ANSI) C84.1-2011 (electric power systems and equipment voltage ratings standard [3]) were imposed on distribution system operators so that the distribution substation behaved in the most advantageous manner to the transmission system point of view. Some examples of transmission imposed distribution substation guidelines are as follows:

- The substation should maintain three-phase balanced loading conditions
- The substation should absorb reactive power or maintain unity power factor
- The substation is the only real and reactive power source in the system
- The path of power from the substation (source) to the customer should be radial and thus uni-directional
- Substation voltages are regulated such that the customers’ voltage is maintained in the range of $\pm 5\%$ of the nominal 120 V.

Implementing the transmission operating guidelines accommodates transmission modeling of a distribution network. The distribution network model is simplified to an individual balanced three-phase aggregated (lumped) load and a step-down transformer.
Conforming to the transmission system model reduces the amount of coordination needed between distribution system and transmission system operators in order to serve the load. Additionally, modeling a distribution network as a three-phase, balanced bulk load allows for transmission system per-phase power flow solvers. These simplifications neglect explicit Distribution Systems (DS) characteristics which are needed to adequately model and study the network.

With characteristics that differ from the transmission systems model distribution systems:

- Are large scale radial systems with multi-phase branches and buses
- Contain single-phase, two-phase and three-phase (multi-phase) loads
- Have unequal conductor (lines) spacing which results in non-equal off-diagonal terms in the impedance matrix (mutually coupled impedances)
- Voltage profiles are not flat and profiles will vary from network to network
- Contain non-utility controlled multi-phase distributed energy resources
- DS power flows bi-directionally, (i.e. from sub. to the consumer and from the distributed energy resources to nearby buses)

In order to evaluate distribution systems properly, a multi-phase, unbalanced power flow solver with detailed network components is needed. Thus, the transmission system should relax some of their distribution substation guidelines and proceed to model distribution networks beyond the substation in order to capture the multi-phase and unbalanced network components of the system.
Federal funding for advanced distribution automation [4], such as two-way communications, measurements, and automated equipment has enabled the modernization of distribution systems. Through these technological advancements there is a potential for many multi-phase network devices to change their control settings (e.g. capacitors, on-load tap changers) or reduce their load during a 24 hour day.

Metrics are needed to provide technical clarity for a DS’s criteria to alter device control settings. This thesis makes use of constraint driven analytics to assess the DS capacitor placement and control problem and investigates load changes on the system’s existing control settings. Furthermore, previously dictated management of distribution systems as directed by transmission systems may no longer be effective for modern systems which include distributed energy resources and advanced metering with bi-directional controls and communications.

The next section provides some background information for the capacitor placement and control problem.

1.2. BACKGROUND

The capacitor placement problem in distribution systems has been widely discussed since the late 1950's [5]. Capacitors are placed in distribution systems and employed by system operators for the purpose of reactive power compensation, voltage regulation, power factor correction and energy loss reduction [6-9].

Previous methods for solving the capacitor placement problem employ dynamic programming techniques [10-13], mixed-integer programming [14], heuristic methods [15], and intelligent algorithms [16-20]. Additionally, combinations of search techniques
and graph search methods are used to arrive at an optimal solution. Others have solved multi-objective capacitor placement problems by employing trade-off analysis or Pareto optimality. In each of these works, the optimal capacitor solution is sought while satisfying the system's operational constraints and loading conditions.

In the works the optimal control settings for capacitors in a distribution system are solved for by defining discrete hourly load settings from a 24 hour forecasted load. Network constraints and the maximum number of switching operations are checked at each load setting in order to arrive at the solution. Similarly to , this thesis used static discrete load levels.

Still, previous works omitted explicit constraints between control actions and transmission system interconnection considerations about the distribution substation (e.g. substation reactive power requirements). Therefore, in this thesis, two constraints are added to the problem formulation and employed in the solution algorithm.

The first constraint monitors the change of bus voltage due to control device switch actions. The voltage rise constraint requires bus voltage magnitudes between pre and post device switch actions to be held to an acceptable change (rise/drop) in bus voltage. It is assumed that all control actions can be enacted before a significant change in load occurs. The voltage rise constraint assists with finding the order of device control actions between multiple load settings that can be applied/realized without creating voltage violations.

The second constraint is on the reactive power at a distribution system’s substation. The substation reactive power constraint is included in order to comply with
the transmission system operating requirements. The transmission system operating requirement is expected to be withheld by distribution system operators yet this constraint is often not included in the capacitor placement and control literature.

In this thesis direct load control participants and photovoltaic generators are included in the control sequence problem for voltage spread reduction objective to achieve state mandated peak load reduction goals set by [34]. The extended control studies devices are similar to the devices selected by [35]. In the work [35], demand response and photovoltaic generation with time of day price signals were used in a residential setting to shift peak load. Here, a focus on distribution system operators reaching previously planned capacitor control settings with the selected network devices is studied. Therefore, the selected network devices are studied separately and no price signals are included in the problem formulation.

This dissertation differs from traditional capacitor placement and control papers, which typically focus predominantly on the quality of the solution. Instead, this thesis studies the implementation of network device control settings. While exact implementation presented may yield local optimal values with respect to the selected objective, the algorithm provides a clear way to realize that local optima. Methodologies, which do not consider constraints between control actions, provide control settings for various load levels, but no feasible path to move between these settings. These solutions are more difficult for distribution system operators to implement as they may cause unintended constraint violations. Hence, it will be shown in this dissertation that constraint driven problem formulation and solution methodologies are needed.
In the next section motivations to study the capacitor placement and control sequence problems for select network devices is provided.

1.3. Motivation

There is a continued focus on advancing and diversifying the US electric energy sector. Federal funding from the American Recovery Investment Act of 2009 has been distributed to improve energy efficiency and integrate renewable sources into the transmission and distribution systems. Additionally, state initiatives have been imposed to reduce peak loading conditions and to provide incentives for residential customers to increase the quantity of renewable generators and demand response participants [40-45].

In this thesis, the voltage spread reduction (VSR) and real power loss reduction (RPLR) objectives are considered sequentially to solve the capacitor placement and control problem. Motivations to examine these two objectives include the utility's desire to meet mandated energy consumption reduction standards [34] and to participate in energy efficiency incentive programs [4].

Motivations to expand the control problem to add direct load control participants and photovoltaic generators include political [4] and economic [36-39] initiatives. In [40] a renewed interest by electric system regulators to use demand side management programs as a method to enhance reliability through a reduction of system wide peak load was indicated.

Research shows that a popular residential and subsidized choice of renewable resource is photovoltaic generation. As summarized from [41] and [42] reports, the U.S. market for PV products has grown in recent years, accounting for about 12% of global
PV installations in 2013 [43]. Additionally, since 1998, installed photovoltaic system prices have fallen by 6-7% per year on average [44]. More residential photovoltaic generation has been added to distribution systems.

In the next section a list of the thesis objectives and contributions is given.

1.4. OBJECTIVES AND CONTRIBUTIONS

In this thesis the capacitor placement and control sequence problem is formulated with a voltage rise constraint in order to identify feasible control sequence and a reactive power constraints in order to adhere to transmission system operating practices. The overall framework of the thesis is provided below in Figure 1.1:

**Note:** The select network devices which are included in the control status & sequence problem are as follows:
1. Direct Load Control (DLC) Participants whose control status is controlled by system operators
2. Photovoltaic (PhV) Generators which are uncontrolled by system operators

Figure 1.1 Capacitor Placement and Control Sequence Problem Framework
The assessment of a distribution system is accomplished by including the following objectives:

- Perform capacitor (cap) placement and cap control for two separate goals
  - Voltage Spread Reduction (VSR)
  - Real Power Loss Reduction (RPLR)
- Examine effects of relaxing transmission system substation reactive power requirements on the VSR and RPLR objectives
- Determine feasible control sequences for VSR cap placements
- Add selected network components and evaluate cap control settings and feasible control sequences for:
  - Capacitors and Direct Load Control (DLC) Participants
  - Capacitors with Photovoltaic (PhV) Generators

These objectives aim to assist distribution system operators in answering the questions:

- Does a network aid in voltage reduction and/or real power loss reduction?
- Which bus and what size of cap should be placed in a distribution system?
- When should caps participate in control (i.e. how long and how often)?
- Are the capacitor solutions affected by DLC participants and PhV generators (how long and how often does the solution hold)?

It is assumed that all capacitor controls can be enacted before a significant change in load occurs. Therefore, a realizable control sequence of device switch actions that transitions from the initial control setting to the final control setting at a given load setting may be found.
This thesis’s main contributions are the following:

- A voltage rise constraint applied to pre and post switch actions to maintain acceptable variations in bus voltages.
- A substation reactive power constraint that binds DS reactive power to conditions accepted by TS operators.

Subsequently, additional contributions in this thesis are:

- An investigation of substation reactive power constraints that satisfy TS reactive power requirements.
- A constraint driven solution algorithm that ensures feasible switching sequences.
- Non-inferior, physically attainable capacitor placement and control results for selected load settings.
- Realistic switching sequences that are required to transition between device actions at a given load setting.
- Comprehensive simulation results.

The list of above contributions’ order is applied to the arrangement of the thesis; an overview of thesis organization is provided next.
1.5. **THESIS ORGANIZATION**

The thesis is ordered as follows:

- In Chapter 2 the cap placement and control sequence problems are stated.
- In Chapter 3 practical considerations used to reduce the search space size are discussed.
- In Chapter 4 detailed methodologies applied to solve the placement and control problems are provided.
- In Chapter 5, selected distribution system results for the problems and solution algorithms are delivered.
- In Chapter 6 a summary of thesis findings and achievements are given and suggested improvements for future work are disclosed.
2. PROBLEM FORMULATION

The main problem addressed in this thesis focuses on realizable, constraint driven capacitor placement and control problem sequences for voltage spread reduction and real power loss reduction. A voltage rise constraint was added to maintain acceptable variations in bus voltages between device control actions. In order to incorporate transmission system operating requirements a reactive power constraint is included. In order to capture emerging distribution systems components, direct load control participants and photovoltaic generation will also be included.

The problem is formulated as a non-linear, non-differentiable, constrained, combinatorial, multi-objective optimization problem. In this thesis, the voltage spread reduction (VSR) and real power loss reduction (RPLR) objectives are considered sequentially to solve the capacitor placement and control sequence problem. Then the control sequence problem is subsequently extended to include direct load control (DLC) participants and photovoltaic (PhV) generation.

In this chapter, the assumptions are stated in section 2.1 followed by problem’s objectives in section 2.2, and constraints in section 2.3. A particular focus on the new limits on voltage rise and reactive power are given and their significance to the problem is stated. In section 2.4, a discussion of engineering rational used in order to reduce the search space size is presented. In section 2.5, a copy of the complete mathematical problem formulation will be repeated as a reference. Lastly, a chapter summary is provided in section 2.6.
2.1. ASSUMPTIONS

There are three network devices that are included in the placement and control studies, they are capacitors, DLC participants, and PhV generators. These devices are considered sources of real and reactive power, respectively noted as $P$ and $Q$, and will be referred to in this thesis as PQ injections. These network devices inject PQ differently from one and other. The capacitors are installed along distribution feeders and inject only reactive power ($Q$) into the system. The DLC participants and PhV generators inject both real power ($P$) and reactive power ($Q$). A diagram illustrating each possible type of PQ injection at a bus is provided in Figure 2.1.

![Diagram of PQ Injections at a Bus: Capacitors, Loads, DLC Participants, and PhV Generators](image)

Figure 2.1 A Diagram Illustrating PQ Injections at a Bus: Capacitors, Loads, DLC Participants, and PhV Generators

In Figure 2.1, an example of in-service three-phase bus with the positive direction of current assigned flowing through the line from left to right is shown with arrows. The bus has four types of PQ source injections, capacitors, loads, DLC and PhV. The net injection from these sources will supply or demand $P$ and $Q$ from the bus. Both loads and DLC draw power from the bus. The capacitors supply $Q$ to buses and PhV generators
typically supply $P$ and $Q$ at a bus. The amount of real and reactive power varies for each location, load type, and PQ source. Thus, for the overall placement and control problem the following assumptions are made:

A1. DLC participants are regulated by system operators and load shedding occurs automatically.

A2. PhV generators are not controlled by the utility.

A3. Device actuating times are minimal so that steady state analysis is valid.

A4. Given placement and device control settings at a load setting, there exists a feasible path of control actions that transition from the initial control setting to the final control setting with respect to voltage constraints.

A constructive algorithm, which can also identify cases where A4 is invalid, was developed; and a discussion of the algorithm’s implementation is provided in the solution algorithm chapter. Next, the objectives for the capacitor placement and control sequences problems are discussed.

2.2. Objectives

The first objective selected for the capacitor placement and control problem is to reduce the voltage spread along a given distribution feeder and at a respective load setting. The second objective is to minimize the total energy losses of the system at all provided load settings. These objectives are studied separately.

Voltage spread reduction’s goal is to minimize the nodal voltage differential along distribution systems feeders by operating feeder capacitors to maintain acceptable
network voltages. VSR enables distribution system operators to lower the substation voltage in order to achieve network load reduction. The process of decreasing a system’s voltage to achieve network load reduction is known as conservation voltage reduction. The real power loss reduction minimizes the total energy losses of the system for the given load settings and time of study.

The problem has been defined per node (bus and phase). The VSR objective and RPLR objectives follow respectively as:

\[
\begin{align*}
\min \ & \max_{u_{s}, u_{p} \in U} \left| V^{p}_{i} (u_{s}, \lambda_{k}) - V^{p}_{j} (u_{s}, \lambda_{k}) \right| \quad \forall \ k \quad k = 1 \ldots N_{LL} \\
\min \ & \sum_{k=1}^{N_{lk}} C_{L,k} T_{Loss,k} (V, u_{s}, \lambda_{k}) 
\end{align*}
\]

where:

\( u_{s} \): discrete control variables; a binary vector of nodal device settings at load setting \( k \), \( u_{s} : \left[ n_{\text{cap}}^{\text{total}} \times 1 \right], u_{s} \in [0, 1] \)

\( U \): search space of capacitor placement and device control schemes

\( p \): set of present phases \( \{a, b, c\} \) at a bus \( i \) or at a bus \( j \)

\( n_{\text{cap}}^{\text{total}} \) is the total number of per phase capacitors at a bus

\( n_{\text{dvc}}^{\text{total}} \) : is the total number of per phase devices at a bus (i.e. capacitor, direct load control participants and photovoltaic generators)

\( N \): set of all buses

\( |V^{p}_{i}|, |V^{p}_{j}| \): voltage magnitude at bus \( i \) and bus \( j \), for a phase \( p \)
$\lambda_k$: continuous load parameters, a vector of nominal nodal complex powers at $k^{th}$ load setting, $\lambda_k \in \mathbb{C}^N$, $\lambda_k = P_k^p + jQ_k^p$

$N_{LL}$: total number of load settings (levels/profiles)

$C_{L,k}$: substation specific cost per kWh due to losses at the $k^{th}$ load setting

$T_k$: time duration in hours per load setting $k$ for losses

$P_{Loss,k}$: a network's total real power losses at $k^{th}$ load setting

$V_k$: continuous state variables, a vector of complex node (bus and phase) voltages at $k^{th}$ load setting, $V_k \in \mathbb{C}^N$

The maximum total size of the problem’s search space can be quite large because there can be many devices to control. Even if each network device is limited to two states, either on or off the search space size is $\left(2^{n_{dev}}\right) \times N_{LL}$. The maximum number of power flows to evaluate each control answer is equal to $\left(2^{n_{dev}}\right)$.

The $u_s$ discrete control variable can be composed of integer variables describing the control devices under study. For capacitor control capacitor bank sizes are important. Table 2.1 is an example of how $u_s$ can be coded from different capacitor bank sizes.
Table 2.1 Example of Nodal Coding of Discrete Control Sizes to Integer Settings \( (u_s) \) Using 100 kVAr Capacitor Banks

<table>
<thead>
<tr>
<th>Size</th>
<th>( u_s ) Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 kVAr</td>
<td>0</td>
</tr>
<tr>
<td>100 kVAr</td>
<td>1</td>
</tr>
<tr>
<td>200 kVAr</td>
<td>2</td>
</tr>
<tr>
<td>300 kVAr</td>
<td>3</td>
</tr>
<tr>
<td>400 kVAr</td>
<td>4</td>
</tr>
</tbody>
</table>

In Table 2.1, the capacitor sizes are listed in column one and the corresponding five possible control settings are provided in column two. Here, the possible control settings for each phase of the capacitor are settings 0 to 4. Setting ‘0’ means the capacitor is off. The final setting ‘4’ means that four 100 kVAr per phase capacitor banks are on. In this thesis, only two settings are considered to limit the control space size. Therefore, after the capacitor location is selected, the maximum kVAr size is determined from the capacitor placement problem.

The search space \( u_s \), can be expanded and collapsed depending on the number and type of network devices included in a distribution feeder. For example, \( u_s : [n_{\text{dvc}}^{\text{total}} \times 1] = [n_{\text{cap}}^{\text{total}} \times 1 : n_{\text{DLC}}^{\text{total}} \times 1] \), where respectively, \( n_{\text{cap}}^{\text{total}} \) is the total number of capacitors and \( n_{\text{DLC}}^{\text{total}} \) is the total number of DLC participants at a bus. Capacitor placement and control is investigated first and is included in all variations of the search space. Then capacitor control and the addition of direct load control participants were studied. Followed by studies of capacitor control and photovoltaic generators. Other
variations, such as, capacitor control with both DLC participants and PhV generation can be studied.

The above problem has been defined per node to accommodate emerging systems such as microgrids where single-phase compensation may be enacted. The application of this thesis was to in-service distribution systems. Thus, the subsequent algorithm and simulations address three-phase reactive power placement and control.

In the following section, the network and operational constraints of the problem are presented.

2.3. CONSTRAINTS

The placement and control problems are solved sequentially. For their solutions to be feasible, all network and operational constraints must be satisfied. Distribution systems are unbalanced and multi-phase; e.g. composed of single-phase, two-phase and three-phase network devices, such as buses, branches, transformers, switches, capacitors, loads and distributed energy resources. Therefore, the following constraints are included.

2.3.1. EQUALITY CONSTRAINTS

The main equality constraint for the capacitor placement and control problem is that an unbalanced multi-phase power flow for a given system is satisfied. Mathematically, the unbalanced power flow equations are written as:

\[ F \left( V_k, u_s, \lambda_k \right) = 0 \quad \forall \ k \quad k = 1 \ldots N_{LL} \]  \hspace{1cm} (2.3.1)
where \( F(V_k, u^s, \lambda_k) \) are the power flow equations, a set of non-linear algebraic equations specific to a distribution system. The solution to the power flow equations are the steady state voltages at every node (bus and phase) of the system.

The second equality constraint of the problem is the assumption that a realizable control sequence \( (u_{cs}(\lambda_k)) \) exists. The control sequence is defined as a feasible set of binary control actions that transition from the initial control setting \( (u_i) \) to the final control setting \( (u_{Nsu}) \) at a given load. The path of control actions for select network devices along a trajectory of load settings is written mathematically as:

\[
\exists \text{ a control sequence: } u_{cs}(\lambda_k) =\begin{bmatrix} u_1(\lambda_1), \ldots, u_{s-1}(\lambda_{s-1}), u_s(\lambda_s), u_{s+1}(\lambda_{s+1}), \ldots, u_{Nsu}(\lambda_{Nsu}) \end{bmatrix} \quad (2.3.2)
\]

where \( u_{cs}(\lambda_k) \) is a matrix of column vectors. The number of columns in the control sequence’s matrix depends on the number of per phase device binary switch operations \( (n_{ops,k}) \) needed to transition from the initial control setting \( (u_i) \) to the final control setting \( (u_{Nsu}) \) at load setting \( k \). A binary switch action for turning a device “on” is indicated by a change in the device status from a ‘0’ to a ‘1’. Likewise, a binary switch action for turning a device “off” is indicated by a change in the binary status from a ‘1’ to a ‘0’. In this thesis, simultaneous device switch actions are permitted. Lastly, the path of control sequences along a trajectory of load settings is written mathematically as

\[
\begin{bmatrix} u_{cs}(\lambda_1), \ldots, u_{cs}(\lambda_{s-1}), u_{cs}(\lambda_s), u_{cs}(\lambda_{s+1}), \ldots, u_{cs}(\lambda_{Nsu}) \end{bmatrix}, \text{ where, } u_{cs}(\lambda_k) \text{ is the control}
\]
sequence at load setting $\lambda_i$ and $u_{ex}(\lambda_{N,LL})$ the control sequence at the final load setting $\lambda_{N,LL}$.

Inequality constraints have been added to the problem formulation that assist in finding a control sequence and will be discussed in subsections of the inequality constraints.

### 2.3.2. Inequality Constraints

The inequality constraints restrict the solution to be within the systems’ physical operating limits. The inequality constraints are generically written in the following form:

$$G(V_k, u_s, \lambda_k) \leq 0 \quad \forall \ k \quad k = 1 \ldots N_{LL} \hspace{2cm} (2.3.3)$$

Here, common distribution system operating constraints are discussed first. Constraints on bus voltages and branch flows and feeder capacities are presented.

The voltage constraint is defined as:

$$|V_i^{\text{min}}| \leq |V_i^p(u_s, \lambda_k)| \leq |V_i^{\text{max}}| \quad \forall \ i \in \{1, \ldots N\}, \ p \in \{a, b, c\} \ & \forall \ k = 1 \ldots N_{LL} \hspace{2cm} (2.3.4)$$

where $|V_i^p(u_s, \lambda_k)|$ is the voltage magnitude at bus $i$, phase $p$, at the current control setting $s$ and at load setting $k$. $|V_i^{\text{max}}|, |V_i^{\text{min}}|$ are respectively, the maximum (max) and minimum (min) operating voltage magnitude at a bus $i$. The constraint is required for all energized buses and for all load settings. The voltage magnitude can be line-to-line or line-to-neutral dependent upon bus grounding.
The current constraint is defined as:

\[
I_{ij}^{\text{max}} \leq I_{ij}^p(V_k, u_s, \lambda_k) \leq I_{ij}^{\text{min}} \quad \forall \ ij \in N_{Br}, \ p \in \{a, b, c\} \ \& \ \forall \ k = 1 \ldots N_{LL} \tag{2.3.5}
\]

where \(I_{ij}^p(V_k, u_s, \lambda_k)\) is the current magnitude through the branch from bus \(i\) to bus \(j\), for phase \(p\), at the current control setting \(s\), and at load setting \(k\). \(I_{ij}^{\text{max}}, I_{ij}^{\text{min}}\) are the maximum and minimum acceptable current magnitude through branch \(ij\). \(N_{Br}\) is the set of all energized branches. The constraint is required at all energized branches and for all load settings.

The feeder or transformer capacity constraint is defined as:

\[
\left(P_i(V_k, u_s, \lambda_k)\right)^2 + \left(Q_i(V_k, u_s, \lambda_k)\right)^2 \leq \left(S_i^{\text{max}}\right)^2 \quad \forall \ i \in N_F, \ \& \ \forall \ k = 1, \ldots, N_{LL} \tag{2.3.6}
\]

where \(P_i(V_k, u_s, \lambda_k)\) and \(Q_i(V_k, u_s, \lambda_k)\) are respectively the real and reactive power flow through a feeder or transformer \(i\), at the current control setting \(s\), and for a load setting \(k\). \(S_i^{\text{max}}\) is the maximum apparent power for the feeder or transformer. \(N_F\) is the set of all feeders in the system. This constraint is required for all feeders or transformers and for all load settings.

2.3.2.1. **Voltage Rise and Reactive Power Constraints**

For the capacitor placement and control problem two new inequality constraints are introduced to the problem formulation. The focus of this thesis is to identify a path to
the solution where the sequence is a set of feasible operating points driven analytically by
the following constraints.

Typical intelligent system applications produced final control states but no
sequence of control actions to arrive at the solution. In [47] the concern for over
simplification of the problem was stated and addressed by adding to the problem the total
energy losses in the system using constant load levels for a set period of time. Inclusion
of the voltage rise constraint differs from previous methods because this constraint
evaluates the nodal voltages between control actions at a given load setting. The voltage
rise constraint is added so that the system operator can reach the solution of the next
control setting through a sequence of control actions from a previous set point. The
voltage rise constraint is defined as:

\[
\left| V_i^p (u_s, \lambda_k) - V_i^p (u_{s-1}, \lambda_k) \right| \leq V_{\text{rise}}^{\text{max}} \quad \forall \ i \in \{2, \ldots, N\}, \ p \in \{a, b, c\}, \ & \forall k = 1 \ldots N_{\text{LL}} \tag{2.3.7}
\]

In equation (2.3.7), the voltage rise is the absolute value of the difference between
the voltage magnitudes at a bus \( i \), and a phase \( p \) at the current control setting \( s \) and the
previous control setting \( s-1 \) for a given load setting \( k \). \( V_{\text{rise}}^{\text{max}} \) is the maximum allowable
voltage rise between a network’s device operations. By maintaining an acceptable voltage
magnitude during device switching operations a point-in-load feasible switching
sequence can be found. A point-in-load, is the load setting \( \lambda_k \). The voltage rise constraint
ensures that a realizable control sequence \( u_{cs} \) can be built. This constraint was first
introduced in [48].
The substation reactive power constraints originates from transmission system operating requirements. These limits are expected to be enforced by the distribution system. Previous literature has yet to investigate the impact of a substation reactive power constraint on the placement and control problem. The substation reactive power constraint is defined as:

\[
Q_{k}^{p,\text{min}} \leq Q_{\text{sub},k}^{p}(V_{k},u_{s},\lambda_{k}) \leq Q_{k}^{p,\text{max}} \quad p \in \{a,b,c\} \& \forall \, k = 1\ldots N_{L} \tag{2.3.8}
\]

where, \(Q_{k}^{p,\text{max}}, Q_{k}^{p,\text{min}}\) are the maximum and minimum reactive power out of the substation into a given feeder/system for a phase \(p\) and load setting \(k\). \(Q_{\text{sub},k}^{p}(V_{k},u_{s},\lambda_{k})\) is the reactive power out of the substation into a given feeder/system, for a phase \(p\) at the current control setting \(s\) and at a load setting \(k\). In order to discern the impacts of this constraint, simulation studies are performed with and without the reactive power limits in Chapter 5.

The reactive power limits in (2.3.8) are defined for each load level and phase. The limits on each phase can be different depending on acceptable phase balance levels for operators. An alternate constraint that can be considered limits the total amount of capacitance that can be placed and controlled in a distribution network. In distribution systems, multiple circuits may connect to a single substation. Thus for control, an individual circuit's reactive power limits may be relaxed provided the substation's total reactive power is within the transmission range at peak load or

\[
Q_{k}^{\text{min}} \leq Q_{\text{sub},k}^{\text{total}}(V_{k},u_{s},\lambda_{k}) \leq Q_{k}^{\text{max}}. \quad \text{Where } Q_{\text{sub},k}^{\text{total}}(V_{k},u_{s},\lambda_{k}) \text{ is the sum of the three phase reactive power magnitudes at the substation } Q_{\text{sub},k}^{\text{total}} = Q_{\text{sub},k}^{a} + Q_{\text{sub},k}^{b} + Q_{\text{sub},k}^{c}. \]

In this thesis,
the driving concern for operators was that the total reactive power output at the substation is within the transmission system's acceptable range. In the next section, the specific constraints for capacitors are presented.

2.3.2.2. Capacitor Constraints

The number of capacitors placed into a network is restricted by the costs to purchase and install a capacitor bank. The maximum number of capacitors to place may also be limited by the number of devices allowed to operate in a network. In papers [16-17] and [19] the number of capacitor placements is limited through a minimum cost objective. However, in this thesis the maximum number of capacitors for placement constraint is written directly as:

\[ n_{\text{cap}}^{\text{new}} \leq n_{\text{cap}}^{\text{max new}} \]  \hspace{1cm} (2.3.9)

where, \( n_{\text{cap}}^{\text{new}} \) is the number of new capacitor locations and \( n_{\text{cap}}^{\text{max new}} \) is the maximum number of new capacitor locations.

The total number of capacitors to control in a distribution system is of interest to a utility because there are costs associated with operating and maintaining each device. Here the limit on the total number of capacitors in a network is defined as:

\[ n_{\text{cap}}^{\text{total}} \leq n_{\text{cap}}^{\text{max}} \]  \hspace{1cm} (2.3.10)
where, \( n_{\text{total}}^{\text{cap}} \) is the total number of remote automated three-phase capacitors existing \( (n_{\text{exist}}^{\text{cap}}) \) and new \( (n_{\text{new}}^{\text{cap}}) \) in the system or \( n_{\text{cap}}^{\text{total}} = n_{\text{cap}}^{\text{exist}} + n_{\text{cap}}^{\text{new}} \). \( n_{\text{cap}}^{\text{max}} \) is the maximum number of remote automated capacitors in a distribution system.

Capacitors are manufactured in discrete nominal bank sizes (e.g. 300 kVAR). Here, a bank is assigned a single standard installation size. The constraint on the number of per phase banks at a bus \( \left(n_{\text{bank},i}^{p}\right) \), is limited by the minimum and maximum number of banks per phase at a bus \( \left(n_{\text{bank},i}^{\text{max}} \text{ and } n_{\text{bank},i}^{\text{min}}\right) \) and this is written mathematically as:

\[
\begin{align*}
    n_{\text{bank},i}^{\text{min}} & \leq n_{\text{bank},i}^{p} \leq n_{\text{bank},i}^{\text{max}} \\
    & \forall \ i \in \{1,\ldots,n_{\text{cap}}^{\text{total}}\}, \ p \in \{a,b,c\} \quad (2.3.11)
\end{align*}
\]

Utilities typically require capacitors to be balanced installations. In this thesis, all capacitor installations are assumed to be balanced across three phase buses.

Electric distribution system operators and planners are concerned with the maintenance and life-time of a switching device. Thus, the number of daily switch operations of a device is limited. The physical limitations on operation time and the number of successful mechanical actuations may be embedded into the problem formulation [48] or controlled by the number of load settings examined [9]. In this thesis, the number of switch operations per capacitor per day is explicitly defined so that the system operator can dictate the maximum number of actions and so that the number of load settings considered is independent of the maximum number of actions. Here the maximum number of switch actions per capacitor per day is written as:
\[ \sum_{k=1}^{N_{\text{fu}}} n_{\text{ops},k} \leq n_{\text{ops}}^{\text{max}} \]  

(2.3.12)

where, \( n_{\text{ops},k} \) is the number of per capacitor switch operations for a control sequence at load setting \( k \). Recall from equation (2.3.2), a control sequence at load setting \( k \) is defined as \( \lambda_1 \), \( \lambda_s \), \( \lambda_{s+1} \), \( \lambda_{s+2} \), \( \ldots \), \( \lambda_{N_u} \). Then, each \( n_{\text{ops},k} \) is a specific capacitor’s count of the number of switch operations to transition the capacitor from its initial control settings \( (u_1) \), to its final control setting \( (u_{N_u}) \) at load setting \( k \). Mathematically the number of operations is counted as follows

\[
 n_{\text{ops},k} = \sum_{s=2}^{N_u} \Delta u_{cs} (\lambda_s) = \sum_{s=2}^{N_u} |u_s (\lambda_s) - u_{s-1} (\lambda_s)| \]

\( n_{\text{ops}}^{\text{max}} \) is the maximum number of allowable switch operations for a capacitor from the first control sequence at the first load setting \( (u_{cs} (\lambda_1)) \) to the control sequence at the final load setting \( (u_{cs} (\lambda_{N_{\text{fu}}})) \).

In the next section, the constraints on direct load control participants are presented.

### 2.3.2.3. DIRECT LOAD CONTROL (DLC) PARTICIPANT CONSTRAINTS

Demand Response (DR) is an economic tool used by the electric utility industry to provide an incentive for an electric customer to reduce their real power electric consumption for a limited period of time (i.e. several hours). Demand response is typically enacted during critical peak load periods when congestion in the network is the greatest or when the reliability of the power system is a concern. DR participants can be
assigned a price forecast based on their location, length of time and amount of load that is reduced.

Demand response loads are categorized as two types; load that is controlled by the utility and load that is controlled by the consumer. This thesis considers the first type of demand response load, the portion of the real power load that can be directly shed by the utility’s system operator. This network operations technique is referred to as Direct Load Control (DLC).

This thesis is concerned with what effect DLC has on established capacitor sizes, steady-state control settings and realizable control sequences for the voltage spread reduction (VSR) objective. A constraint on the amount of participating per phase maximum power at DLC load buses was included. All DLC participants should be within acceptable per phase real power ($P$) and reactive ($Q$) power limits at their respective load locations. This constraint is written mathematically as:

\[
\left( P_D(V_k,u_s,\lambda_k) \right)^2 + \left( Q_D(V_k,u_s,\lambda_k) \right)^2 \leq \left( S_D^{\text{max}} \right)^2 \quad \forall \, D \in D_{\text{DLC}}, \forall \, k = 1,..,N_{\text{LL}} \tag{2.3.13}
\]

where $P_D(V_k,u_s,\lambda_k)$ and $Q_D(V_k,u_s,\lambda_k)$ are respectively the real and reactive power flow at a DLC load bus $D$, at the current control setting $s$, and for a load setting $k$. $S_D^{\text{max}}$ is the maximum apparent power for the DLC load bus $D$, for current control setting $s$, and for a load setting $k$. $D_{\text{DLC}}$ is the set of DLC participant buses. This constraint is required for all DLC load buses.

Next, a discussion on reducing the size of the problem’s search space is given.
2.4. REDUCING THE PROBLEM'S SEARCH SPACE

The problem's search space is very large. Consequently, in this thesis, practical engineering constraints were used to reduce the search space size and the problem's complexity. This thesis has been applied to PPL Electric Utilities in-service distribution systems. Thus PPL's existing equipment, associated control and communications system and maintenance requirements were considered to reduce the search space size. In particular,

- Underground buses are removed from the capacitor placement search space.
- Capacitors are placed at three-phase buses.
- Capacitors are controlled as switchable banks; limited to two states, either on or off.
- Control sequences that exceed the maximum number of switch operations are eliminated.

Similar reductions of the search space size are made when direct load control participants are included in the control problem. For instance, all DLC injections are controlled in groups by their respective customer classification and are limited to two states, either on or off. The above practical engineering drivers are presented in the following chapter.

The overall problem formulation is repeated in the next section.
2.5. COMPREHENSIVE PROBLEM FORMULATION

The problem formulation and respective equations numbers is repeated below in its entirety.

\[
\min \max_{u_i \in U} \left\| V_i^p (u_s, \lambda_k) - V_j^p (u_s, \lambda_k) \right\| \quad \forall \ k \quad k = 1 \ldots N_{LL} \tag{2.3.1}
\]

\[
\min \sum_{k=1}^{N_{th}} C_{L,k} T_k P_{Loss,k} (V_k, u_s, \lambda_k) \tag{2.2.2}
\]

Subject to:

\[
F (V_k, u_s, \lambda_k) = 0 \quad \forall \ k \quad k = 1 \ldots N_{LL} \tag{2.3.1}
\]

\[
\exists \text{ a control sequence:}
\]

\[
u_{cs} (\lambda_k) = [u_1 (\lambda_k), \ldots, u_{s-1} (\lambda_k), u_s (\lambda_k), u_{s+1} (\lambda_k), \ldots, u_{N_{th}} (\lambda_k)] \quad \forall \ k, \ k = 1 \ldots N_{LL} \tag{2.3.2}
\]

\[
\left| V_i^{\text{min}} \right| \leq \left| V_i^p (u_s, \lambda_k) \right| \leq \left| V_i^{\text{max}} \right| \quad \forall \ i \in \{1, \ldots, N\}, \ p \in \{a, b, c\} \ & \forall \ k = 1 \ldots N_{LL} \tag{2.3.4}
\]

\[
\left| I_{ij}^{\text{min}} \right| \leq \left| I_{ij}^p (V_k, u_s, \lambda_k) \right| \leq \left| I_{ij}^{\text{max}} \right| \quad \forall \ ij \in N_{br}, \ p \in \{a, b, c\} \ & \forall \ k = 1 \ldots N_{LL} \tag{2.3.5}
\]

\[
\left( P_i (V_k, u_s, \lambda_k) \right)^2 + \left( Q_i (V_k, u_s, \lambda_k) \right)^2 \leq \left( S_i^{\text{max}} \right)^2 \quad \forall \ i \in N_F, \ & \forall \ k = 1, \ldots, N_{LL} \tag{2.3.6}
\]

\[
\left| V_i^p (u_s, \lambda_k) - V_i^p (u_{s-1}, \lambda_k) \right| \leq V_{\text{rise}}^{\max} \quad \forall \ i \in \{2, \ldots, N\}, \ p \in \{a, b, c\}, \ & \forall k = 1 \ldots N_{LL} \tag{2.3.7}
\]
\[ Q_{p}^{\min} \leq Q_{\text{sub},k}(V_{k},u_{s},\lambda_{k}) \leq Q_{p}^{\max} \quad p \in \{a,b,c\} \quad \& \quad \forall \ k = 1\ldots N_{LL} \]  

(2.3.8)

\[ n_{\text{cap}}^{\text{new}} \leq n_{\text{cap}}^{\text{max,new}} \]  

(2.3.9)

\[ n_{\text{cap}}^{\text{total}} \leq n_{\text{cap}}^{\text{max}} \]  

(2.3.10)

\[ n_{\text{bank},i}^{\min} \leq n_{\text{bank},i}^{p} \leq n_{\text{bank},i}^{\max} \quad \forall \ i \in \{1\ldots,n_{\text{cap}}^{\text{total}}\}, \quad p \in \{a,b,c\} \]  

(2.3.11)

\[ \sum_{k=1}^{N_{LL}} n_{\text{ops},k}^{p} \leq n_{\text{ops}}^{\max} \]  

(2.3.12)

\[ \left(P_{D}(V_{k},u_{s},\lambda_{k})\right)^{2} + \left(Q_{D}(V_{k},u_{s},\lambda_{k})\right)^{2} \leq \left(S_{D}^{\max}\right)^{2} \quad \forall \ D \in D_{DLC}, \& \quad \forall \ k = 1\ldots, N_{LL} \]  

(2.3.13)

The notation used in the equations above is repeated below in alphabetical order.

\[ C_{L,k} \] : substation specific cost per kWh due to losses at the \( k \)th load setting

\[ D_{DLC} \] : is the set of DLC participant buses

\[ F(V_{k},u_{s},\lambda_{k}) \] : are the power flow equations a set of non-linear algebraic equations specific to a distribution system

\[ \left|I_{ij}^{\max}\right|, \left|I_{ij}^{\min}\right| \] are the maximum and minimum acceptable current magnitude through branch \( ij \)

\[ \left|I_{ij}^{p}(V_{k},u_{s},\lambda_{k})\right| \] is the current magnitude through the branch from bus \( i \) to bus \( j \), for phase \( p \), at the current control setting \( s \), and at load setting \( k \).
\( \lambda_k \): continuous load parameters, a vector of nominal nodal complex powers at \( k^{th} \) load setting, \( \lambda_k \in \mathbb{C}^{3N}, \lambda_k = P_k^p + jQ_k^p \)

\( (n_{\text{bank},i}^p) \): the number of per phase banks at a bus \( i \)

\( (n_{\text{bank},i}^{\text{max}}, n_{\text{bank},i}^{\text{min}}) \): are the maximum and minimum number of banks per phase at a bus \( i \)

\( n_{\text{cap}}^{\text{exist}} \): is the total number of existing capacitor locations prior to placement

\( n_{\text{cap}}^{\text{max}} \): is the maximum number of allowed remote automated three-phase capacitors in a system

\( n_{\text{cap}}^{\text{max},\text{new}} \): is the maximum number of allowed new capacitor locations

\( n_{\text{cap}}^{\text{new}} \): is the number of new remote automated capacitor locations

\( n_{\text{cap}}^{\text{total}} \): is the total number of remote automated capacitors in a system, \( n_{\text{cap}}^{\text{total}} = n_{\text{cap}}^{\text{exist}} + n_{\text{cap}}^{\text{new}} \)

\( n_{\text{ops},k} \): is the number of per capacitor switch operations for a control sequence at load setting \( k \) where \( u_{cs}(\lambda_k) = [u_1(\lambda_k), \ldots, u_{s-1}(\lambda_k), u_s(\lambda_k), u_{s+1}(\lambda_k), \ldots, u_{N_5}(\lambda_k)] \)

\( n_{\text{ops}}^{\text{max}} \): is the maximum number of allowable switch operations for a capacitor from the first control sequence at the first load setting \( (u_{cs}(\lambda_1)) \) to the control sequence at the final load setting \( (u_{cs}(\lambda_{N_5})) \)

\( N \): is the set of all buses

\( N_{Br} \): is the set of all energized branches

\( N_F \): is the set of all feeders or transformers in the system
$N_{Li}$ : is the total number of load settings (levels/profiles)

$p$ : are the set of present phases \{a,b,c\} at a bus i or at a bus j

$P_D (V_k, u_s, \lambda_k)$: is the real power flow at a DLC load bus $D$, for control settings $s$, and for a load setting $k$.

$P_i (V_k, u_s, \lambda_k)$: is the real power flow through a feeder or transformer $i$, at the current control setting $s$, and for a load setting $k$.

$P_{Loss,k}$: is a network's total real power losses at $k^{th}$ load setting

$Q_D (V_k, u_s, \lambda_k)$: is the reactive power flow at a DLC load bus $D$, for control settings $s$, and for a load setting $k$.

$Q_i (V_k, u_s, \lambda_k)$: is the reactive power flow through a feeder or transformer $i$, at the current control setting $s$, and for a load setting $k$.

$Q_k^{p,\text{max}}, Q_k^{p,\text{min}}$ are the maximum and minimum reactive power out of the substation into a given feeder/system for load for a phase $p$ and load setting $k$.

$Q_{sub,k} (V_k, u_s, \lambda_k)$ is the reactive power out of the substation into a given feeder/system, for a phase $p$, for the control settings $s$ and a load setting $k$.

$S_{D}^{\text{max}}$ : is the maximum apparent power of a DLC participant at a bus $D$

$S_i^{\text{max}}$ is the maximum apparent power for the feeder or transformer $i$

$T_k$ : time duration in hours per load setting $k$ for losses

$u_i$ : the initial control settings; control settings at the previous load level $u_s (\lambda_{k-1})$

$u_{N,s}$ : the final (optimal) control settings at the current load level $u_s (\lambda_k)$
$u_{cs}(\lambda_k)$: is a realizable control sequence, a feasible set of binary control actions that transition the device control settings from $u_1$ to $u_{Ns}$ at a given load setting $k$

$u_{cs}(\lambda_k) = \left[ u_1(\lambda_k), \ldots, u_{s-1}(\lambda_k), u_s(\lambda_k), u_{s+1}(\lambda_k), \ldots, u_{Ns}(\lambda_k) \right]$

$u_{cs}(\lambda_{k_1})$: is the control sequence at the first load setting $\lambda_{k_1}$

$u_{cs}(\lambda_{N_{kl}})$: is the control sequence at the final load setting $\lambda_{N_{kl}}$

$\Delta u_{cs}(\lambda_k)$: is the change in control settings from the initial control settings $(u_1(\lambda_k))$, to the final control settings $(u_{Ns}(\lambda_k))$, at a load setting $k$

$u_s$: are the discrete control variables; a binary vector of nodal device settings at load setting $k$, $u_s: [n_{dis} \times 1], u_s \in [0,1]$

$U$: is the search space of capacitor placement and device control schemes

$|V_i^p|, |V_j^p|$: are the voltage magnitudes at bus $i$ and bus $j$, for a phase $p$

$V_i^{\text{max}}, V_i^{\text{min}}$: are respectively, the maximum (max) and minimum (min) operating voltage magnitude at a bus $i$

$V_k$: are the continuous state variables, a vector of complex node (bus and phase) voltages at $k^{\text{th}}$ load setting, $V_k \in \mathbb{C}^{3N}$

$|V_i^p(u_s, \lambda_k)|$: is the voltage magnitude at a bus $i$, and a phase $p$ at the current control setting $s$ for the load setting, $k$

$|V_i^p(u_{s-1}, \lambda_k)|$: is the voltage magnitude at a bus $i$, and a phase $p$ at the previous control setting $s-1$ for the load setting, $k$

$V_{\text{rise}}^{\text{max}}$: is the maximum allowable voltage rise between network device operations
In the last section a chapter summary is provided.

2.6. SUMMARY

In this chapter the problem formulation for realizable constraint driven capacitor placement and control sequences for the voltage spread and real power loss reduction objectives was presented. Selected network resources such as direct load control participants and photovoltaic generation injections were identified as additional injections to be studied. A discussion on these selected network devices individual modifications and impacts to the problem formulation was provided. Additionally, two new practical constraints were introduced to typical problem formulation, a voltage rise constraint and a substation reactive power constraint.

The formulation provided in this thesis differs from other approaches for solving capacitor control problems because other methodologies do not consider constraints between control actions. Existing formulations and subsequent techniques provide control settings for various load levels, but do not provide a feasible path of actions to move between the devices’ control settings. Thus the provided solutions from existing strategies may be unattainable.

It was stated that the placement and control problems are solved sequentially. Still the individual placement and control problems are very large. In the following chapter the engineering logic used to reduce the problem’s complexity is presented.
3. ALGORITHM CONSIDERATIONS

In this chapter, algorithmic considerations are presented for the capacitor placement problem and control problem. The placement and control problems are very large and engineering reasoning is applied to limit the size of the search space. There are considerations that are used for both placement and control problems and there are some logic that only applies to each individual problem. In order to restrict the search space size the following rationale is discussed:

- network and utility operating limits
- feasible switching sequences
- mutual proximity of the capacitor installations
- method of load data collection and selection for placement
- organizing and categorizing a systems’ load

In the following subsections, the considerations identified for placement and control are discussed. Distribution and transmission system interconnection considerations are provided first. Then constraint considerations that facilitate finding a sequence of control actions to transition across multiple load settings for distribution system operators are provided. Subsequently, the considerations that limit capacitor placement, the logic used for load level identification and selection, and the rational used to classify distribution system load which limits the number of direct load control options are provided. Lastly, the chapter summary in section 3.6 reviews the highlighted algorithm viewpoints.
3.1. Utility Substation Reactive Power Requirements

In this thesis, substation reactive power constraints are employed and relaxed in order to investigate the effects of their inclusion on the objectives. Reactive power in transmission systems is highly regulated by Independent System Operators and Regional Transmission Organizations in accordance with North American Electric Reliability Corporation (NERC) standard VAR-001-3 [1] and TOP-001-3 [2]. Synchronous generators, capacitors, static var compensators are sources of reactive power used in a transmission system to maintain acceptable transmission bus voltages and to reduce real power losses. With respect to transmission systems, distribution systems are modeled in aggregate, as loads. Previous works omitted transmission system interconnection considerations.

The substation reactive power in a distribution network can be calculated as a quantity or as a ratio of real power to apparent power (power factor). Regardless of the method chosen to measure reactive power, the transmission system operator restricts the distribution system reactive power at the substation in order to absorb reactive power in order to maintain a power factor of one.

Sustaining a lagging reactive power at distribution substations ensures that transmission system voltages are held to acceptable limits and prevents voltage dropping in the sub-transmission system which would lead to an increased current and possible line overloading. Yet additional reactive power could possibly be supplied from the distribution system to assist the transmission system when there is heavy loading.

Adhering to reactive power guidelines set by the transmission system operators limits the amount of capacitance that can be placed in a distribution system. In
distribution systems multiple circuits may connect to a single substation. For control purposes, an individual circuit's reactive power limits could be relaxed provided the substation's total reactive power is within the transmission range at peak load. Therefore in the problem formulation, a substation’s per phase reactive power limits (Eq. 2.3.8) are relaxed and instead the substation’s total reactive power, \( Q_{t}^{\min} \leq Q_{\text{total},k}^{\max} \left( V_{k}, u_{k}, \lambda_{k} \right) \leq Q_{k}^{\max} \) is held to the transmission specified range at peak load.

3.2. Feasible Switching Sequences

Given a capacitor placement, many global optimization schemes have been shown to be successful at finding high quality solutions to the capacitor control problem [5]. In many cases, however, these results provide no means for system operators to realize the computed solutions. Algorithms must then be designed to provide such a sequence directly or utilize post-processing techniques to constructively determine if a feasible sequence exists [49]. In this thesis, enforcing voltage rise from [48] and requiring reactive power constraints allows for a functional check that the solution provided will also be physically realizable by system operators.

This thesis studies the implementation of a capacitor control setting and differs from previous works which:

- omitted explicit constraints between control actions
- provide control settings for various load levels, but no feasible path to move between the load settings
The solutions found using traditional capacitor and placement control methodologies are more difficult for distribution system operators to implement as they may cause unintended constraint violations.

3.3. Placement Proximity

Considering the proximity in which capacitors are installed in a network will reduce the placement search space size because available locations near existing, replaced, and newly placed capacitors are eliminated. Capacitor installation spacing requirements may be specified as a number of electrically connected buses or as a computed electrical or physical distance (Z).

Here, the placement proximity is defined as the number of electrically connected buses between any two buses with capacitors. For example, Figure 3.1 provides a network diagram for a portion of a distribution system and the minimum placement proximity is defined as at least two buses away from existing or newly placed capacitors. The diagram enumerates buses upstream and downstream of an existing capacitor and identifies the distance required for a new capacitor placement.
In the following section the impacts of available load data and load models on the placement and control problem is discussed.

### 3.4. Load Level Identification and Selection

The type of load measurements available and the load model plays a significant role in identifying and selecting load levels. For the presented cases, access to, automatic meter reading (AMR) data of multi-phase loads is available [50]. Therefore, in this thesis all loads are represented in the system at their measured locations. Additionally, by using measured load data, distinct load levels over which to place capacitors and to compute capacitor control actions were identified.

In this thesis, load settings are defined via AMR and scaling as follows:

- a peak and a 70% load profile is created using AMR load data
- a minimum load level is produced from scaling the peak profile
These load settings are applied to the control problem which includes direct load control participants and photovoltaic generators. Here a static set of annual measured load data which disregards seasonal and daily load changes is used. Other approaches may utilize sets of seasonal or daily load profiles to determine control decisions.

In the following section, distribution system load classification is presented.

3.5. DISTRIBUTION SYSTEM LOAD CLASSIFICATION

Distribution system load classification is considered in order to manage direct load control participants and photovoltaic generators in the control problem’s solution search space. In this thesis, a system’s peak load data is used to:

- determine the type of customers that are present
- limit the number of participants in a given class
- reduce the number of direct load control variables applied to finding a control sequence.

The Pennsylvania Utility Commission’s (PUC) 2012 monthly electric usage ratepayer categories and PPL customer count data was used to divide the provided AMR peak load data for a given system into customer classifications. The PUC’s ratepayer classifications and usage assumptions listed in their commission rate comparison report are displayed in Table 3.1.
Table 3.1 Bureau of Technical Utility Services, "Pennsylvania Public Utility Commission Rate Comparison Report" Pennsylvania PUC, April 30, 2012 [51]

<table>
<thead>
<tr>
<th>PUC’s Ratepayer Classification</th>
<th>Monthly Electric Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>500 kWh</td>
</tr>
<tr>
<td>Residential Heating</td>
<td>2,000 kWh</td>
</tr>
<tr>
<td>Small Commercial</td>
<td>5 kW demand 1,000 kWh</td>
</tr>
<tr>
<td>Medium Commercial</td>
<td>25 kW demand 10,000 kWh</td>
</tr>
<tr>
<td>Large Commercial</td>
<td>500 kW demand 200,000 kWh</td>
</tr>
<tr>
<td>Industrial</td>
<td>1,000 kW demand 400,000 kWh</td>
</tr>
<tr>
<td>Public Street Lighting (sodium vapor)</td>
<td>250 W</td>
</tr>
</tbody>
</table>

The monthly usage for each classification is used to define the lower bound on a customer class and ranges up to the next customer class’ monthly usage. The data in Table 3.1 makes it possible to categorize the customer loads of a distribution network by their size and count. The following table lists the distribution system customer classification (DS Class), their corresponding kW range and customer type grouping. Four DS Class categories are used in the extended control problem (DLC & PhV) in order to select and assign parameters to groups and to limit the number of control variables. The loads are classified as either a residential customer (R), a small commercial customer (S), a medium commercial customer (M), or as a large commercial/industrial customer (L/I). The (L/I) customers are grouped because residential distribution systems do not typically contain many customers with loads greater than 500 kW and some may lack (L/I) customers entirely.
Table 3.2 Distribution System Customer Classification and Ranges (kW)

<table>
<thead>
<tr>
<th>Distribution System Customer Classification</th>
<th>Range (kW)</th>
<th>Customer Type Grouping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Street Lighting</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>Residential</td>
<td>0.25 - 5</td>
<td>(R)</td>
</tr>
<tr>
<td>Small Commercial</td>
<td>5 - 25</td>
<td>(S)</td>
</tr>
<tr>
<td>Medium Commercial</td>
<td>25 - 500</td>
<td>(M)</td>
</tr>
<tr>
<td>Large Commercial</td>
<td>500 - 1000</td>
<td>(L/I)</td>
</tr>
<tr>
<td>Industrial</td>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

Each distribution network’s load composition is unique and may not include all ratepayer categories. Additionally, the above electric usage ranges and categories are specific to Pennsylvania. Distribution system classification of load enables the following:

- Loads to be categorized solely on kW size by applying Table 3.2 kW ranges together with customer count data (i.e. No customer private data).
- Customer type group assignment of parameters and control actions instead of individual customer location specific parameters and controls.

As a result of grouping customers into customer classes, the number of direct load control variables in the solution search space is reduced (i.e. \( n_{DLC}^{\text{total}} = 4 \times 1 \)) and

\[
\mathbf{u}_s : [n_{DVC}^{\text{total}} \times 1] = \left[ \left[ n_{cap}^{\text{total}} , n_{DLC}^{\text{total}} \right] \times 1 \right]
\]

The following section reviews the key points of the algorithmic considerations chapter.
3.6. **Summary**

In this chapter the algorithmic considerations for capacitor placement and control were provided. The following main points from each section are:

- The amount of capacitance that is placed in a distribution system is inhibited by the reactive power guidelines.
- A constraint on the substation’s total reactive power is used in the solution algorithm instead of a substation’s per phase reactive power limits.
- Physically realizable control sequences are ensured by performing voltage rise and reactive power constraints validation in the solution algorithm.
- The number of locations for placements are reduced using placement proximity.
- All loads in the network exist at their measured locations and the load levels are defined using both AMR and scaling.
- Classification of load reduces the number of direct load control variables by eliminating individual parameter and controls assignments.

The next chapter presents the solution algorithm for the capacitor placement and control sequence problems.
4. SOLUTION ALGORITHM

In this thesis, the capacitor placement and control sequence problem is divided into three stages. The first stage is the capacitor placement problem, the second stage is the control settings problem and the third stage is the control sequence determination problem. Figure 4.1 shows the hierarchy of the solution algorithm stages.

The voltage spread reduction (VSR) and real power loss reduction (RPLR) objectives are considered separately for the capacitor placement and control problems. The placement and control setting results are used to determine feasible control sequences for the given load settings. Subsequently, the control sequence problem is extended to include direct load control (DLC) participants and photovoltaic (PhV) generation.

This chapter is outlined as follows:

- Section 4.1 provides the capacitor placement algorithm
- Section 4.2 presents the control settings algorithm
- Section 4.3 applies a dynamic programming framework to the control sequence problem
- Section 4.4 provides the feasible control sequence algorithm
- Section 4.5 gives methods used to include DLC and PhV in the problem
- Section 4.6 is the chapter summary
Solution Algorithm Hierarchy:

Capacitor Placement & Control Sequence Determination for Selected Network Devices

1. Capacitor Placement Problem
   - Objectives:
     1. Voltage Spread Reduction (VSR)
     2. Real Power Loss Reduction
   - Sub-analysis: Test Reactive Power Constraints
   - Answers:
     - Where the caps should be installed.
     - How much (kVar) to install.

2. Capacitor Control Sequence Problem
   - Analysis:
     1. Cap Control Settings at all Load Profiles/Levels for VSR/RPLR
     2. Determine Realizable Cap Control Sequence
     - Answers:
       - When should capacitors participate in control (i.e. how long, how often, order in which caps switch)
       - Can a feasible control sequence for given control settings be found?

3. Extended Control Problem: Caps w/ DLC & Caps w/PhV
   - Separate Problems to Study:
     1. Add DLC to Circuit
     2. Add PhV to Circuit
   - Analysis:
     1. Device Control Settings at all Load Profiles/Levels for VSR
     2. Determine Realizable Device Control Sequence
     - Answers:
       - Do capacitors control statuses and sequences change? (i.e. who, when, what is the new order to switch)
       - How do the values of objectives and constraints change? (i.e. voltage spread, real power loss, reactive power)
       - Can capacitor operations or installations be postponed?

Note: The select network devices which are included in the control status & sequence problem are as follows:
1. Direct Load Control (DLC) Participants whose control status is controlled by system operators
2. Photovoltaic (PhV) Generators which are uncontrolled by system operators

Figure 4.1. Hierarchy of Solution Algorithm Stages
4.1. Capacitor Placement Algorithm

The capacitor placement algorithm determines the locations and sizes of capacitors to be installed at the peak load setting. The peak load setting has been selected for placement because the greatest amount of reactive power will be needed to maintain the voltage along a feeder in comparison to other load settings.

In this thesis, greedy heuristics for VSR and RPLR were developed and used to place capacitors. VSR placement was greedy with respect to the voltage spread. RPLR placements were sensitivity based [52] and greedy with respect to real power losses. The greedy heuristic methods are presented after the placement algorithm.

The steps for the capacitor placement algorithm are:

1. Initialize the capacitor size to be placed to the largest available capacitor size, 
   \[ n_{\text{bank},i}^p = n_{\text{bank},i}^{\max}. \]

2. Initialize, \( n_{\text{cap}}^{\text{new}} = 0 \)

3. Run a multi-phase unbalanced power flow using peak load data.

4. Post-process the solution. Calculate and record the voltage spread/real power losses for the network. Check for bus voltages, branch flows, and transformer/feeder violations, if none exist, go to Step 5. Else go to Step 4a.

4a. If \( n_{\text{cap}}^{\text{new}} = 0 \), stop exit the placement algorithm. Base case violations are outside the scope of work in this thesis. Else, follow capacitor size reduction steps then return to Step 3.
Step 5. Check for substation reactive power $Q_{k}^{\min} \leq Q_{\text{sub},k}^{\text{total}} (V_k, u_s, \lambda_k) \leq Q_{k}^{\max}$ violations. If $Q_{\text{sub},k}^{\text{total}} (V_k, u_s, \lambda_k) > Q_{k}^{\max}$ then more capacitance is needed, go to Step 6. Else if, $Q_{\text{sub},k}^{\text{total}} (V_k, u_s, \lambda_k) > Q_{k}^{\min}$ then less capacitance is needed. Follow capacitor size reduction steps then return to Step 3. Else, reactive power constraints are satisfied, go to Step 6.

Step 6. Apply the greedy heuristic method for voltage spread reduction (VSR) or real power loss reduction (RPLR) objectives to determine the location of a new or replaced capacitor.

Step 7. Get the saved power flow results for new capacitor (location and size). Calculate and record the voltage spread/real power losses according to the objective.

Step 8. Compare new capacitor case results to the previous case results. Does the placement cause the voltage spread/real power loss solutions increase? If yes exit the program, the previous case results are placement solution. Else continue to Step 9.

Step 9. Update $\left( n_{\text{cap}}^{\text{new}} = n_{\text{cap}}^{\text{new}} + 1 \right)$. Check $n_{\text{cap}}^{\text{exist}} + n_{\text{cap}}^{\text{new}} \leq n_{\text{cap}}^{\max}$? If the maximum number of capacitors have been placed into the network, stop and exit the program the solution has been found. Else, more capacitors can be placed return to Step 6.

In the next sub-section the steps for reducing a capacitor’s size are given. Section 4.1.1 and Section 4.1.3 present the VSR & RPLR placement algorithms.
4.1.1. **Capacitor Size Reduction Steps**

The steps to reduce a capacitor’s size are listed below:

Step 1. Locate the farthest existing capacitor downstream of the substation or last added capacitor. Go to Step 2.

Step 2. Perform capacitor size reduction. If the reduced capacitor size is less than the minimum size \(n_{\text{bank},i}^p < n_{\text{bank},i}^\text{min}\), Stop, remove the capacitor and decrease the max number of capacitors \(n_{\text{cap}}^{\max}\) and corresponding number of existing or new capacitors \(n_{\text{cap}}^{\text{exist}} / n_{\text{cap}}^{\text{new}}\). Else, apply the reduced capacitor size to the existing or new placement.

In Step 1, the farthest existing capacitor downstream of the substation is found using bus level and lateral indexing from [53] and the last added automatic capacitor location is retrieved from a stored list. Once the capacitor to reduce is located, the location’s total capacitor size is reduced by 300 kVAR.

A flow chart which outlines the steps for the capacitor placement algorithm are given in Figure 4.2. The greedy heuristic methods for the VSR and RPLR objectives which are used to determine the location of new or replaced capacitor are presented in the next section of this chapter.
Start

Program settings:
\[ k = 100\% \text{ profile} \]

\[ n_{\text{cap},i} = n_{\text{cap},i}^{\text{max}} \]

\[ n_{\text{cap}} = n_{\text{cap}}^{\text{max}} \]

\[ n_{\text{cap}} = 0 \]

Run unbalanced multi-phase power flow

\[ V_i, \theta_i, (u_i) \]

Post-process the solution. Calculate \(|I|, |S|, Q_{\text{out}}, P_{\text{loss}}, \) & Max Voltage Spread metrics

Are the \(|V|, |I|, \) & \(|S|\) constraints satisfied?

Are \(Q\) constraints satisfied?

Apply greedy heuristic method for VSR and/or RPLR objective to find a 3-phase capacitor placement location. Internally, \(n_{\text{cap}}\) is incremented.

\[ n_{\text{cap}} + n_{\text{cap}} = n_{\text{cap}} \]

Stop & Exit Program

Store power flow solution and calculated metrics

Figure 4.2 Capacitor Placement Algorithm Flow Chart
4.1.2. **Voltage Spread Reduction Heuristic Algorithm**

In this thesis, three-phase buses are used to place capacitors. It is assumed that an improvement at the minimum per phase bus voltage (nodal voltage) in the system will improve the system’s overall voltage spread. Therefore, once the minimum node voltage is located the nearest three-phase bus is found for placement using bus level and lateral indexing from [53]. The algorithm is greedy with respect to the voltage and the procedure to find the capacitor locations for placement are as follows.

Step 6.1. Compile a list of all buses.

Step 6.2. Remove all underground buses from the list.

Step 6.3. Find the minimum per phase bus voltage and store to a list. Sort the list in ascending order and select the node at the top of the sorted list.

Step 6.4. Locate the nearest three-phase bus using bus level and lateral indexing from [53]. Pick this three-phase bus location for placement.

Step 6.5. Check if the bus is located near other capacitors using the fixed placement proximity. If yes, remove this three-phase bus location and the per phase bus location from the list go to Step 6.5.a. Else go to Step 6.6.

Step 6.5.a. Check have all available bus locations been removed? If no more available locations exist, stop, all available new locations for placement have been checked. Replacement steps are not included in this algorithm. Exit VSR placement algorithm. Else return to Step 6.3 to select next available location.

Step 6.6. Place a capacitor at this bus location.
Step 6.7. Run a multi-phase unbalanced power flow with peak load data. Post-process the solution. Save results and continue.

Step 6.8. Check for substation reactive power violations.

\[ Q_{k,\text{min}}^p \leq Q_{\text{sub},k}^p \left(V_k, u_k, \lambda_k\right) \leq Q_{k,\text{max}}^p. \]

If \( Q_{\text{sub},k}^p \left(V_k, u_k, \lambda_k\right) > Q_{k,\text{max}}^p \) then more capacitance is needed go to Step 6.9. Else if \( Q_{\text{sub},k}^p \left(V_k, u_k, \lambda_k\right) > Q_{k,\text{min}}^p \) then less capacitance is needed. Follow capacitor size reduction steps and return to Step 6.7. Else the substation reactive power limits are satisfied, continue.

Step 6.9. Check for bus voltages, branch flows, and transformer/feeder violations. If violations exist, and \( n_{\text{bank},i}^p > n_{\text{bank},i}^{\text{min}} \) then reduce the capacitor size according to capacitor size reduction steps then return to Step 6.7 Else if \( n_{\text{bank},i}^p < n_{\text{bank},i}^{\text{min}} \), remove the capacitor, remove this capacitor location from available locations list and return to Step 6.3. Else the constraint limits are satisfied go to Step 6.10.

Step 6.10. Stop, a placement location for VSR is found. Exit VSR placement algorithm, and go to Step 7 of the main Capacitor Placement Algorithm.

Next the method used to place capacitors to minimize the real power losses in a network is presented.
4.1.3. **Real Power Loss Reduction Heuristic Algorithm**

The partial derivative of the real power loss with respect to reactive power is used to place new capacitors and to relocate capacitors for the real power loss reduction objective. The algorithm is greedy with respect to the real power losses and the procedure to find the capacitor locations for placement are as follows.

Step 6.1. Compile a list of all three-phase buses.

Step 6.2. Remove all underground buses from the list.

Step 6.3. Calculate the nodal sensitivity matrix according to [52]. Obtain the real power loss with respect to the reactive power \( \frac{\partial P_{loss}}{\partial Q} \) vector. It is noted that the vector is predominately negative with exception of zeros for buses near the substation.

Step 6.4. Find the minimum across all three-phases and store to a list. Sort the list in ascending order and select the bus at the top of the sorted list.

Step 6.5. Check if the bus is located near other capacitors using the fixed placement proximity. If yes, remove this bus location from the list go to Step 6.5.a. Else go to Step 6.6.

Step 6.10.a. Check have all available bus locations been removed? If no more available locations exist, all available new locations for placement have been checked. Replacement steps are not included in this algorithm. Exit RPLR placement algorithm. Else return to Step 6.4 to select next available location.

Step 6.6. Place a capacitor at this bus location.
Step 6.7. Run a multi-phase unbalanced power flow with peak load data. Post-process the solution. Save results and continue.

Step 6.8. Check for substation reactive power violations.

\[ Q_{k,p}^{\min} \leq Q_{sub,k}^{p}(V_k, u_s, \lambda_k) \leq Q_{k,p}^{\max}. \]

If \( Q_{sub,k}^{total}(V_k, u_s, \lambda_k) > Q_{k,p}^{\max} \) then more capacitance is needed go to Step 6.9. Else if \( Q_{sub,k}^{total}(V_k, u_s, \lambda_k) < Q_{k,p}^{\min} \) then less capacitance is needed. Follow capacitor size reduction steps and return to Step 6.6. Else the substation reactive power limits are satisfied, continue.

Step 6.9. Check for bus voltages, branch flows, and transformer/feeder violations. If violations exist, and \( n_{bank,i}^{p} > n_{bank,i}^{\min} \) then reduce the capacitor size according to capacitor size reduction steps then return to Step 6.7 Else if \( n_{bank,i}^{p} < n_{bank,i}^{\min} \), remove the capacitor, remove this capacitor location from available locations list and return to Step 6.4. Else the constraint limits are satisfied go to Step 6.10.

Step 6.10. Stop a placement location for RPLR is found. Exit RPLR placement algorithm, and go to Step 7 of the main Capacitor Placement Algorithm.

The algorithms used to determine capacitor placements have been presented. The following sections focuses on capacitor control settings and realizable sequences determination for all load settings.
4.2. CONTROL SETTINGS ALGORITHM

In Stage 2 of the capacitor placement and control sequence problem, the placement results from Stage 1 are supplied to the control problem in order for the control settings at all load settings can be determined.

The primary assumption for the control setting algorithm is that the total number of controllable devices in the network is small. Economic and other practical engineering issues may limit the total number of controlled devices in a system. When large numbers of controllable devices exist, other optimization routines can be implemented which may yield suboptimal yet feasible control settings. Given that the number of controllable devices is small, the exhaustive search of device control actions taken 1-at-a-time is performed to determine the control actions at a given load setting.

The assumptions for the control settings algorithm are:

- The number of devices in a network for control is less than 10 devices
  $\left(n_{dvc}^{total}\right) \leq 10$.
- All devices for control are automated, and are limited to 2 control actions, either on or off. Thus binary control settings with size $2^{n_{dvc}^{total}}$.
- All capacitors at the peak load setting are turned on.
- Direct load control participants are controlled as groups of customers.

In the problems addressed, the number of remote automated capacitors in a network may be limited by the Distribution Management System (DMS) communications equipment which is required to enable and support switchgear. Furthermore, due to combinatorics the DLC participant locations are grouped by customer types so that the number of devices available for control is tractable. Using the stated assumptions, the
procedure to determine the controls settings at each load setting is given as a flow chart in Figure 4.3.
Figure 4.3 Control Settings Algorithm Flow Chart
Given that the control settings at the provided load settings for all switchable devices are found, then the next stage of the solution algorithm is to find a feasible control sequence. In the next section, concepts from dynamic programming are borrowed in order to explain the foundation (set-up) of the feasible control sequence algorithm.

4.3. Dynamic Programming Framework

The framework of solving a problem via dynamic programming is similar to the structure of the algorithm used to determine feasible control sequences. Therefore, in order to explain the structure of the control sequence algorithm the dynamic programming theory and framework is briefly reviewed.

Dynamic Programming is a search algorithm developed in the 1950’s and was formulated using Richard Bellman’s Principal of Optimality [54] which states:

“There optimal policy has the property that, whatever the current state and decision, the remaining decisions must constitute an optimal policy with regard to the state resulting from the current decision.”

Dynamic programming (DP) divides an optimization problem into a sequence of sub-problems. Each sub-problems’ optimal solution is sought and the solution to a sub-problem is dependent on the solution of a previous sub-problem. The total of all sub-problem solutions is the optimal policy [55]. In the next section the sub-problems for control sequences are defined.
4.3.1. **Control Sequence Sub-problems and Stages**

In this thesis, the control sequence problem is to find a feasible sequence of on/off device control action to transition along a given load setting that minimizes the maximum voltage spread. This optimization problem is divided into sequential sub-problems whose solution defines the next stage of the sequence.

For example, the problem is to find a feasible sequence of 1-at-a-time control actions for that minimizes the maximum voltage spread over a day. In Figure 4.4 a 24 hour day is defined into 3 load settings and the goal is to determine $u_{cs} = [u_1, u_2, u_3]$. Here, the load settings are assumed to be symmetric around the third load setting and therefore the forward control sequence path will be identical and opposite for the reverse control sequence or $[u_1, u_2, u_3] = [u_3, u_2, u_1]$.

![Figure 4.4 A 24 Hour Day Defined as Load Settings](image)

Given the placements and control settings at each load setting are known a control sequence $u_{cs}$, is found by dividing the initial sequence problem into two sub-problems, $u_{cs1}$ and $u_{cs2}$.
Sub-problem $u_{cs1}$: Find a feasible path of 1-at-a-time device switch actions to transition from the $u_1$ load setting control positions to $u_2$ load setting control positions and minimizes the maximum voltage spread.

Sub-problem $u_{cs2}$: Find a feasible path of 1-at-a-time device switch actions to transition from the $u_2$ load setting control positions to $u_3$ load setting control positions and minimizes the maximum voltage spread.

This can be visualized through the following example. Given there are a total of 4 controllable devices and there are zero devices on at load setting $u_1$, there are three devices on at load setting $u_2$ and all four devices are on at load setting $u_3$. Then the first sub-problem $u_{cs1}$ with the forward and reverse control sequence path concept is shown in Figure 4.5.

![Figure 4.5 Example of sub-problem $u_{cs1} = [u_1, u_2]$](image-url)

The individual sub-problems are then divided further into sequential smaller sub-problems called stages. Here the following assumptions are applied:

- 1-at-a-time device switch/control actions \( n_{\text{action}}^{\text{stage}} = 1 \), a stage is concerned with solving the problem of turning on or off an individual control device
- the number of switch/control actions per device is limited to two states, either all on or all off

Given \( n_{\text{dvc}}^{\text{total}} = 4 \), the total number of device control actions is calculated as \( 2^{n_{\text{dvc}}^{\text{total}}} = 2^4 = 16 \). Continuing with the same example, the total number of stages is calculated as

\[
C\left(\frac{n_{\text{dvc}}^{\text{total}}}{n_{\text{action}}^{\text{stage}}}\right) = \binom{n_{\text{dvc}}^{\text{total}}}{n_{\text{action}}^{\text{stage}}} = \frac{4!}{1!(4-1)!} = 4 \text{ stages}.
\]

Here, stage 1 is concerned with solving the problem of turning on 1 controllable device, the second stage is concerned with turning on a 2nd device, and so forth until all devices are turned on in the fourth stage.

**4.3.2. CONTROL SEQUENCE STATES**

Within each specific stage the set of possible device control settings are called states. The set of possible device control settings in a stage is a subset of all device control settings. The number of possible states per stage \( n_{\text{States}}^{\text{Stage}} \), can be calculated as:

\[
n_{\text{Stage}}^{\text{States}} = \binom{n_{\text{dvc}}^{\text{total}}}{(\text{Stage} - 1)!\left(\frac{n_{\text{dvc}}^{\text{total}}}{(\text{Stage} - 1)!}\right)}
\]

where a Stage is the individual stage under study. Adding all states for the stages gives the total number of states. In this example, the number of states from the first to fourth stage is 15. When the start stage which has all devices turned off is added to the count the total becomes 16 states. The binary states and individual state counts for each stage are shown below in Figure 4.6.
In DP, states transfer information to make future decisions without regard to how the process reached the current state [55]. Typically, weights are assigned to each state and the weight is independent of how the state is reached. When weights are dependent they cannot be assigned a priori they must be assigned in-situ. For example, [31] solves the capacitor control problem to minimize losses over a 24 hour period using feeder losses as weights. The feeder losses are a function of which capacitors on/off states at each stage. The previous states feeder losses will not hold for the next stage and must be recalculated for each stage of the problem.

In this thesis, weights could be assigned using the voltage spread for each state in a stage, and the voltage spread would need to be recalculated at each stage of the problem. Here, the next step (stage and control states) that yields the minimum voltage spread was selected. However, here the application is for determining the sequence at the same load setting. Other metrics could be used to select the next control state and stage of the sequence. Additionally, using 1-at-time device switch actions automatically provides
the order of devices in the sequence and removes the need of additional weighting to identify device order. In the next section the concept of stepping is defined.

### 4.3.3. **Control Sequence Steps**

A step is defined as the movement from the current state (device control positions) and stage to the next stage and respective state. In DP, each step moves the problem forward (uni-direction) until the final stage, reverse moves are typically not allowed. However, in this thesis, the control sequence algorithm differs from the DP method because reverse steps are included in the set of possible per stage states. Including bi-directional steps increases the search space size, by adding reverse binary states and stages.

Continuing with the example where $n_{dvc}^{total} = 4$, including reverse steps increases the total number of problem stages from 4 to 10, the total number of states (stage and control settings) from 16 to 64 and number of possible steps from 32 to 236. Allowing bi-directional steps to the search space, expands the device control search by three identical groups/collections of the steps, states, and stages. Each collective includes the forward and reverse steps, states, and stages. The three bi-directional groups are identified by the enclosed in dashed lines in Figure 4.7. The initial reverse action of each of the additional bi-directional actions groups occurs at the second, fourth and sixth stages. Thus each additional bi-directional action group shifts the search space to the right by an additional two stages.
Figure 4.7 Diagram Illustrating Expansion of Control Sequence Search Space When Bi-Directional Steps are Permitted Using $n^\text{total}_{disc} = 4$ and $n^\text{stage}_{action} = 1$

3 Additional Bi-directional Groups of States and Stages are Enclosed in Dashed Lines, Total Stages = 10, Binary States = 64, & Bi-Directional Steps = 236

In Figure 4.7 for clarity only the forward uni-directional steps are drawn explicitly for the stages 1 to 4. An example of forward and reverse action steps for stages 1 to 6 and using the first bi-directional group of steps is shown in Figure 4.8 on page 64.

In Figure 4.8, all sixteen binary control settings are shown at each given stage. The states which are the set possible control actions at a stage are identified by shaded rectangles. The forward and reverse steps that indicate a directed move from the current state and stage to the next state and stage are displayed as arrows between the shaded rectangles. For reference, the uni-directional forward steps was presented in Figure 4.7 can be traced in Figure 4.8. Table 4.1 provides the number of forward and reverse steps for each stage given in Figure 4.8.
Figure 4.8 Diagram Illustrating Bi-Direction Steps, Where a Reverse Action to the Initial Start State [0 0 0 0] Occurs at 2nd Stage and Consequence Reverse Steps Occur up to Stage 6 When $n_{dvc}^{total} = 4$ and $n_{action}^{stage} = 1$ are Used
Table 4.1 A Count of Forward and Reverse Steps Illustrated in Figure 4.8 Using $n_{dvc}^{total} = 4$
and $n_{action}^{stage} = 1$

<table>
<thead>
<tr>
<th>Stage Number</th>
<th>Bi-Directional States Number of Steps</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>Forward</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>Not Shown</td>
</tr>
</tbody>
</table>

In Table 4.1 the current stage numbers are given in the first column, the remaining columns are the count of forward, reverse and total actions that occur in order to move between the stages. In the next section, the concept of a feasible path of device switch actions for a given load setting is presented.

4.3.4. CONTROL SEQUENCE PATH AND MULTIPLE DEVICE AT-A-TIME ACTIONS

In this thesis, a path is defined as the group of steps that transitions the control devices from the initial state and stage to the final state and stage at a given load setting. A feasible path is a path of control actions that does not cause constraint violations. Here, the control sequence algorithm’s aim is to minimize the steps taken in the feasible path or to use the least number of steps to move from the starting stage to the final stage. The algorithm will be provided in the next chapter section.
The number of device stage actions per stage may need to be expanded to include multiple device at-a-time operations and to operate a mix of 1-at-time and multiple-at-time actions (e.g. 2-at-a-time device operations). This concept is applied if a search of all 1-at-time switch actions fails to produce a feasible path. It is assumed that the multiple-at-a-time control actions are clock synchronized and therefore are simultaneously actuated. Figure 4.9 on page 66 provides a diagram depicting 2-at-a-time bi-directional switch actions using $n_{\text{dvc}}^{\text{total}} = 4$.

![Diagram](image)
In Figure 4.9, all sixteen binary control settings are shown at each given stage. The states at each stage are identified by shaded rectangles and the steps are displayed as arrows between the states. The total number of stages shrinks from 10 to 4 when comparing the number of stages required for 1-at-time bi-directional switch action to the number of stages required for 2-at-time bi-directional switch actions. Hence, 2-at-a-time operations appear to be superior to 1-at-time bi-directional switching because it reduces the total number of steps taken to determine a feasible path. However, multiple-at-a-time-actions is not the preferred method of operation since additional coordination among protection devices and clock synchronization is needed in order to realize simultaneous switch actions. Therefore in this thesis, sequences that contain multiple-at-time-actions are only considered after 1-at-time-switch operation sequences have been exhaustively checked. In the next section, the heuristic based greedy algorithm used to determine control sequences is provided.

4.4. Feasible Control Sequences

When solving the control sequence problem the capacitor placement and device control setting results are examined across all load profiles/levels to locate switchable devices that experience a change in status (on/off). The devices and the corresponding load profile/level where the shift in control setting occurs are saved to the control settings list.

The control settings list is examined to determine feasible control sequences. A feasible sequence is a path of device control actions that transitions the control devices
from the initial state and stage to the final state and stage at a given load setting without
constraint violations. The following assumptions were presented in previous chapters and
are applied to the control sequence algorithm.

- Capacitors are controlled as switchable banks; limited to two states, either
  on or off.

- Control sequences that exceed the max number of per capacitor switch
  operations are eliminated \( n_{ops,k} \leq n_{ops}^{\max} \).

- All capacitors at the peak load setting are turned on.

- All direct load control participants are controlled as groups with two
  states, actuated or not actuated.

- At the peak load setting, the direct load control participants are operated
  after all capacitors are turned on.

The basic feasible control sequence algorithm using 1-at-time switch operations
for the voltage spread reduction objective is:

Step 1. Start with the first two consecutive load settings: \( u_{CS1} = [u_i (\lambda_1), u_i (\lambda_2)] \).

Step 2. From the optimal control setting results: \( u_i (\lambda_2) = u_i (\lambda_1) \) and
\( u_{N_0} (\lambda_2) = u_i (\lambda_2) \), identify devices that change their status (i.e. capacitors or
capacitors and direct load control groups). Store these devices to a change
status list, \( \Delta u_i (\lambda_1, \lambda_2) \).

Step 3. Pick the initial \( u_i (\lambda_2) \) and final \( u_{N_0} (\lambda_2) \) control settings. Get \( u_{N_0} (\lambda_2) \) power
flow results. Pick one of the capacitors on the \( \Delta u_i (\lambda_1, \lambda_2) \) list.
Step 4. Set count of devices studied equal to 1. Set the #at-time switch actions equal to 1.

Step 5. Operate the capacitor to create a list of #at-a-time-possible capacitor switch actions that alters the device from \( u_i(\lambda_2) \) to match the final control setting results \( u_{\infty}(\lambda_2) \). Run a power flow for each operation and save the results.

Step 6. From the #at-a-time-switch actions list calculate the voltage rise and check if
\[
\left\| V^p_i (u_s, \lambda_2) - V^p_i (u_{s-1}, \lambda_2) \right\| \leq V_{rise}^{\max}.
\]
If a voltage rise violations exist, remove the switch action from #at-time-switch actions list. Go to Step 6a. Else go to Step 7.

Step 6a. Have all possible #at-time switch actions been removed? If yes go to Step 6b. Else go to Step 7.

Step 6b. Are there any more capacitors on \( \Delta u_s(\lambda_1, \lambda_2) \) list? If yes, then get the next capacitor on the \( \Delta u_s(\lambda_1, \lambda_2) \) list. Increment count and return to Step 5. If no capacitors remain on the \( \Delta u_s(\lambda_1, \lambda_2) \) list, then remove the sequence and increase the number of switch-actions-at-a-time to find a feasible sequence, reset count equal to 1 and go to Step 5.

Step 7. Arrange the #at-a-time switch actions by the minimum voltage spread (ascending order) and select the settings that corresponds to the top of the ranked list.

Step 8. Is the sequence, \( u_{cs}(\lambda_s) \) complete? If yes go to Step 9. Else go to Step 8a.
Step 8a. Are there any more capacitors on $\Delta u_s(\lambda_1, \lambda_2)$ list? If yes, then get the next capacitor on the $\Delta u_s(\lambda_1, \lambda_2)$ list. If no capacitors remain on the $\Delta u_s(\lambda_1, \lambda_2)$ list, go back to Step 5 and increase the number of switch-actions-at-a-time to find a feasible sequence. Else, exit the control sequence algorithm.

Step 9. Count the #-at-a-time per device switch actions in the sequence. Check $n_{ops,k} \leq n_{ops}^{max}$. If the number of per device switch operations exceeds the maximum, remove the sequence. Go to Step 10. Else, a feasible sequence is found, stop. Exit the control sequence algorithm.

Step 10. Repeat steps 1-9 and increase the number of switch-actions-at-a-time to find a feasible sequence. If all devices are operated simultaneously and still no sequence is found, stop and exit the control sequence program.

The simplest form of the feasible control sequence algorithm has been presented. Additional steps which include reverse moves (backing up to a previous state) and operating devices that do not change their status across two consecutive load settings were not provided because they are straightforward extensions to the algorithm. A flow chat which outlines the steps for the feasible control sequence algorithm are given in Figure 4.10.
Start

Get 2 consecutive load levels & their control settings $u_{cs} = [u(\lambda_1), u(\lambda_2)]$

Set $u_i(\lambda_2) = u(\lambda_1) \& u(\lambda_2) = u(\lambda_2)$

Make a list of devices that change $\Delta u_{cs} = [u(\lambda_3), u(\lambda_3)]$

Select a cap from $\Delta u_{cs}$
Set count = 1 Set # actions = 1

Operate caps #at-time until $u_i(\lambda_2)$ to $u(\lambda_2)$

Run power flow for given control settings & load setting

Calculate & check Voltage Rise Constraint for each transition

Is voltage rise limit violated?

YES

NO

Are all the #at-time sequences removed?

YES

NO

Sort ascending by min voltage spread the #at-time switch action list. Select the 1st Sequence.

Is the sequence $u_{cs}$ complete?

YES

NO

# Per Cap Switch Actions < Max?

YES

Stop & Exit Program

NO

Get the next capacitor on the $\Delta u_{cs}$ list.

Then increase the #at-time switch actions

Can #at-time switch actions increase?

YES

NO

Any more caps on $\Delta u_{cs}$ list?

YES

NO

Remove the action from #at-time-operation list

A power flow is run. Post-process the solution. Perform Data Calculations & Record Results
1. Calculate the maximum voltage spread.
2. Compute the total real power losses of the network.
3. Calculate the substation's total reactive power.

Figure 4.10 Feasible Control Sequences Algorithm Flow Chart
DLC groups are considered only at peak loading. Capacitor control sequences are found first. Once a sequence of actions to turn on all capacitors at peak load is found, the DLC group control actions are taken. The DLC groups are arranged in reverse order of customer sizes. This means DLC control sequence actions will be actuated in the following order, large commercial/industrial customers, medium commercial customer, small commercial customers and finally residential customers.

While finding all feasible control sequences is not necessary, for illustrative purposes, the control sequences were exhaustively examined at all load settings and the findings are in the results chapter of this thesis.

4.5. METHODS USED TO INCLUDE DISTRIBUTED ENERGY RESOURCES

The considerations that were presented in Chapter 3 in order to organize and categorize loads are applied to a system to determine the type of customers that are present in a distribution system.

The control sequence problem was extended to include direct load control (DLC) participants and photovoltaic (PhV) generation. Each type of PQ source was added separately to the capacitor control settings and sequence problems. Stage 2 device control settings with either, capacitors and DLC participants or capacitors settings with PhV generators included in the distribution system are found for given load settings. The device setting results from Stage 2 are then supplied to the Stage 3 control sequence problems which include DLC participants and PhV generators. The goal of each control sequence problem is to determine a feasible sequence of on/off device control actions that minimizes the maximum voltage spread.
In this thesis the following terms that are used in the procedures to determine the locations for DLC participants and for PhV generators are defined.

- **Class** refers to a distribution system customer category; i.e. Residential, Small Commercial, Medium Commercial, or Large Commercial and Industrial classes.
- **Class count** is the sum of the number of loads in given class.
- **Customer count** is the number of customers at a bus and is assumed to be equal to one if no count was provided.
- **The participation level percentage (level %)** is the amount (%) of real power at an individual load to be reduced.
- **Participant count** is the calculated integer number of PQ sources for a class. Participation count is found by rounding the product of the study level percentage and the class count to an integer value.

### 4.5.1. Direct Load Control Participants Selection Steps

Direct load control is typically enacted during critical peak load periods when congestion in the network is the greatest or when the reliability of the electric system is a concern. Direct load control actions occur after distribution system operators have employed all capable control devices (i.e. capacitors). Therefore, in this thesis, DLC injections are applied only at peak load conditions and are operated after all capacitors are actuated.

The direct load control participants are actuated as DS class groups which eliminates individual customer location specific parameters and controls. This manner of operation is desired to reduce the number of control devices in the control sequence.
problem and to reduce the size of the combinatorial solution space. As long as the number of control devices is small the control sequence algorithm is applicable to determine a feasible control sequence.

Here, either the DLC are not actuated and the group load is at its nominal load, or the DLC loads in a group are actuated and the group load is decreased from the nominal peak load to the desired reduced operating point. The number of control states is restricted in order to align with the framework of the capacitor control sequence algorithm.

The procedure to create a direct load control participant load data file for a given distribution system and its respective peak load profile data is provided in Figure 4.11:
Figure 4.11 - Direct Load Control Participant Load Data File Procedure Flow Chart
In the next section the procedure to include photovoltaic generation in the control settings and sequence problems is provided.

4.5.2. **Photovoltaic Generators Selection Steps**

Photovoltaic generators (PhV) are another type of PQ resource that is included in the control problems. Unlike DLC, it is assumed that the PhV generators are not owned or controlled by the utility. Additionally, it is assumed that all PhV’s are rooftop systems with common sizing that was set for each type of distribution customer. Lastly, no power system supporting equipment such as batteries are included in the PhV systems. Given these assumptions, the PhV generator’s real power output \( P_{phV} \) varies only with the time of day, here represented by load settings. A PhV generator’s \( P_{phV} \) output is fixed and assigned to the load settings as follows:

- The maximum \( P_{phV} \) output occurs during peak loading.
- A PhV supplies 50% of its maximum \( P_{phV} \) output at the 70% load profile.
- Each PhV’s \( P_{phV} = 0 \) during the minimum load setting.

The PhV injections are applied at the load settings before capacitor control actions are taken. Therefore, the control settings and sequences for capacitors are re-analyzed to determine if the PhV generators caused capacitor control settings and sequences to change. The next section provides a review of the algorithms presented in this chapter.
4.6. Chapter Summary

In this chapter, the capacitor placement and control sequence problem was separated into three consecutive stages and the solution algorithm for each stage was presented. Additionally, methods used to include DLC participants and PhV generation in the problem were provided. The following main points from each solution algorithm are:

- The capacitor placement algorithm for VSR uses the minimum bus voltage across three-phase buses to place/replace capacitors.
- The capacitor placement algorithm for RPLR uses the partial derivative of the real power loss with respect to reactive power \( \frac{\partial P_{\text{loss}}}{\partial Q} \) \([52]\) to place/replace capacitors.
- The control settings algorithm uses an exhaustive search of 1-at-a-time device control actions to find the control settings at a given load setting.
- The control sequence algorithm differs from dynamic programming because reverse steps are included in the set of possible states.
- A feasible control sequence must satisfy the voltage rise constraint for all device actions \([48]\).

Results of the above algorithms applied to in-service distribution networks are presented next.
5. SIMULATION RESULTS

In this chapter, simulation results for the capacitor placement and control problems are presented. In the following subsections, the simulation set-up, test systems, study parameters, placement results, control results and their respective observations are given. Results from the extended control problem using the selected network devices, specifically the inclusion of direct load control loads and photovoltaic generation, are then provided. Subsequently, the impacts of these selected network devices on the voltage spread reduction objective, network operating constraints, and realizable control sequences are discussed.

All simulation results come from a set of thirty-four existing in-service circuits provided by PPL Electric Utilities. The provided in-service circuits are in the Harrisburg Region service territory which is geographically located in the central part of the state of Pennsylvania. In this thesis PPL Electric Utilities will be referred to as PPL. In this chapter, selected in-service PPL circuits are presented to illustrate the capacitor placement and control applications. Basic circuit details, study parameters, and the solution for each selected circuit are provided.

The details of the simulation set-up are provided in the next section.

5.1. SIMULATION SET-UP

The simulations were conducted on a PC with a Windows 7 Enterprise, 64-bit operating system. The computer contained an Intel Xenon processor with a rated clock speed of 3.2 GHz. The PC contained 6 gigabytes of random access memory. The multi-
phase power flow solver from [56] was run on MATLAB version 7.9.0.529 (R2009b) for Windows 64-bit machines. Additionally, the capacitor placement and control programs were also created and run on the above listed version of MATLAB.

The test systems for the simulation results are presented in the next section.

5.2. TEST SYSTEMS

Thirty-four PPL circuits were studied. Placement and control for these circuits were examined separately. Here, results from two selected circuits are used to highlight placement, control, and the impacts on control with additional network devices. The selected circuit results begin in section 5.3 and are presented in the following order:

- Section 5.3 - Capacitor placement: Circuit I
- Section 5.4 - Capacitor control: Circuit II
- Section 5.5 - Direct load control participation: Circuit II
- Section 5.6 - Photovoltaic generation injections: Circuit II

In the following subsections the general study parameters for capacitor placement and control problems are given followed by specific study parameters for direct load control participants and photovoltaic generators.

5.2.1. STUDY PARAMETERS

Here, in order to meet Pennsylvania energy consumption reduction standards (e.g. PA Act 129 [34]), it was desired to reduce substation voltages by approximately 2.5%. Thus capacitor placement and control along the feeder was investigated to maintain
system voltage. Here, base case data refers to the circuit’s components at the desired reduced substation voltage and before capacitor placement and control actions were taken.

Capacitor placements were determined after substation voltage reduction and at peak load conditions. The following location considerations that reduced the placement search space were applied:

- All underground buses were removed from the search space.
- Capacitors were placed at three-phase buses.
- Placement proximity was defined as two buses away.

Figure 5.1 illustrates the minimum placement proximity requirements for a capacitor to an existing or newly placed capacitor. For these studies the placement proximity was chosen as two buses away (upstream or downstream). This “distance” was requested by PPL to satisfy their spacing requirements and maintenance guidelines.

![Figure 5.1 Network Diagram with Placement Proximity Set to 2 Buses Away](image-url)
Two types of capacitors exist in the circuits: manual and switchable. Manual capacitors are on at all load levels. Switchable capacitors are limited to two states, either all on or all off. Newly placed and replaced capacitors are switchable and automated.

Capacitor control sequences were determined for a typical load day, which consisted of three load settings. PPL supplied data for two load profiles from their AMR system [53]: peak ("100\%") and "70\%" loading. Scaling the peak load condition created a third and minimum (min) load level. Figure 5.2 shows how the three load settings are applied to a summer day with peak hours occurring from 1PM to 6PM [57]. Moving from one load setting to the next load setting is defined as a load transition. Using Figure 5.2, there are a total of four load transitions in a typical load day. The day can be divided differently.

![Figure 5.2. A Typical Load Day (24 Hours) Defined Using Load Profiles/Levels](image)
PPL has verified its AMR data to constant power load models. Thus, constant power load models were requested by PPL and used for these studies. Table 5.1 contains the list of constraints values in column one and their respective equation number in column two. The limit for each constraint is applied to all studies. $n_{\text{oper}}^\text{max}$, was set to 5 operations per capacitor so that each capacitor can operate once for each of the four load transitions in a load day. Then a single and additional switch action was added for the instance when a capacitor needs to operate a second time during a load transition. It is necessary to limit the total number of switch actions across the load settings so that a device’s switch lifetime is preserved, switch failures are limited and maintenance guidelines are met.

### Table 5.1. Constraint Values for Capacitor Placement and Control Problems

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Equation Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.95 \text{ per unit} \leq</td>
<td>V_i^p(u_s, \lambda_k)</td>
</tr>
<tr>
<td>$V_{\text{rise}}^{\max} = 0.016 \text{ per unit (p.u.)}$</td>
<td>(2.3.7)</td>
</tr>
<tr>
<td>$700 \text{kVar leading} \leq Q_{\text{sub}}^p(V_k, u_s, \lambda_k) \leq 200 \text{kVar leading}$</td>
<td>(2.3.8)</td>
</tr>
<tr>
<td>$n_{\text{oper}}^\text{max} = 5 \text{ operations per capacitor per day}$</td>
<td>(2.3.9)</td>
</tr>
<tr>
<td>$n_{\text{cap}}^\text{max new} = 5 \text{ new capacitors per circuit}$</td>
<td>(2.3.10)</td>
</tr>
<tr>
<td>$n_{\text{cap}}^\text{max} = 5 \text{ automated capacitors per circuit}$</td>
<td>(2.3.11)</td>
</tr>
</tbody>
</table>

In order to include in the control problem distributed energy resources, such as direct load control (DLC) customers and photovoltaic generators (PhV), the circuits were
categorized by the Distribution System Classification (DS Class) as presented in Chapter 3. Once the selected circuit was divided into classes, assumptions are applied and parameters are assigned to each present category. In this thesis, the DLC and PhV injections are studied separately.

General assumptions applied to the control settings and sequence problems for both DLC and PhV injections are:

- The peak load profile will have the greatest variation in the number of existent customer classes.
- Injection locations are picked by the consumer; not determined by a utility.
- Economic factors limit the number of participant locations.
- Injection size is limited by the location’s respective load or building type.

Given the above assumptions the following simulation parameter selections were made.

- DS load classification is applied to a circuit at the peak load profile.
- The number of participants and injection sizes are fixed for each DS class.
- Participant locations are selected randomly.
- PQ injections are balanced across the present phases of its respective bus.

Assumptions and simulation parameters selections specific to DLC and PhV’s are discussed in the following subsections respectively.
5.2.2. DIRECT LOAD CONTROL (DLC) INJECTIONS PARAMETERS

Each selected circuit was divided into DS class categories then the DLC injections were assigned. An integer number of contributors in each customer class was determined prior to participant selection. Pennsylvania (PA) Act 129 of 2008 set a direct load control of 2% statewide single-year peak demand (MW) [58]. In an effort to achieve this goal the number of participants by DS class category were set as follows.

- Residential customers’ contribution levels are 10%, 25%, and 33%.
- Non-residential customers contribute at a fixed level of 20%.

Each DS class category uses electricity differently, and DS class group load reduction percentages were created using Energy Information Administration’s residential, commercial and industrial data [59-61]. The electrical consumption contribution from each customer category changes by climate, season, size and type of industry, building, or dwelling, the age of electric equipment, plug-in devices and an individual sector’s behavior [59-61]. The load reduction percentages for each DS class are given in Table 5.2.

<table>
<thead>
<tr>
<th>Customer Type</th>
<th>Minimum %</th>
<th>Middle %</th>
<th>Maximum %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Small Commercial</td>
<td>5</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>Medium Commercial</td>
<td>5</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>Large Commercial/Industrial</td>
<td>10</td>
<td>-</td>
<td>20</td>
</tr>
</tbody>
</table>
In Table 5.2 the customer type is given in the first column, the remaining columns provide three levels for real power load reduction. The following considerations limit the amount of real power ($P$) and reactive power ($Q$) reduced at a load:

- The real power load reduction percentage of every participant is fixed.
- The power factor at all participating loads is preserved.

Residential customer load reductions are assumed to have the greatest variability. Therefore, in Table 5.2 the residential customers are given three load reduction percentages. For example Figure 5.3, presents the 2009 annual electric use of PA residential customers: here, 50% of electric usage originates from non-thermostatic loads. However, it is recognized that not all of non-thermostatic loads are available for direct control and a more reasonable maximum load reduction percentage of 30% was chosen.

![Pennsylvania Annual Residential Electric Consumption By End Use](image)

Figure 5.3. EIA’s 2009 Annual Household Energy Use in Pennsylvania (PA) [62]
The other class categories are restricted by how much load they can reduce because commercial and industrial customers must maintain stricter levels of operation during work hours. Thus in Table 5.2 they are only allotted two reduction percentages, a minimum and maximum.

The provided PPL circuits are largely residential, using Table 5.2 the majority of the load was classified as small or residential customers. It is noted that when classifying loads in a circuit, not all customer types from Table 5.2 may exist. If it is assumed all customer types are present in the distribution network, then there are a maximum of 81 real power load reduction levels to apply to a circuit and study. Therefore, for the control settings and sequence problems, it is assumed that all existent customer categories are actuated as a group. Customer category group actuations reduces the number of control variables and reduces the size of the combinatorial solution space of the control settings and sequences problems. Here, either the DLC loads are not actuated and the group load is at its nominal load, or the DLC loads in a group are actuated, the group load is decreased from the nominal peak load to the desired reduced operating point.

In the next section assumptions and parameters that are specific to photovoltaic generator injections are presented.

### 5.2.3. PHOTOVOLTAIC GENERATION (PhV) INJECTION PARAMETERS

Pennsylvania’s Utility Commission (PUC) Alternative Energy Portfolio Standard Act of 2004 requires that 8% of electricity sales in Pennsylvania (PA) must come from renewable resources by the year 2020. Subsequently, Act 213 of 2004 provided a specific
target of, 0.5% of the mandated 8% for solar photovoltaic technologies by June 1, 2020 [58].

In this thesis, the effect of non-utility PhV injections on the voltage spread and established realizable control sequences is studied. A utility PhV system is very large (100’s of MW) and is typically ground mounted. Non-utility PhV systems are often rooftop installations and classified by their size, National Renewable Energy Laboratory in [63] defines the following sizes and two categories:

- Residential rooftop PhV systems are generally between 2 kW – 10 kW.
- Commercial rooftop PhV systems are greater than 10 kW.

In the United States, these rooftop PhV’s are not owned or operated by electric distribution utilities. Furthermore, PhV’s are intermittent resources and therefore in this thesis the injections are considered uncontrolled PQ injections. In this thesis, battery storage is not considered to be installed with PhV systems.

Also, the type of inverters will vary with the cost and size of a PhV system. Here, it is assumed that all inverters:

- are grid-tied with maximum power point tracking
- do not allow for voltage regulation
- will maintain or improve the existing power factor at their respective load

A grid-tied inverter will match the phase angle with the utility supplied sine wave and will disconnect the PhV system during an outage. Maximum power point tracking (MPPT) ensures maximum power during various weather conditions. From [64], voltage regulation in distribution systems is normally performed at the distribution substation level, and distribution voltage regulation by distributed resources is not allowed by IEEE standard
Normally, distributed resources operate with fixed power factor with respect to the local system. Newer inverters may supply some reactive power, therefore an improvement in the power factor at a load bus is accepted.

The size of an individual PhV generator is assigned to DS class categories. The size of an average residential rooftop PhV installation was found to be 8 kW for Public Service Electric & Gas in New Jersey [66]. Thus, the 8 kW size is used for both residential and small commercial customers. Separate PhV sizes could be applied to each category. For medium commercial customers and large commercial/industrial customers the size of 10 kW and 20 kW were picked arbitrarily.

The integer number of PhV generators in each ratepayer category is set prior to the randomized location selections. For each DS class low, medium and high percentages of PhV installations was created. The number of generators per class was calculated by taking the product of the total number of customers in a class and a given participation count percentage. Then, the number of participants was rounded to the closest integer. Table 5.3 below, provides the customer type, PhV installation size, and percentages for low medium and high levels of PhV penetration.
Table 5.3. DS Class Categories’ Individual PhV System Size and Percentages for Given Levels Used to Calculate Number of PhV Participants (Participation Count %)

<table>
<thead>
<tr>
<th>Customer Type</th>
<th>Individual PhV Size (kW)</th>
<th>Participation Count (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td>Residential</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Small Commercial</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Medium Commercial</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Large Commercial/Industrial</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

As an example in these studies, a maximum of 8% reduction in total substation load was chosen to coordinate with PA Act 213 guidelines. Therefore, using Table 5.3, the participation count percentages were checked to ensure that the total real power injected into a circuit by PhV generation \( \sum (P_{phV}) \) at peak load would not exceed an 8% reduction in the circuit’s total nominal peak load or \( \sum P_{phV} \leq (\sum P_{sub})(92\%) \).

The number of PhV systems in a circuit is limited by a customer’s economic factors (affordability) and physical rooftop area (property) limitations. Therefore, the following selections were made. Three levels of participation counts were chosen for residential and small customers. The medium and large customer’s participation (i.e. number of customers with PhV generation) is fixed to 10% and their PhV system sizes are respectively 10 kW and 20 kW.

The load settings have been assigned to a time-of-day in Figure 5.2. A PhV generator’s \( P_{phV} \) output is fixed and assigned to the load settings as follows:
- the maximum $P_{PhV}$ output occurs during peak loading
- a PhV supplies 50% of its maximum $P_{PhV}$ output at the 70% load profile
- each PhV’s $P_{PhV} = 0$ during minimum load level

The next section discusses results for the capacitor placement problem.

5.3. **CAPACITOR PLACEMENT RESULTS:**

In this section, the capacitor placement discussion will compare the voltage spread reduction (VSR) and real power loss reduction (RPLR) objectives.

- For VSR, thirty-four circuits were studied for capacitor placement and new capacitor locations were selected using the VSR reduction placement method.
- For RPLR, fifteen of these thirty-four circuits were studied for capacitor placement and new capacitor locations were selected using the RPLR placement method.

The real power loss placements for seven of the fifteen circuits (46%) were identical to the placements found via the voltage spread reduction method. This indicates that the voltage spread objective and real power loss objectives are linked but the placement solutions may differ and placement should be directly given by the most desired objective.

A circuit was selected that highlights the impact of the reactive power and network operating constraints. The next subsections provide circuit details and results pertaining to capacitor placement for the chosen circuit.
5.3.1. Circuit I: Base Case Characteristics

The components for the selected circuit are shown below in Table 5.4. The substation voltage was reduced from 12.75 kV to 12.44 kV. The circuit’s nominal peak load is 9235.03 kW and 2604.14 kVAR. Evaluating the circuit at the reduced substation voltage and at peak load conditions resulted in the subsequent observations:

- the substation reactive power output is lagging and capacitance is needed
- 315 under voltage violations occurred

As such, a study of the differences when transmission system reactive power constraints are applied and when they are not applied was performed. A network diagram for Circuit I is displayed on page 92 in Figure 5.4.

Table 5.4: Circuit I Base Case Components and Counts

<table>
<thead>
<tr>
<th>Component</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buses</td>
<td>2025</td>
</tr>
<tr>
<td>Nodes</td>
<td>2466</td>
</tr>
<tr>
<td>Branches</td>
<td>2024</td>
</tr>
<tr>
<td>Capacitors</td>
<td>3</td>
</tr>
<tr>
<td>Loads</td>
<td>426</td>
</tr>
</tbody>
</table>
Figure 5.4 Circuit I Case 4, 2025 Bus, 2466 Node Multi-Phase Unbalanced Distribution System
3 New Capacitor Bank Locations Are Circled
5.3.2. **OBJECTIVE AND CONSTRAINTS**

The effects of the reactive power constraints on placement are considered separately for VSR and RPLR objectives. Five cases are studied. Case 1 is the base case: results at the reduced substation voltage and with the existing capacitor placements. For cases 2 to 5, the algorithms selected to add capacitors to the circuit. Case 2 and Case 3 are results with the VSR and RPLR objectives applied respectively without $Q$ constraints. Case 4 and Case 5 are results when $Q$ constraints are applied.

Here, existence of violations, the substation reactive power output ($Q_{out}$); total real power losses ($P_{Loss}$) and the per unit (p.u.) voltage spread ($VS$) at peak loading with all switchable capacitors turned on are provided for the five cases. Please see Table 5.5. Capacitor bus locations and respective sizes are provided in Appendix A, Table A.1.

All placement methods produced solutions that improved the corresponding objective function with respect to the base case and resolved the under voltage violations. However, noticeable differences occur with respect to $Q$ limits. Here, the percent change is calculated with respect to the base case as follows:

$$\% \text{ Change} = \left( \frac{\text{Base\_Value} - \text{New\_Value}}{\text{Base\_Value}} \right) \times 100\% \quad (5.4.1)$$

Below are some remarks with respect to the base case:

- Case 2 (VSR) reduced the $VS$ by 38.01% yet increased the $P_{Loss}$ by 7.23%.
- Case 3 (RPLR) decreased $P_{Loss}$ by 0.025% and reduced the $VS$ by 16.08%.
- Case 4 (VSR) reduced the $VS$ by 31.99% but had a 4.75% increase in $P_{Loss}$.
- Case 5 (RPLR) decreased $P_{Loss}$ by 0.006% and reduced the $VS$ by 20.88%.
- Case 2 (VSR) and Case 3 (RPLR) would violate $Q$ limits.

Table 5.5: Results at Peak Loading and with All Switchable Capacitors Turned On:
Circuit I with All Cases at 12.44 kV
VSR - Voltage Spread Reduction
RPLR - Real Power Loss Reduction

<table>
<thead>
<tr>
<th>Circuit I Case Settings</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>Objective</td>
</tr>
<tr>
<td>1</td>
<td>base case</td>
</tr>
<tr>
<td>2</td>
<td>VSR</td>
</tr>
<tr>
<td>3</td>
<td>RPLR</td>
</tr>
<tr>
<td>4</td>
<td>VSR</td>
</tr>
<tr>
<td>5</td>
<td>RPLR</td>
</tr>
</tbody>
</table>

Although Case 3 and Case 5 both reduced $P_{Loss}$ by very similar amounts, their capacitor placements and resulting $VS$ for the RPLR objective differed. For Case 3, a single 1200 kVAr capacitor was placed into the network. Case 5 had three 300 kVAr capacitors placed into the circuit. Comparing their $VS$ results, Case 5 is 3.39% less in $VS$ than Case 3.
In this instance, Case 5 showed that the $Q$ constraint assisted the placement algorithm, a placement was found in which a greater reduction in $P_{Loss}$ was achieved.

Comparing, Case 2 and Case 4 placement results for VSR objective, the previously observed relationship between $Q$ constraints and the $VS$ is contradicted; Case 4 showed that the $Q$ constraint restricts the reduction in $VS$ by 8.85%.

These results illustrate perhaps less common but theoretically expected characteristics and highlight the need for thorough analysis:

- an increase in reactive power might not significantly reduce $P_{Loss}$ or significantly impact the $VS$ (i.e. Case 3 and Case 5)
- compensation for $VS$ can lead to a very high total $Q_{out}$ thus violating $Q$ limits (i.e. Case 2)
- the substation reactive power ($Q$) constraints bind the placement solution and therefore should be included in capacitor placement and control problems

Lastly, capacitor control sequences were found for Case 4 and Case 5. Case 4 and Case 5 were selected because both cases met the $Q$ constraints and both cases minimized their individual objectives (i.e. VSR or RPLR). For both cases the control sequence had no voltage rise violations.

In the next section, capacitor placement and control results are provided for Circuit II.
5.4. **CAPACITOR CONTROL RESULTS: CIRCUIT II**

In this section, results are provided for capacitor control settings and sequences. Here, only the voltage spread objective will be considered. Six of the thirty-four circuits would have had voltage rise violations in capacitor switching sequences (seq.) if constraint (2.3.7) was not included [48]. The voltage rise constraint guides the control sequence and therefore should be included in the capacitor placement and control problems. Here, a circuit was selected for illustration and capacitor switching sequences are given.

5.4.1. **CIRCUIT II: CHARACTERISTICS**

Circuit II has 948 buses, 947 branches, 1224 nodes multi-phase unbalanced distribution network and contains 5 capacitors and 282 loads. The circuit's nominal peak load is 8214.87 kW and 2980.95 kVAr. The substation voltage was reduced from 12.9 kV to 12.6 kV. A network diagram for the circuit has is provided in Figure 5.5 on page 98.

5.4.2. **CAPACITOR PLACEMENT**

The capacitor placement for Circuit II, which resulted in the minimum voltage spread, was chosen to demonstrate the control algorithm. The capacitor on/off statuses that resulted in the minimum voltage spread and met the \( Q \) constraints was chosen at each load setting. No violations occurred for this placement. The placement results and control settings for the VSR objective are provided in Table 5.2.
Table 5.6: Circuit II Capacitor Placement & Control Results for VSR Objective

<table>
<thead>
<tr>
<th>Capacitor Bus Number</th>
<th>Size (kVAr)</th>
<th>New/Existing Capacitors</th>
<th>Type Manual/Switchable</th>
<th>Control Operations for Given Load Level/Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>26%</td>
</tr>
<tr>
<td>1333</td>
<td>600</td>
<td>Existing</td>
<td>Manual</td>
<td>on</td>
</tr>
<tr>
<td>1937</td>
<td>600</td>
<td>Existing</td>
<td>Switchable</td>
<td>on</td>
</tr>
<tr>
<td>1292</td>
<td>600</td>
<td>Existing</td>
<td>Switchable</td>
<td>off</td>
</tr>
<tr>
<td>1177</td>
<td>1200</td>
<td>New</td>
<td>Switchable</td>
<td>off</td>
</tr>
<tr>
<td>1015</td>
<td>900</td>
<td>New</td>
<td>Switchable</td>
<td>off</td>
</tr>
</tbody>
</table>
Figure 5.5 Circuit II 948 Bus, 1224 Node Multi-Phase Unbalanced Distribution System
2 New Capacitor Bank Locations Are Circled
5.4.3. Capacitor Control Settings

In Table 5.6, the optimal capacitor control statuses at each load profile/level are shown for the Circuit II placement:

- all switchable capacitors are off for the min load level except at bus 1937
- all switchable capacitors are on at the 70% and 100% load levels

Table 5.7 results were obtained by applying each set of control states from Table 5.6 to the respective capacitors and assigning the corresponding load level/profile to the loads. Then the power flow equations were solved with these settings implemented. The power flow solution was post-processed to calculate $Q_{out}$, $P_{Loss}$ and the $VS$.

Table 5.7: $Q_{out}$, $P_{Loss}$ & $VS$ Metrics at Specified Load Settings for Circuit II Placement and Control Results for VSR Objective

<table>
<thead>
<tr>
<th>Metric</th>
<th>Given Load Level/Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>26%</td>
</tr>
<tr>
<td>$Q_{out}$ (kVAR)</td>
<td>426.68 leading</td>
</tr>
<tr>
<td>$P_{Loss}$ (kW)</td>
<td>10.34</td>
</tr>
<tr>
<td>$VS$ (p.u.)</td>
<td>0.00255</td>
</tr>
</tbody>
</table>

5.4.4. Effects of the Voltage Rise Constraint on Control

In this section, the effects of network operating constraints on capacitor control sequences are examined more closely. The 1-at-a-time switching sequences were
exhaustively checked. For Circuit II, the switching sequences are non-unique. Here, both feasible (Table 5.8) and infeasible (Table 5.9) capacitor switching sequences were readily obtained. Since, control statuses between the 70% and 100% profiles do not change, only switching sequences for the 26% to 70% load transition are investigated.

**Table 5.8: Circuit II Selected Feasible Switching Sequence (Seq.) as: Capacitors Transition from the 26% Load Level’s Control Settings to the 70% Profile’s Control Settings (\( u_{cs}(70\%) \))

<table>
<thead>
<tr>
<th>Seq.</th>
<th>Order of Operation</th>
<th>Capacitor Switching Sequence</th>
<th>Switch Action (on/off)</th>
<th>Capacitor Bus Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1937 1177 1292 1015 1937</td>
<td>off</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1937 1177 1937 1015 1292</td>
<td>on</td>
<td>2</td>
</tr>
</tbody>
</table>

**Table 5.9: Circuit II Selected Infeasible Switching Sequence (Seq.) as: Capacitors Transition from the 26% Load Level’s Control Settings to the 70% Profile’s Control Settings (\( u_{cs}(70\%) \)):

<table>
<thead>
<tr>
<th>Seq.</th>
<th>Order of Operation</th>
<th>Capacitor Switching Sequence</th>
<th>Switch Action (on/off)</th>
<th>Capacitor Bus Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1292 1015 1177</td>
<td>on</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1177 1292 1015</td>
<td>on</td>
<td>2</td>
</tr>
</tbody>
</table>
Below are some remarks with respect to the capacitor switching sequences:

- The order in which capacitors actions are taken throughout a day is significant and should be guided by the problem's constraints.
- The voltage rise constraint adds an additional two control actions for Circuit II.
- In this case, all 1-at-a-time feasible switch sequences require the capacitor at bus 1937 to be turned off before turning on the capacitor at bus 1177.
- The infeasible switching sequences caused 13 voltage rise violations.

Providing only control status results can lead to seemingly intuitive yet infeasible switch sequences. Here, an intuitive sequence was deemed to be turning the 3 remaining capacitors on at 70% profile which also results in a desirable minimal number of switch actions. Thus, it has been demonstrated in this thesis that the voltage rise constraint (Eq. 2.3.7) should be included in capacitor problem formulations. Additionally, operating constraints should be actively implemented in the solution algorithm as they assist in obtaining realizable switching sequences.

In the next section, results from a further investigation on the effects of increasing the number of load levels on the number of capacitor control actions are provided.

5.4.5. Effect of the Number of Load Levels on Control

In this section, a study on the number of load levels and number of switch operations for capacitor control was performed and results are provided. Additional load levels were calculated by arbitrarily adding increments of 2.5% to the 26% load level.
Supplementary load data was created by scaling the 100% profile’s data proportionally to a load level percentage. A total of 20 load settings were analyzed.

Table 5.10 provides a list of capacitor control actions for the VSR objective at selected load levels. The last row in the table is the $V_S$ for a set of capacitor statuses at a selected load setting. Below are some remarks.

- Capacitors fluctuate between on and off states in order to achieve the optimal voltage spread at a given load level.
- The number of control actions per capacitor varies depending on the number of load settings examined.

As load data availability increases the results indicate that the number of load settings to include in a study should be thoroughly investigated. This is left for future work.

Table 5.10: Circuit II Capacitor Control Results for the VSR Objective with an Increased Number of Load Levels Added from the 26% Level to 70% Profile (Selected Load Levels Shown)

<table>
<thead>
<tr>
<th>Capacitor Bus Number</th>
<th>Load Levels (Scaled Proportionally from Peak Profile)</th>
<th>Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>26</td>
<td>33.5</td>
</tr>
<tr>
<td>1333</td>
<td>on</td>
<td>on</td>
</tr>
<tr>
<td>1937</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>1292</td>
<td>off</td>
<td>off</td>
</tr>
<tr>
<td>1177</td>
<td>off</td>
<td>off</td>
</tr>
<tr>
<td>1015</td>
<td>off</td>
<td>on</td>
</tr>
<tr>
<td>$V_S$ (p.u.)</td>
<td>0.00254</td>
<td>0.00621</td>
</tr>
</tbody>
</table>
In the next section, additional feasible capacitor control sequences are identified using an exhaustive search. The exhaustive search results are compared to the results obtained using the greedy heuristic control sequence algorithm.

5.4.6. **Comparison of Exhaustive Search and Greedy Heuristic Control**

An exhaustive search of 1-at-a-time control actions was performed on Circuit II as the capacitors transition from the 26% load level’s control settings to the 70% profiles control settings. The search was performed in order to validate the sequence results found using the greedy heuristic control algorithm. Six feasible sequences out of sixty sequences for the VSR objective were found. Please see Table 5.11. The first row lists the sequence order of operation, the second row in the table is the switch action and the remaining rows are the sequences.

Table 5.11: A Set of Feasible Switching Sequences (Seq.) as: Capacitors Transition from the 26% Load Level’s Control Settings to the 70% Profile’s Control Settings \( (u_{cs} (70\%)) \)

<table>
<thead>
<tr>
<th>Seq.</th>
<th>Capacitor Bus Number</th>
<th>Switch Action (on/off)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>off</td>
<td>on</td>
<td>on</td>
<td>on</td>
<td>on</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>1937</td>
<td>1177</td>
<td>1937</td>
<td>1292</td>
<td>1015</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>1937</td>
<td>1177</td>
<td>1292</td>
<td>1937</td>
<td>1015</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>1937</td>
<td>1177</td>
<td>1292</td>
<td>1015</td>
<td>1937</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>1937</td>
<td>1177</td>
<td>1015</td>
<td>1937</td>
<td>1292</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>1937</td>
<td>1177</td>
<td>1015</td>
<td>1292</td>
<td>1937</td>
</tr>
</tbody>
</table>
Table 5.11 above is a list of 1-at-a-time feasible switching sequences that minimize the number of switch actions per capacitor. Exhaustively, sixty sequences 1-at-a-time sequences exist. In comparison, the greedy heuristic successfully found sequence one directly.

In the next section, Circuit II results with direct load control participants included into the capacitor control settings and sequence problem is given.

5.5. DIRECT LOAD CONTROL (DLC) PARTICIPATION RESULTS

In this section, results are provided for DLC participation. Here, only the voltage spread objective was considered. Five circuits that had voltage rise violations and required ordered capacitor switching sequences were studied. These five circuits were chosen because the voltage concerns provided worst case test scenarios in which the DLC participation could be assessed.

Prior to adding DLC, capacitance in 2 circuits was reduced in order to meet the \( \min Q \) constraint. Consequently, under voltage violations returned when the capacitance was reduced. For these 2 circuits, DLC aided in resolving the under voltage violations. General observations of the five circuits are summarized next.

For all five circuits, the minimum voltage spread at peak loading was achieved:

- when all automated capacitors were turned on
- with the greatest number of DLC participants included
- with a maximum percentage of reduction in real power load was applied

However, when all capacitors are turned on and DLC is fully enacted, the \( Q \) constraint’s minimum \( Q \) limit was exceeded. This consequence was expected and further
supports the desire to reevaluate minimum $Q$ limits. Additionally, this overcompensation in reactive power lead to a decrease in the substation power factor. Still, no new operating violations occurred when DLC was included in the control settings and sequence studies.

From a capacitor placement and control sequences viewpoint adding DLC to the selected five circuits proved to be beneficial for the following reasons:

- No voltage rise violations occurred due to DLC switch operations.
- No changes to established capacitor control settings & sequences were needed.
- Two of five circuits had network violations which were resolved when all automated capacitors were turned on and DLC was fully employed.
- Four of five circuits showed that DLC can provide $|V|$ support so that existing capacitors may be turned off at peak loading.
- Two of five circuits showed that DLC can be used to delay the purchase and installation of new capacitors.

Next Table 5.12 provides a summary of results for the five circuits when the minimum voltage spread at peak load was achieved. The percent change is calculated with respect to the base case using Eq. 5.4.1:
Table 5.12. Selected Five Circuits Minimum Voltage Spread ($VS$) Results with Greatest Amount of DLC Participation and Maximum Percentage (%) of Load Reduction Applied

<table>
<thead>
<tr>
<th>Circuit</th>
<th>Peak Nominal Load $P$ (kW) $Q$ (kVar)</th>
<th>$VS$ Results (Per Unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No DLC Case</td>
<td>Max DLC Case</td>
</tr>
<tr>
<td>Circuit I</td>
<td>$P$ 9235.0</td>
<td>8790.1</td>
</tr>
<tr>
<td></td>
<td>$Q$ 2604.1</td>
<td>2475.6</td>
</tr>
<tr>
<td>Circuit II</td>
<td>$P$ 8214.9</td>
<td>7694.9</td>
</tr>
<tr>
<td></td>
<td>$Q$ 2981.0</td>
<td>2756.8</td>
</tr>
<tr>
<td>Circuit III</td>
<td>$P$ 8032.5</td>
<td>7366.0</td>
</tr>
<tr>
<td></td>
<td>$Q$ 2932.6</td>
<td>2710.5</td>
</tr>
<tr>
<td>Circuit IV</td>
<td>$P$ 6668.5</td>
<td>6221.3</td>
</tr>
<tr>
<td></td>
<td>$Q$ 2658.2</td>
<td>2437.9</td>
</tr>
<tr>
<td>Circuit V</td>
<td>$P$ 4919.8</td>
<td>4658.3</td>
</tr>
<tr>
<td></td>
<td>$Q$ 1308.9</td>
<td>1238.6</td>
</tr>
</tbody>
</table>

In Table 5.12 the circuit number is listed in the first column. Circuit I and Circuit II are the same circuits provided in results sections 5.4 and 5.5. The second column provides the nominal peak load ($P$ (kW) and $Q$ (kVar)) without DLC participation (No DLC Case). Column three is the circuit’s nominal peak load with the greatest number of direct load control participants and with the maximum (max) percentage of load reduction applied. In this thesis, this case is referred to as the max DLC case. The percent decrease between the nominal peak load without DLC included and the nominal peak load for the max DLC case is calculated in column four. The next two columns list the per unit voltage spread results for the no DLC case and the max DLC case at peak load.
with all capacitors turned on. Lastly, the percent change in the voltage spread was calculated. Some remarks about the results are:

- Circuit I and Circuit V do not have large customer loads in their circuit so the reduction in total nominal load is less than circuits II, III and IV.
- Circuit II has the largest amount of nominal peak load \( P \) (kW), the greatest change in voltage spread (27.83%) at the peak load and only 6.3% reduction in nominal real power load.
- Circuit III has the greatest reduction in real-power load (8.3%) but only half the reduction in voltage spread in comparison to Circuit II.

The above results infer that not all of the randomly selected DLC locations are equal in assisting in minimizing the voltage spread. Therefore, load locations that do not contribute to the objective, resolve constraint violations or assist in the delay of operating capacitors result in different benefits for the system. The value applied to a respective load location could reflect the contribution of that load to the system. Thus, the study of respective load location value may be a topic of future work. In the next section, detailed DLC participation results for Circuit II are given.

### 5.5.1. Direct Load Control Results: Circuit II

Circuit II was selected to illustrate the effects of DLC participation on the voltage spread reduction (VSR) objective and network constraints. Each load in Circuit II was classified as either a residential customer, a small commercial customer, a medium commercial customer or as a large commercial/industrial customer. In Circuit II all customer types were represented. The per phase real power nominal load (kW) at a given
bus was divided by the number of customers at the bus in order to calculate the individual customer’s per phase real power load at a node. If no customer count was provided at a bus, the customer count was assumed to be one.

Table 5.13 contains the DS Class categories and a breakdown of the circuit’s load by customer type. The class percentage of total customers (% of DS Class), the class’ respective number of loads (load count), nominal peak real power (Nom. $P$), nominal peak reactive power (Nom. $Q$) and the class’ real power load percentage ($%P$) of total nominal real power load ($P_{Load}$) are provided in the table.

<table>
<thead>
<tr>
<th>Customer Type</th>
<th>% of DS Class</th>
<th>Load Count</th>
<th>Nom. $P$ (kW)</th>
<th>Nom. $Q$ (kVar)</th>
<th>$%P$ of Total $P_{Load}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential (R)</td>
<td>46.45</td>
<td>131</td>
<td>2351.9</td>
<td>729.39</td>
<td>28.63</td>
</tr>
<tr>
<td>Small Commercial (S)</td>
<td>47.87</td>
<td>135</td>
<td>3349.2</td>
<td>1148.5</td>
<td>40.77</td>
</tr>
<tr>
<td>Medium Commercial (M)</td>
<td>5.32</td>
<td>15</td>
<td>1513.7</td>
<td>483.37</td>
<td>18.43</td>
</tr>
<tr>
<td>Large/Industrial (L/I)</td>
<td>0.35</td>
<td>1</td>
<td>1000</td>
<td>619.74</td>
<td>12.17</td>
</tr>
<tr>
<td>Totals</td>
<td>100</td>
<td>282</td>
<td>8514.9</td>
<td>2981</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 5.13 shows that the majority of customer loads were classified as small commercial customers (40.77%). Residential customers make up 28.63% of the total real power load. Only a single large/industrial customer exists in the circuit and draws 12.17% of the total real power load.

After DS classification of the circuit’s peak load, the participation levels and load reduction percentages from Table 5.2 on page 84 were applied to create DLC
participation cases for study. Table 5.14 provides the number of participants (load count) and load reduction percentages (reduction \( \% P \)) for the DLC participation cases. In Table 5.14, the participation level percentage (level \%), participation load count (load count), the total nominal peak real power (Nom. \( P \)), and total nominal reactive power (Nom. \( Q \)) at the randomly selected loads. The last two columns in the table are the total load reduction real power percentage (reduction \( \% P \)) and the total real power load reduction in kW, \( P_{DLC}^{Reduce} \) for the randomly selected load locations.

Table 5.14. Circuit II: Nominal Peak Load DLC Participation Case Data and DLC Total Real Power Load Reduction, \( P_{DLC}^{Reduce} \) (kW) using Randomly Selected Load Locations

<table>
<thead>
<tr>
<th>Customer Type</th>
<th>Total Load Count</th>
<th>Participation</th>
<th>Totals at Randomly Selected Load Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Level (%)</td>
<td>Load Count</td>
</tr>
<tr>
<td>Residential (R)</td>
<td>131</td>
<td>10%</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25%</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>33%</td>
<td>44</td>
</tr>
<tr>
<td>Small (S)</td>
<td>135</td>
<td>20%</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medium (M)</td>
<td>15</td>
<td>20%</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large/Industrial (L/I)</td>
<td>1</td>
<td>20%</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
From Table 5.14, 81 DLC participation cases were created. A DLC case with minimum reduction in real power was created by including only 13 randomly selected residential customers as DLC participants and reducing their respective individual loads by 10%. The 10% reduction in residential load sheds 18.61 kW from the circuit which equals a 0.20% decrease in the circuit’s total nominal real power load.

Similarly, a case with maximum reduction in real power was created by including all customer classes with the greatest number of DLC participants and with the maximum (max) percentage of real power load reduction applied. This case is referred to as the max DLC case. The max DLC case for Circuit II contains the following DLC participation counts (load count) and reduction percentages (%P):

- 44 residential (R) loads with each load reducing their real power by 30%
- 27 small (S) loads with each load reducing their real power by 10%
- 3 medium (M) loads with each load reducing their real power by 10%
- 1 large/industrial (L/I) load with a 20% reduction in its real power

The total load reduction due to DLC for the max DLC case is 520.02 kW which equals a 6.33% decrease in Circuit II’s total nominal real power load.

In the next section, a summary of results for Circuit II’s 81 DLC participation cases is presented.

### 5.5.1.1. Circuit II: Summary of 81 Real Power Reduction Load Cases

For Circuit II, the effects of DLC participation the voltage spread, voltage rise and network constraints were observed. Bulleted observations with respect to all 81 DLC real power reduction load cases are listed below.
- 8 cases with DLC participation (9.87%) met the 700 kVAr leading \( Q \) constraint
- no new voltage or current violations occurred at any level of DLC reduction
- voltage rise violations were not caused by the DLC load reduction cases

The reduction in real power from DLC participation consequently led to excess reactive power at the substation. An increase in reactive power is expected since all switchable capacitors are turned on at peak loading and the DLC participants directly reduce their respective load. In all cases, after capacitors are enacted, the DLC participant can be enacted in any order without violating the voltage rise constraint.

For the max DLC case the 700 kVAr leading (min \( Q \)) limit was not met. Therefore, analysis was performed in order to meet the substation reactive power limit. For these studies, max DLC participation was applied and switchable capacitors were individually turned off. Table 5.15, provides Circuit II results at the peak load setting without DLC participation (No DLC Applied), with max DLC participation applied (Case 1 to Case 5). Case 1 are the max DLC case results, Case 2 to Case 4 are new results with max DLC participation applied and one of four switchable capacitors turned off.
Table 5.15. Circuit II: Peak Load Capacitor Control Results without DLC and Peak Load Capacitor Control Results using the Max DLC Case for the VSR Objective

<table>
<thead>
<tr>
<th>Capacitor Bus Number</th>
<th>No DLC Applied</th>
<th>Max DLC Participation and Max Load Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td></td>
<td>All Caps</td>
<td>No 1937</td>
</tr>
<tr>
<td>1333</td>
<td>on</td>
<td>on</td>
</tr>
<tr>
<td>1937</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>1292</td>
<td>on</td>
<td>on</td>
</tr>
<tr>
<td>1177</td>
<td>on</td>
<td>on</td>
</tr>
<tr>
<td>1015</td>
<td>on</td>
<td>on</td>
</tr>
</tbody>
</table>

| VS (p.u.) | 0.01207 | 0.00876 | 0.01456 | 0.01200 | 0.02163 | 0.01661 |
| Q_{out} (kVAR) | 612.23 leading | 894.03 leading | 267.14 leading | 274.38 leading | 350.58 lagging | 37.14 lagging |

In Table 5.15 the capacitor on/off status, per unit (p.u.) voltage spread (VS) and total substation reactive power output (Q_{out}) in kVAR are given. Capacitor at bus 1333 is manual. Some remarks comparing results of cases 1 to 5 to the no DLC participation case are:

- A tradeoff between a reduction in real power and an increase in the total reactive power in Circuit II was observed.
- Cases 2, 4 and 5 all experienced an increase in their VS when their designated capacitor was turned off.
- Case 3, is the only case which had a reduction in the VS and met the Q constraints; but its control settings required the capacitor at bus 1292 to be turned off.
• Turning off capacitor at bus 1292 for Case 3 extended the realizable control sequence by an additional capacitor operation.

Therefore, the min $Q$ constraint could be relaxed so that DLC can be enacted with the greatest number of direct load control participants and with the maximum (max) percentage of real power load reduction applied. If the min $Q$ constraint is held then the capacitor control settings at peak load would need to be investigated to determine which capacitors to turn off.

In the next section the results for the addition of photovoltaic generation into the problem is presented.

5.6. PHOTOVOLTAIC GENERATION (PhV) STUDY RESULTS

In this section, results are provided for photovoltaic generation studies. Circuit II was selected to illustrate the inclusion of PhV generation on the VSR objective.

5.6.1. CIRCUIT II: PhV RESULTS

Circuit II was divided into DS classes and PhV generator locations were randomly assigned by category. The individual load’s PhV system size and participation count percentages from Table 5.3 were used to create low, medium and high PhV participation cases to study. Data for the low, medium and high PhV injection cases is provided in Table 5.16. Column one of Table 5.16 lists the DS Class customer types. Then, the individual PhV system size (kW) which was assigned to a specific customer type is provided in column two. Column three gives each class’ total load count. The remaining
columns are separated into levels of PhV participation cases (low, medium, and high). Within each level the number of PhV injections (count) and total injected nominal real power load \( (P_{PhV}) \) in kW for the cases is provided. The final row in the table lists the percentage of total reduction in nominal real power peak load \( (%P_{Load} \text{ Reduction}) \) for each case.

Table 5.16. Circuit II: PhV Parameters for Low, Medium, and High Participation Cases

<table>
<thead>
<tr>
<th>Customer Type</th>
<th>PhV Size (kW)</th>
<th>Total Load Count</th>
<th>Low Count</th>
<th>Medium Count</th>
<th>High Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R)</td>
<td>8</td>
<td>131</td>
<td>13</td>
<td>104</td>
<td>26</td>
</tr>
<tr>
<td>(S)</td>
<td>8</td>
<td>135</td>
<td>14</td>
<td>112</td>
<td>27</td>
</tr>
<tr>
<td>(M)</td>
<td>10</td>
<td>15</td>
<td>2</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>(L/I)</td>
<td>20</td>
<td>1</td>
<td>1</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>-</td>
<td>282</td>
<td>30</td>
<td>256</td>
<td>56</td>
</tr>
</tbody>
</table>

\[ \%P_{Load} \text{ Reduction} \]

<table>
<thead>
<tr>
<th></th>
<th>1.47%</th>
<th>4.38%</th>
<th>5.65%</th>
</tr>
</thead>
</table>

The reductions in total nominal real power peak load from the low, medium, and high PhV injection cases were less than 8%. The PhV injections locations were selected for Circuit II using a uniformly distributed random number generator.

The results for PhV injection participation cases are similar to the results from DLC participation cases. As expected, the PhV low, medium, and high participation cases achieved the minimum voltage spread at peak load setting with all capacitors turned and
PhV injections at their maximum real power output \( (P_{phV}) \). Next, Table 5.17 provides Circuit II’s results with and without PhV included. For these results, all switchable capacitors are turned on at the 100% and 70% profiles. \( Q_{out} \) in kVAr and \( V_S \) in p.u. are listed in the rows.

Table 5.17: Circuit II: Results without PhV Participation and with PhV Participation for the VSR Objective. \( Q_{out} \), & \( V_S \) Metrics are given at the Peak & 70% Load Profiles. All Switchable Capacitors are Turned On

<table>
<thead>
<tr>
<th>Case Metric</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
<th>Case 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_{out} ) (kVAr)</td>
<td>612.23 leading</td>
<td>543.34 leading</td>
<td>723.59 leading</td>
<td>596.06 leading</td>
<td>767.7 leading</td>
<td>616.9 leading</td>
<td>812.7 leading</td>
<td>638.07 leading</td>
</tr>
<tr>
<td>( V_S ) (p.u.)</td>
<td>0.01207</td>
<td>0.00926</td>
<td>0.01055</td>
<td>0.00941</td>
<td>0.01016</td>
<td>0.00920</td>
<td>0.00928</td>
<td>0.00906</td>
</tr>
</tbody>
</table>

In Table 5.17, the no PhV results are repeated from Table 5.7. The remaining cases are respectively the 100% and 70% load profiles with low, medium, and high PhV participation. The PhV injections at peak loading used their assigned maximum (max) real power output \( (P_{phV}) \). For the 70% load profiles, the real power output of the PhV injections \( (P_{phV}) \) was reduced to 50% of its max real power output.

The low, medium and high PhV participation case results are compared respectively to Case 1 and Case 2 which did not include PhV generators.
• All peak cases (3, 5, and 7) reduced the $V_S$ and exceed the $Q$ min constraint.

• Case 4 increased the $V_S$ by 1.62% yet met the min $Q$ limit.

• Case 6 and Case 7 both decreased the $V_S$ and met the min $Q$ requirement.

• Case 7 had a 23.12% decrease in $V_S$ and this was the largest decrease observed.

Some remarks on case results that are not explicitly shown in Table 5.17 are:

• Turning off any capacitor for the peak load PhV participation cases (3, 5, and 7) would cause the max $Q$ limit to be exceeded.

• For all PhV cases (3-8) the capacitor control settings are the same as the no PhV cases from Table 5.6.

• For the 100% and 70% profile cases the feasible control sequence from Table 5.8 is valid and the order of capacitors does not change.

A summary of all results is given in the next chapter section.

5.7. Chapter Summary

In this chapter, results for the capacitor placement and control problems were presented. Additionally, results using DLC participants and PhV generation were given. Effects of the voltage rise constraint and a substation reactive power constraint were discussed. The constraints were utilized to obtain realizable placements, control settings and sequences for given load settings.

For capacitor placement, the effects of the $Q$ constraint on the VSR and RPLR objectives were studied. Circuit I showed that the $Q$ constraints bound the placement
solution for both objectives. A larger reduction in $VS$ occurred when the lower limit of the $Q$ constraint was exceeded. If reactive power limits are relaxed, then there is potential to operate distribution substations as sources of reactive power for the transmission system during peak load conditions.

The coupling of constraints, especially within the capacitor control problem was also highlighted in this thesis. For Circuit II, multiple 1-at-at-time feasible switch sequences were found with special attention on the voltage rise/drop between discrete load variations. The problem's constraints guided the order in which capacitors actions were taken throughout the day. It is shown that constraint-driven methodologies are needed to generate control sequences, which can realize the objectives.

Overall these studies showed:

- The transmission system reactive power requirements for the VSR and RPLR objectives significantly impact the placement and control results.

- Clarity in optimal control settings are needed with respect to capacitor switching sequences that transition between load settings for a typical load day.

- A comparison of capacitor control sequence results using an exhaustive search and capacitor control sequence results using the control sequence algorithm which is based on a greedy heuristic.

- DLC and PhV results showed that a tradeoff exists between a reduction in real power and increase in the total reactive power in the circuit.

The following chapter will review the presented problem and conclude this thesis.
6. CONCLUSION

The objectives of this thesis was to study the capacitor placement problem, the control settings and sequences problems which considered select network devices within a distribution system. Voltage spread reduction (VSR) and real power loss reduction (RPLR) objectives were considered. The devices included in the control settings and sequence problems were capacitors, direct load control participants and photovoltaic generators. It was demonstrated that in order to achieve the desired objective (VSR/RPLR), it is necessary to provide distribution system operators with a feasible sequence of control actions that transition the system’s control devices along given load setting.

This thesis made use of constraint driven analytics to assess the DS capacitor placement and control problems and investigates load changes on the system’s existing control settings. Practical control sequences for given load settings were determined by including a voltage rise constraint and a substation reactive power constraint in the problem formulation. Subsequently, heuristic based greedy algorithms were developed that implemented the constraints in order to find a solution.

In this chapter, the thesis contributions made towards the above objectives is presented followed by a summary of related and future work.
6.1. Contributions

Specifically, the following contributions were provided in this thesis.

- A non-linear, non-differentiable, constrained, combinatorial, multi-objective optimization problem formulation that includes device control settings for varying load parameters.
- A substation reactive power constraint was introduced in order to satisfy transmission system reactive power requirements
- A voltage rise constraint was included at each load setting
- Constraint driven algorithms were applied to pre and post device switch actions (between control operations) in order to identify feasible control sequences
- Capacitor, DLC and PhV simulation results indicated implications of reactive power and transmission system requirements on distribution systems
- Simulation results showed optimal control algorithms must provide a control sequence of device actions in order to realize the solution

Given the constraint driven problem formulation and solution algorithms, a comprehensive simulation assessment was performed and results were presented on two in-service distribution circuits.

In the next section some related work and ideas to extend the capacitor placement and device control settings and sequence problems are presented.
6.2. **Related and Future Work**

Related work includes distributed capacitor control for Circuit II, which was presented in this thesis, and can be found in [67]. In [67], an analytical partitioning method based on capacitor reactive power domains was presented, then a distributed control algorithm was employed to support distribution operation applications with a focus on VSR and real power loss minimization. Capacitor reactive power domains are a function of the capacitor location, capacitor size, system component parameters, and load distribution [67]. The analytical partitioning method could be applied to capacitor control sequence problems. This method of control could be investigated to determine if it aids in reducing voltage rise violations in control sequence problems.

Some recommendations for future work on the presented problems are given in this chapter section. Given that the number of control actions per device will vary depending on the following:

- the number of load settings included in a study
- the length of time for a load setting dictates the frequency of device operations

Then, it is recommended to apply load capability to the control settings and sequence problems in order determine the numbers of load settings and length of time for a load setting that is required to minimize the number of device control actions in a typical load day. Furthermore, load control sensitivity analysis can performed by taking the partial derivative of the real power losses with respect to the partial derivative of real power and evaluating how sensitive the losses are at a bus with respect to real power injections. Lastly, other transmission system operator constraints on DS automation could
be investigated to determine the interaction between TS and DS and the requirement’s effects on the DS.

Improvements could also be made to methods used to include and model DLC participants and PhV generators. For instance, load variation vectors for each specific DLC participant group and for each size of PhV injection could be introduced into the problem. Additionally, the types of emerging distribution system components could be expanded for the control problems. For example, thermostatically controlled loads, voltage regulating PhV systems and energy storage (battery) systems could be used.

Additionally, the revenue and DS market values are changed by including DLC participants at peak load conditions. The DLC participant locations should be evaluated to determine their contribution to reducing the load of a system. Locational pricing which is dependent on the level of contribution needs to be created and should be applied to the DLC participants. As a result, the value added by the DLC participant can be realized from a market prospective.

Lastly, as the number of distributed energy resources increases in the distribution system, the number of inverters used to enable DC-AC resources also increases, there is a potential for these inverters to become a source of frequency disturbances. Therefore, distribution system operators may desire to investigate the effects of device control actions on total harmonic distortion within a distribution system.


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APPENDIX A

A.1. CIRCUIT I: CAPACITOR PLACEMENT BUS LOCATIONS AND TOTAL SIZES

In the results chapter, section 5.3 the capacitor placement results for VSR and RPLR objectives using Circuit I were provided. Five cases were presented. The bus locations to install capacitors and their respective sizes for each case are provided in Table A.1. A description for each of the cases are as follows:

- Case 1: base case, results at the reduced substation voltage and with existing capacitor placements only.
- Case 2: VSR results without $Q$ constraints applied and with existing and new capacitor placements.
- Case 3: RPLR results without $Q$ constraints applied and with existing and new capacitor placements.
- Case 4: VSR results with $Q$ constraints applied and with existing and new capacitor placements.
- Case 5: RPLR results with $Q$ constraints applied and with existing and new capacitor placements.

Here, the case number, the three-phase bus number where the capacitor was placed, each capacitors respective total size (kVAr), the capacitor type of switching (manual or automated), and capacitor type (existing or new) is provided for each of the five cases. The remaining columns in the table provide the number of under voltage violations, $Q_{out}$ (kVAr), $P_{Loss}$ (kW), and the per unit $VS$ at peak loading with all automated capacitors turned on.
Table A.1. Placement Results at Peak Loading with All Switchable Capacitors Turned On with (w/) and without (no) \( Q \) limits applied for each objective:

Circuit I with All Cases at 12.44 kV,
VSR – Voltage Spread Reduction
RPLR – Real Power Loss Reduction

| Case        | Cap Bus Number | Size (kVAr) | Switch Type | Cap Type | Number of Bus w/ \( |V| \leq 0.95 \) (p.u.) | \( Q_{\text{out}} \) (kVAr) | \( P_{\text{Loss}} \) (kW) | \( VS \) (p.u.) |
|-------------|----------------|-------------|-------------|----------|-----------------------------|----------------|------------------|--------------|
| 1. Base Case | 2025           | 600         | Manual      | Existing | 315                         | 957.18 lagging | 282.42           | 0.05423      |
|             | 1278           | 900         | Automated   |          |                             |                |                  |              |
|             | 1350           | 900         | Automated   |          |                             |                |                  |              |
| 2. VSR no \( Q \) Limits | 2596          | 600         | Automated   | New      | 0                           | 818.28 leading | 302.84           | 0.03405      |
|             | 2595           | 600         | Automated   |          |                             |                |                  |              |
|             | 2602           | 600         |             |          |                             |                |                  |              |
| 3. RPLR no \( Q \) Limits | 2849          | 300         | Automated   | New      | 0                           | 75.162 lagging | 282.35           | 0.04609      |
|             | 2793           | 300         | Automated   |          |                             |                |                  |              |
|             | 2602           | 300         |             |          |                             |                |                  |              |
| 4. VSR w/ \( Q \) Limits | 2596          | 300         | Automated   | New      | 0                           | 517.11 leading | 295.83           | 0.03735      |
|             | 2595           | 600         | Automated   |          |                             |                |                  |              |
|             | 2602           | 600         |             |          |                             |                |                  |              |
| 5. RPLR w/ \( Q \) Limits | 2827          | 1200        | Automated   | New      | 0                           | 225.37 leading | 282.39           | 0.04454      |
APPENDIX B

B.1. ABBREVIATIONS FREQUENTLY USED IN THESIS

In Table B.1 a list of frequently used terms and their respective abbreviations that were applied throughout the thesis are provided. Metric terms have abbreviations that are italicized.

<table>
<thead>
<tr>
<th>Term</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advanced Distribution Automation</td>
<td>ADA</td>
</tr>
<tr>
<td>Capacitor(s)</td>
<td>Cap(s)</td>
</tr>
<tr>
<td>Distributed Energy Resource</td>
<td>DER</td>
</tr>
<tr>
<td>Direct Load Control Participant</td>
<td>DLC</td>
</tr>
<tr>
<td>Distribution System</td>
<td>DS</td>
</tr>
<tr>
<td>Photovoltaic Generator</td>
<td>PhV</td>
</tr>
<tr>
<td>Real Power (kW)</td>
<td>$P$</td>
</tr>
<tr>
<td>Total Real Power Losses (kW)</td>
<td>$P_{Loss}$</td>
</tr>
<tr>
<td>Reactive Power (kVar)</td>
<td>$Q$</td>
</tr>
<tr>
<td>Substation Reactive Power Output (kVar)</td>
<td>$Q_{out}$</td>
</tr>
<tr>
<td>Real Power Loss Reduction</td>
<td>RPLR</td>
</tr>
<tr>
<td>Transmission System</td>
<td>TS</td>
</tr>
<tr>
<td>Voltage Spread (per unit)</td>
<td>VS</td>
</tr>
<tr>
<td>Voltage Spread Reduction</td>
<td>VSR</td>
</tr>
</tbody>
</table>
VITA

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