Model-Based Controller Design for General Nonlinear Processes

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Dedications

To my parents
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Non-minimum-phase behavior of a process limits the degree of achievable control quality and complicates the controller design for the process. This behavior can be due to the presence of an unstable mode in the zero dynamics, a time delay, or both. To achieve greater profitability, process designers have been creating designs in regions involving complex nonlinearity where the controllers continue to face stiff challenges. Operation is often more profitable at an unstable steady state or a stable steady state close to an unstable one, sometimes involving non-minimum-phase behavior. This project aims to advance the existing non-minimum-phase control theory and to improve the operation of processes that are unstable and exhibit non-minimum-phase behavior.

The research project has two main objectives: (i) the derivation of a general model-based control system and (ii) the development of a software package for model-based controller design. Two novel nonlinear control laws that are applicable to general multivariable processes, whether non-minimum- or minimum-phase, were developed. The first control law ensures closed-loop stability by forcing all state variables to follow their corresponding linear reference trajectories. The second control law guarantees the closed-loop stability by using a Lyapunov hard constraint that requires all state variables to evolve within a shrinking state ‘tube’.

A prototype controller design software package was developed to simplify the implementation of differential geometric, model-based controllers. The software package carries out symbolic manipulations to automatically generate the analytical model-based controllers and subsequently test the designed controllers. It has a user-friendly interface
that allows the user to enter process model equations and process parameters easily. Using the controller design software, one can design the model-based controllers with much less effort, which is expected to result in more industrial applications of the controllers.
Chapter 1: Introduction

1.1 Motivation

In an effort to achieve maximum profitability, chemical and biochemical processes have to find a compromise between product quality and operating cost. Process optimization has become a preferred choice for a process engineer to achieve this goal. Often operation is more profitable at either an unstable steady state or at a stable steady state in the proximity of an unstable steady state, sometimes involving non-minimum-phase behavior (inverse response). In processes exhibiting such behavior, the controlled output initially responds in the opposite of the desired direction. Examples of these processes include chemical reactors [1-5], fermentation reactors [6-9], fluidized catalytic crackers, reactor-separator-recycle plants [10], azeotropic distillation columns [11, 12], and reboilers [13]. Non-minimum-phase behavior limits the degree of achievable control quality and complicates the controller design for the process. This behavior can be due to the presence of an unstable mode in the zero dynamics (finite right-half-plane zero in the linear case), a time delay (infinite right-half-plane zero in the linear case), or both. In model-based control, significant advances have been made in an effort to address the control problem of non-minimum-phase processes. Differential-geometric control is a model-based control method which involves direct controller synthesis. However, the majority of the existing differential-geometric methods that are applicable to non-minimum-phase processes are not easy to implement because these methods require solving a set of partial differential equations. In addition, there are a few control methods in differential-geometric approach that are able to control the unstable non-minimum-
2

phase processes. These methods are limited to a few classes of nonlinear processes. The existing nonlinear differential-geometric model-based control methods to handle the general multivariable processes whether minimum-phase or non-minimum-phase are still inadequate. In addition, unavailable is a software package that fully automates the design of differential-geometric controllers, given a process model in the form of differential and algebraic equations. The advances in the control of unstable, non-minimum-phase systems will allow for tighter control of severely nonlinear processes with complex dynamics and is expected to lead to greater interest in model-based control. Furthermore, the advances in controller design software are expected to increase the industrial use of differential-geometric control.

1.2 Previous Work

The relevant research work either in the nonlinear model-based control for non-minimum-phase processes or in the software for analytical model-based controller design are briefly discussed in the following subsections.

1.2.1 Nonlinear Model-Based Control

In the area of nonlinear model-based control, the main interest has been focusing on the frameworks of model-predictive control, differential-geometric control, and Lyapunov-based control. In model-predictive control, controller action is the solution to a constrained optimization problem that is solved on-line, involving numerical model inversion [14]. Local closed-loop stability is guaranteed by imposing designed constraints or penalty terms on optimization problems. In the differential-geometric control, the controller is derived by requesting a desired closed-loop response in the absence of input
constraints [15], involving analytical model inversion. In Lyapunov-based control, closed-loop stability plays a fundamental role in the controller design [16-18]. The asymptotic decay of a norm of the state variables is ensured by applying a proper Lyapunov function in the controller design. More details in each framework are discussed in the following subsections.

1.2.1.1 Differential-Geometric Control

The differential-geometric control is the dominant approach in nonlinear model-based control due to direct synthesis of the controllers and the proof on closed-loop stability. Input-Output (I-O) linearization has been the most widely used differential-geometric control, because it is easy to implement (i.e. it does not require solving a set of partial differential equations). Initially, I-O linearization was developed for unconstrained, minimum-phase (MP) processes [19, 20]. The I-O linearization cannot be applied directly to non-minimum-phase (NMP) processes because it includes an inverse of the process model, causing closed-loop instability in the case of NMP processes.

During the past fifteen years, several successful attempts have been made to develop differential-geometric controllers applicable to NMP nonlinear processes [1, 2, 15, 21-33]. For example, total linearization [15, 27, 28], equivalent outputs [21, 29], approximate I-O linearization [1, 25, 32], trajectory linearization [31], $H_\infty$ optimal controller [33], and stable inversion [22-24, 26]. Isidori [15], Isidori and Byrnes [28], and Isidori and Astolfi [27] studied total linearization, which is applicable to a limited class of minimum- and non-minimum-phase processes and requires solving a nonlinear partial differential equation. Kravaris and Daoutidis [2] present a nonlinear state-feedback controller for second-order, non-minimum-phase, nonlinear systems. Niemiec
and Kravaris [21] propose a systematic procedure for the construction of statically equivalent outputs with prescribed transmission zeros. They then design a nonlinear state-feedback controller on the basis of the synthetic outputs. Kanter et al. [1] developed nonlinear control laws for input-constrained, multiple-input, multiple-output stable processes, whether their delay-free part was minimum- or non-minimum-phase. They addressed the nonlinear control of the processes by inducing an approximately linear, input-output response that exploited the connection between model-predictive control and I-O linearization. Kravaris et al. [29] present a systematic method of arbitrarily assigning the zero dynamics of a nonlinear system by constructing the requisite synthetic output maps. The minimum-phase, synthetic output maps constructed can be made statically equivalent to the original output maps, and therefore, they can be directly used for non-minimum-phase compensation purposes. The method requires solving a system of first-order, nonlinear, singular PDEs. Mickle et al. [31] develop a tracking controller for unstable, non-minimum-phase, nonlinear processes by using trajectory linearization. Tomlin and Sastry [32] derived tracking control laws for non-minimum-phase, nonlinear systems with both fast and slow, possibly unstable, zero dynamics. van der Schaft [33] develops a nonlinear state feedback $H_\infty$ optimal controller. Devasia et al. [24] and Devasia [23] introduce an inversion procedure for nonlinear systems that constructs a bounded input trajectory in the pre-image of a desired output trajectory. The pre-image trajectory is noncausal (rather than unstable) in the non-minimum-phase case. Hunt and Meyer [26] show that under appropriate assumptions the bounded solution of the partial differential equation of Isidori and Byrnes [28] for each trajectory of an exosystem must be given by an integral representation formula of Devasia et al. [24]. Chen and Paden
[22] studied the stable inversion of non-minimum-phase nonlinear systems. They derive a stable, but non-causal, inverse that can be incorporated into a stabilizing controller for output tracking. Doyle et al. [25] present a control synthesis scheme for nonlinear, single-input, single-output systems which have completely unstable zero dynamics. The approach is similar to linear approaches for non-minimum-phase systems and involves the derivation of an input-output linearizing controller for a suitably-defined, nonlinear, minimum-phase approximation to the original system. The linearizing controller achieves an approximately linear input-output response and internal stability. McLain et al. [30] propose a controller design strategy for nonlinear systems with more manipulated inputs than controlled outputs. The controller can be used for non-minimum-phase processes. While controllers in [1, 22-26, 30] are applicable to multi-input, multi-output (MIMO), NMP processes, either sets of partial differential equations must be solved [21, 27, 29], or the controllers are applicable to a very limited classes of processes [2, 15, 22-28].

1.2.1.2 Model-Predictive Control

Model-predictive control is a discrete-time control in which control action is the solution to an open-loop constrained optimization problem at each time step. The current values of the process states are used for calculating the control action, and the optimization process is repeated at the next time horizon. A small time interval and a long prediction horizon are required to maintain closed-loop robustness. However, the smaller the time interval is, the heavier the computational load. The model-predictive control generally formulates the optimization problem over a finite prediction horizon to decrease the computational load. In general, the nonlinear model-predictive control leads to a non-convex optimal control problem. The solution of the optimization problem is a local
optimum. However, the model-predictive control can achieve a global optimum by using suboptimal model-predictive control [34].

The model-predictive control has been used widely in the process industries because of its many appealing features such as handling multivariable systems with time-delays. In addition, the constraints on manipulated inputs, states and output variables are explicitly handled in the formulation of the optimization problem. However, the closed-loop stability and feasibility are major concerns in the model-predictive control. The local optimization in a finite horizon does not guarantee closed-loop stability [34, 35]. Thus, the model-predictive control formulates the optimization problem with special constraints or penalty terms based on a Lyapunov function to ensure the closed-loop stability. The stability is guaranteed by imposing an equality or inequality constraint on the final state in the prediction horizon (the terminal state constraint [36-38]), adding a weight on the final state in the objective function (the terminal state penalty [35]), or using an infinite prediction horizon with a finite control horizon [39]. However, the measurable process states, perfect process-model, and high computational load to determine the attraction domain boundaries of the linear controller are required. Furthermore, a large number of tunable parameters are needed when the optimization problem of the terminal state constraint is complex. To avoid the high computational load, the contractive constraint method [40] introduces a stabilizing state constraint and requires the process states at the end of the prediction horizon to be norm-contracted with respect to the process states at the beginning of the prediction horizon. The stability of the closed-loop system in the model-predictive control can be tested by employing a Lyapunov function, or a sequence
of monotonic objective functions only when the prediction horizon is infinite or when a terminal state constraint is applied.

The control problems of unstable processes and non-minimum-phase processes have drawn attention for many years. The control problem of the unstable process requires availability of sufficient control action to keep the process variables within a limited region around the desired operating point. Gobin et al. [41] employed a dynamic matrix control technique that requires linearized process model around an unstable operating point. Nagrath et al. [42] proposed state-estimation-based model-predictive control with no terminal state penalty term to control the unstable processes. The control method employed a concept of cascade control that uses the secondary measurement to aid disturbance rejection. However, the stability and stabilizing constraint are dependent on proper tuning parameters. The control problem of non-minimum-phase processes is handled simply by requesting a sufficiently large prediction horizon and a sufficiently short control horizon [43, 44]. Hernandez and Arkun [45] proposed a $p^{th}$ inverse controller with long prediction horizon for processes. The Jacobian of the closed loop system comes close to the Jacobian of the open-loop system as prediction horizon approaches infinity.

Although the differential-geometric and model-predictive control are respectively formulated in the continuous and discrete time, the relationship between the two approaches has been discussed [46, 47]. Soroush et al. [46, 47] proposed a continuous-time model-predictive control method that exploited the connections between model-predictive and input-output linearizing control methods. The derived model-predictive control laws have the shortest possible prediction horizon and explicit analytical form.
The controller is derived by minimizing a function norm of the deviations of the controlled outputs from desired linear reference trajectories. However, this method is not applicable to unstable, non-minimum-phase processes.

1.2.1.3 Lyapunov-Based Control

In Lyapunov-based control, a concept of Lyapunov stability is integrated into control design. Two dominant methods of Lyapunov stability are used to investigate the stability of nonlinear systems, called linearization method and direct method. The linearization method or indirect method determines the stability of the nonlinear system through the eigenvalues of the Jacobian matrix of the linearized system around equilibrium. The direct method determines the stability of nonlinear system by constructing a Lyapunov function formulated as real positive definite. If the Lyapunov function is continuously dissipated, then the system must eventually settle down to an equilibrium point, whether minimum- or non-minimum-phase systems. Lyapunov-based control control methods are usually of the following types [48]:

- Assume a suitable control law and then find a proper Lyapunov function to prove closed-loop stability, and
- Assume a Lyapunov function candidate and then find a control law to make this candidate a real Lyapunov function, usually called converse Lyapunov theorem.

Most Lyapunov-based control methods are of the second type. Examples are the controllers in [49-51] derived by solving inverse optimal control problem exploited with the concept of control Lyapunov functions. Complementary to the Lyapunov direct
method are feedback control with backstepping [52, 53], energy-predictive control [54, 55] and other approaches [56-60].

For the control problem of non-minimum-phase processes, Kazantzis and Kravaris [61] developed the control method for unstable non-minimum-phase nonlinear processes by integrating both feedback linearization and pole replacement design in a single step via the adapted Luenberger approach. The Lyapunov auxiliary theorem is employed to guarantee the existence and uniqueness of solution. However, this method is required to truncate the control problem formulated in the form of a singular partial differential equation system. In addition, it is restricted to the first order ODE system. Wu [62] proposed the Lyapunov-based linearization method to ensure the asymptotic output regulation. This method uses differential-geometric approach with the Lyapunov function, and the partial state feedback is obtained by observer-based. However, the method presented by Wu is applicable for non-minimum-phase nonlinear processes with the actuator constraint and one controlled output. In addition, the application of this technique requires the definition of the error variables and chooses the appropriate Lyapunov function to stabilize the closed-loop system.

A problem that has received attention in recent years is how to expand the domain of attraction for the closed-loop system in Lyapunov control. To solve this problem, the use of combined control methods [18, 63, 64] and multiple Lyapunov functions in the controller design [17] have been proposed.

1.2.2 Software for Analytical Model-Based Controller Design

Conventional linear controllers, PI and PID, were widely used in the industrial processes for many decades. However, not all processes can be controlled with such controllers.
More advanced control technique such as the model-based control becomes a preferred controller design approach for processes exhibiting high nonlinearity. Model-predictive control and differential-geometric control have been the most widely used model-based control methods. The model-predictive control involves numerical model inversion, while differential-geometric control involves analytical model inversion. The analytical inversion required in differential-geometric control requires analytical partial derivatives and symbolic manipulations, which become cumbersome as the relative order and/or the level of complexity of the model increases. Indeed, the burden of taking analytical partial derivatives and performing symbolic manipulations have prevented this theoretically-sound, efficient controller from being implemented widely in the process industries. In addition, many steps are involved in the design and implementation of differential-geometric controllers including process modeling, model verification, process analysis, controller design, controller verification, and code implementation. To complete the design tasks, there are very elaborate and error prone steps. Furthermore, the differential-geometric controller design still has a gap between the controller verification (by simulation) and the code implementation. The problem with the target platform is not found until the implementation begins. Thus, there are increasing needs for a systematic approach to support controller design algorithms that would lead to reduction in the amount of hand written code, reduction of redundant work in redesigning a new process, and decrease in testing time. The design of the model-based controllers can be used effectively and economically to reduce overall effort.

The increased role of computers in solving complex scientific and engineering problems has led to software packages that facilitate the design of model-based
controllers [65-74]. Symbolic manipulation software for controller design has developed using packages such as MATHEMATICA [75], MAPLE [76], and the Symbolic Toolbox in MATLAB [77]. In one approach, the software is designed as a toolbox for an existing symbolic manipulation package [68, 70, 72, 73]. For example, Blankenship et al. [68] developed a package for the design of nonlinear controllers using MATHEMATICA to provide the controller equations in the format suitable for simulation in SIMULINK. However, the software lacks a visual interface and cannot be used directly for controller design and implementation. In general, existing software has numerical or symbolic computation limitations and requires users to implement lengthy sequences of input commands that follow specific patterns.

To avoid such limitations, an alternate approach to the development of software for the controller design uses multiple calculating engines [69, 72]. For example, Kitamoto et al. [72] created $H_\infty$ controller-design software in which the design algorithm is programmed in MATHEMATICA and the front-end interface is written in MATLAB. Users input the control system in the form of block diagram. Unfortunately, however, the software is unable to analyze the performance of the designed controller, and the capabilities of the symbolic computation are not fully utilized. The software developed by Campbell et al. [69] uses MAPLE to provide symbolic computation of the feedback linearization design for mechanical multi-body systems and Scilab [78] to carry out the numerical simulation.

Although symbolic computation software packages for model-based controller design have been available for many years, their applications have been limited due to deficiencies in available computer-aided design tools and user-interface platforms. For
example, they do not design the controller fully when the process model is a set of ordinary differential and/or algebraic equations. Furthermore, they are not user-friendly for design integration and closed-loop simulation. Thus, the development of a software package devoid of these limitations is of interest in the practice of process control.

1.3 Scope and Objectives

The I-O linearization methods have been widely used because these methods do not require solving a set of partial differential equations and require fewer number of tuning parameters. However, the I-O linearization could not be applied directly to unstable non-minimum-phase processes because of an inverse of the process model causing closed-loop instability. In addition, there are a few control methods in differential-geometric approach that are able to control the unstable non-minimum-phase processes. These methods are limited to a few classes of nonlinear processes.

Thus, one of the objectives of this research effort is to develop new control laws that do not possess the limitations of I-O linearization. The specific research objectives are as follows.

(1) To develop new control laws that are applicable to general nonlinear processes within the framework of the differential-geometric control. The control laws should:

- Guarantee offset-free response in the presence of constant disturbances and process-model mismatch,
- Guarantee the asymptotic stability of the closed-loop system, and
- Be applicable to processes with input constraints.
(2) To develop a software package for designing model-based controllers for general nonlinear processes. The newly developed software will help control engineers to design conveniently model-based controllers for general nonlinear processes. The software should:

- Automate the design of differential-geometric controllers,
- Have a user-friendly visual interface, and
- Be capable of generating controller equations in C, FORTRAN, or MATLAB formats and also carry out closed-loop simulations.

Two novel control laws are presented to achieve the first objective, approximate input-state linearization and input-output linearization with a stability constraint. The controllers derived based on the developed control laws are applicable to stable and unstable processes, whether non-minimum- or minimum-phase.

1.4 Significance of this Research

The impact of this research is two-fold. First, the problem of controller design for general nonlinear processes is studied. The unstable non-minimum-phase behavior limits the degree of achievable control quality of the process. The new control law allows complex processing plants to accomplish greater profitability. Second, the new controller design software enables control engineers to implement differential-geometric nonlinear controllers with little effort. This is expected to lead to an improvement in the industrial popularity of differential-geometric nonlinear controllers and better control of severely nonlinear processes. This work can positively impact the profitability of processing plants through the production of higher quality products at lower costs and improved control quality.
1.5 Organization of the Thesis

The dissertation is organized into two parts. The first part comprises Chapters 2-3 that address the theoretical issues concerning two new nonlinear model-based control laws for general multivariable nonlinear processes with input constraints. The second part comprises Chapter 4 that describes the development of software for analytical model-based controller design.

In the part one, the approximate input-state linearization is introduced in Chapter 2. The proposed method ensures the closed-loop stability by forcing all process state variables to follow their corresponding reference trajectories that have orders higher than the state-variable relative orders. The control system includes a nonlinear state feedback, integrator and a reduced order nonlinear state observer. The scope of the study, mathematical preliminaries of the approximate input-output linearization and reduced-order state observer, and nonlinear feedback control method are discussed in the subsections of Chapter 2. Furthermore, Chapter 2 also demonstrates the performance of the controller by numerical simulation of a bioreactor and chemical reactors.

In Chapter 3, the input-output linearization with stability constraint is discussed. The second control method formulates the optimal output-tracking controller based on I-O linearization and guarantees closed-loop asymptotic stability by including a Lyapunov function as a hard constraint. The scope of the study, mathematical preliminaries of I-O linearization and the Lyapunov’s direct method, and nonlinear control method are discussed in the subsections of Chapter 3. In addition, Chapter 3 also demonstrates the performance of the control method by numerical simulation of several process examples.
In part two, the focus is shifted to the analytical model-based controller design software. The software automates the design of differential-geometric controller for general nonlinear processes, thus avoiding laborious mathematical derivations. The scope of the study, mathematical preliminaries of geometric model-based control, the details of the software, and application of the software are discussed in the subsections of Chapter 4. Finally, Chapter 5 presents the overall conclusions of this work and the suggestions for future research directions.
2.1 Introduction

Differential-geometric control is a direct synthesis approach in which the controller is derived by requesting a desired closed-loop response in the absence of input constraints. Input-Output (I-O) linearization has been the most widely used differential-geometric control method because it does not require solving a set of partial differential equations. However, I-O linearization cannot be applied directly to non-minimum-phase processes (NMP) because it includes an inverse of the process model, thereby causing closed-loop instability in the case of NMP processes. The differential-geometric control requires a special treatment to handle the non-minimum-phase behavior.

During the past fifteen years, differential-geometric controllers have been extended to NMP nonlinear processes [1, 2, 15, 21-30, 32, 33, 79]. While controllers in [22-26, 30] are applicable to multi-input multi-output (MIMO) NMP processes, either sets of partial differential equations must be solved [21, 27, 29], or the controllers are applicable to a very limited class of processes [2, 15, 22-28, 30].

This chapter presents a new control method that uses the same continuous-time model-predictive control framework employed in Kanter et al. [1] and Soroush and Soroush [80]. However, it is conceptually different from those presented previously in [1, 80] as follows:

a) The method is now applicable to processes operated at an unstable non-minimum-phase steady state in the presence of input constraints.
b) The new controller is derived by requesting desired responses for the state variables (instead of outputs).

c) The nonlinear state feedback is derived by minimizing a function norm of the deviations of state variables from linear reference trajectories in each time instant, that have orders higher than the state-variable relative orders.

In the proposed method, the closed-loop stability is ensured by forcing all process state variables to follow their corresponding reference trajectories. The proposed control system includes a nonlinear state feedback, integrator and a reduced order nonlinear state observer. The resulting state feedback approximately induces linear responses to the state variables in the presence of constraints. The integral action is added to ensure offset-free output responses in the presence of constant disturbances and process-model mismatch. The state observer is used to estimate state variables that are not measured.

This chapter is organized as follows. The scope of the study and some mathematical preliminaries are given in Section 2.2 Section 2.3 presents the nonlinear feedback control method. Finally, in Section 2.4, the application and performance of the control method are illustrated by numerical simulation of a bioreactor and two chemical reactors with multiple steady states.

### 2.2 Scope and Mathematical Preliminaries

Consider general, square, multi-input, multi-output processes having a mathematical model in the form:

\[
\frac{dx}{dt} = f(x,u) \quad x(0) = x_0
\]

\[
y = h(x)
\]

(2.1)
where \( x = [x_1 \cdots x_n]^T \in \mathbb{R}^n \) is the vector of state variables, \( u = [u_1 \cdots u_m]^T \in \mathbb{R}^m \) is the vector of manipulated inputs, \( y = [y_1 \cdots y_n]^T \in \mathbb{R}^m \) is the vector of controlled outputs, \\
\( f(x,u) = [f_1(x,u) \cdots f_n(x,u)]^T \) and \( h(x) = [h_1(x) \cdots h_m(x)]^T \) are smooth vector functions.

The relative order (degree) of a state variable, \( x_i \), is denoted by \( r_i \), where \( r_i \) is the smallest integer for which \\
\[
\frac{\partial}{\partial u} \left( \frac{d^{r_i} x_i}{dt^{r_i}} \right) \neq 0.
\]

It is assumed that the relative orders, \( r_1, \ldots, r_n \), are finite, and the process is controllable and observable locally around a desired steady state.

Let \( H_i^0(x) = x \), and define the following notation:

\[
H_i^j(x) = \frac{dx_i}{dt} \\
\vdots \\
H_i^{r_i-1}(x) = \frac{d^{r_i-1}x_i}{dt^{r_i-1}} \\
H_i^r(x,u) = \frac{d^r x_i}{dt^r} \\
H_i^{r+1}(x,u^{(0)},u^{(1)}) = \frac{d^{r+1}x_i}{dt^{r+1}} \\
\vdots \\
H_i^{j}(x,u^{(0)},u^{(1)},\ldots,u^{(j-r)}) = \frac{d^{j}x_i}{dt^{j}}
\]

where \( j_i \geq r_i \), \( i = 1, \ldots, n \), and \( u^{(i)} = \frac{d^i u}{dt^i} \)

For a given output set-point, \( y_{sp} \), the corresponding desired steady-state pair \((x_{ss}, u_{ss})\) satisfy:
These relations are used to describe the dependence of a nominal (desired) steady state, \( x_{ss} \), on the set-point, say \( x_{ss} = F(y_{sp}) \).

### 2.2.1 Approximate Input-Output Linearization

To place the new control method in perspective, it is instructive to review briefly the control method presented in [1]. For a process in the form of (2.1), in [1], closed-loop process output responses of the linear form:

\[
\begin{align*}
(\tau_1 D + 1)^{R_1} y_1 &= y_{sp_1} \\
(\tau_m D + 1)^{R_m} y_m &= y_{sp_m}
\end{align*}
\]  

are requested. Here \( D = \frac{d}{dt} \), \( P_1 \geq R_1, \cdots, P_m \geq R_m \) (\( R_1, \cdots, R_m \) are the relative orders of the process outputs, \( y_1, \ldots, y_m \), respectively), and \( \tau_1, \cdots, \tau_m \) are positive constants that set the speed of the responses of the process outputs, \( y_1, \ldots, y_m \), respectively. Taking the output time-derivatives in (2.3) leads to the derivation of a dynamic state feedback in the compact form:

\[
\Phi_p(x, u^{(0)}, u^{(1)}, \cdots, u^{(P-R)}) = y_{sp}
\]  

where \( P=\max(P_1, \ldots, P_m) \) and \( R=\min(R_1, \ldots, R_m) \). Setting all the time derivatives of \( u \) in (2.4) to zero results in to the static state feedback:

\[
\phi_p(x, u) = \Phi_p(x, u^{(0)}, 0, \cdots, 0) = y_{sp}
\]  

If (2.5) has a real root for \( u \) at each time instant and \( \partial \phi_p(x, u)/\partial u \neq 0 \) at the root, a state feedback of the form exists:
Theorem 1 [1]. The closed-loop system under the state feedback of (2.6) is asymptotically stable, if the following conditions hold:

a) The nominal equilibrium pair of the process corresponding to \( y_{sp} \) is hyperbolically stable;

b) The tunable parameters \( P_1, \ldots, P_m \) are chosen to be sufficiently large; and

c) The tunable parameters, \( \tau_j = \tau_1, \ldots, \tau_m \), are chosen such that for every \( l = 1, \ldots, m \), all eigenvalues of \([I + \tau_l J_{ol}(x_0, u_0)]\) lie inside the unit circle.

Furthermore, as \( P_1, \ldots, P_m \to \infty \), the eigenvalues of the Jacobian of the closed-loop system \([J_{cl}(x_0, u_0)]\) approach the eigenvalues of the Jacobian of the open-loop process \([J_{ol}(x_0, u_0)]\).

2.2.2 Reduced-Order State Observer Design

This subsection presents a brief review of the nonlinear, reduced-order, state observer described in [81]. The state observer will be used in this paper to reconstruct unmeasured state variables of the process under consideration. Consider a nonlinear process in the form of (2.1) with additional output measurements \( Y_1, \ldots, Y_q \); that is,

\[
\begin{aligned}
\frac{dx}{dt} &= f(x, u) \quad x(0) = x_0 \\
y &= h(x) \\
Y &= K(x)
\end{aligned}
\]

The non-redundancy of the measured outputs ensures the existence of a locally-invertible state transformation of the form
where \( \eta = [\eta_1, \ldots, \eta_{n-m-q}]^T \), and \( Q \) is a constant \((n - m - q) \times n\) matrix which for the sake of simplicity, is chosen such that:

a) Each row of \( Q \) has only one nonzero term equal to one; and

b) Locally

\[
\text{rank} \left\{ \frac{\partial}{\partial u} \begin{bmatrix} Qx \\ h(x) \\ K(x) \end{bmatrix} \right\} = n
\]

Thus, the state transformation \([\eta \ y \ Y]^T = T(x)\) is locally invertible. The system of (2.1), in terms of the new state variables, \( \eta_1, \ldots, \eta_{n-m-q}, y, Y \), takes the form

\[
\begin{align*}
\dot{\eta} &= F_\eta(\eta, y, Y, u) \\
\dot{y} &= F_y(\eta, y, Y, u) \\
\dot{Y} &= F_Y(\eta, y, Y, u) 
\end{align*}
\]

(2.7)

where

\[
F_\eta(\eta, y, Y, u) = Qf\left[T^{-1}(\eta, y, Y), u\right]
\]

\[
F_y(\eta, y, Y, u) = \left. \frac{\partial h(x)}{\partial x} \right|_{x = T^{-1}(\eta, y, Y)} f\left[T^{-1}(\eta, y, Y), u\right]
\]

\[
F_Y(\eta, y, Y, u) = \left. \frac{\partial K(x)}{\partial x} \right|_{x = T^{-1}(\eta, y, Y)} f\left[T^{-1}(\eta, y, Y), u\right]
\]
A closed-loop, reduced-order observer is then designed in the form:

\[ \dot{z} = F_\eta(z + L_1 y + L_2 Y, y, Y, u) - L_1 F_\eta(z + L_1 y + L_2 Y, y, Y, u) - L_2 F_\eta(z + L_1 y + L_2 Y, y, Y, u) \]

\[ \hat{x} = T^{-1}(z + L_1 y + L_2 Y, y, Y) \]  

(2.8)

where the constant \([ (n - m - q) \times m ]\) and \([ (n - m - q) \times q ]\) matrices, \(L_1\) and \(L_2\), are the observer gains. The observer gains should be set such that the observer error dynamics are asymptotically stable, which requires all eigenvalues of the \([ (n - m - q) \times (n - m - q) ]\) matrix

\[ \frac{\partial F_\eta(\eta, y, Y, u)}{\partial \eta} - L_1 \frac{\partial F_\eta(\eta, y, Y, u)}{\partial \eta} - L_2 \frac{\partial F_\eta(\eta, y, Y, u)}{\partial \eta} \]

evaluated at the nominal steady-state pair, to be in the left half of the complex plane.

2.3 Nonlinear Control Method

A state feedback that induces approximately linear responses to the state variables is first derived. A reduced-order state observer is then designed to reconstruct unmeasured state variables from the output measurements. To add integral action to the state feedback-state observer system, a dynamic subsystem is finally added.

2.3.1 State Feedback Design

For a process in the form of (2.1) with input constraints:

\[ u_i \leq u \leq u_i, \quad i = 1, \ldots, m \]

a linear response is requested of the following form for each of the state variables:

\[ (\epsilon_i D + 1)^{p_i} x_i = x_{ss_{y_i}} \]

\[ \vdots \]

\[ (\epsilon_n D + 1)^{p_n} x_n = x_{ss_{y_n}} \]  

(2.9)
where \( p_i \geq r_i, \ldots, p_n \geq r_n \), and \( \varepsilon_1, \ldots, \varepsilon_n \) are positive constants that set the speed of the state responses. The state responses in (2.9) can be achieved only when \( m \geq n \). However, since in many processes \( m < n \) (there are more state variables than manipulated inputs), the state responses in (2.9) can rarely be achieved. To relax the requirement of linear responses for all state variables, state responses are requested that are as close as possible to the linear ones described by (2.9). To derive a state feedback that achieves relaxed state responses, the following constrained optimization problem is solved at each time instant:

\[
\min_u \sum_{i=1}^n \theta_i \left[ \frac{(\varepsilon_i D + 1)^p_i x_i - x_{m_i}}{\varepsilon_i^{p_i}} \right]^2
\]

(2.10)

subject to:

\[
u_i^{(l)} = 0, \quad l \geq 1, \quad i = 1, \ldots, m
\]

\[
u_i \leq u \leq u_h, \quad i = 1, \ldots, m
\]

where \( \theta_1, \ldots, \theta_n \) are adjustable, positive, scalar weights whose values are set according to the relative importance of the state variables: the higher the value of \( \theta_i \), the smaller the deviation of the \( x_i \) response from the desired linear response for \( x_i \).

For a process in the form of (2.1), the optimization problem in (2.10) takes the form:

\[
\min_u \sum_{i=1}^n \theta_i \left[ \frac{x_i + \sum_{l=1}^{r_i} p_i \left( H_i^l(x) + \sum_{l=r_i}^n p_i \left( H_i^l(x, u, 0, \ldots, 0) - x_{m_i} \right) \right)}{\varepsilon_i^{p_i}} \right]^2
\]

(2.11)

subject to

\[
u_i \leq u \leq u_h, \quad i = 1, \ldots, m
\]
where

\[
\begin{pmatrix}
  p_i \\
  l
\end{pmatrix} = \frac{p_i^l}{(p_i - l)^l!}
\]

In the case that \( m \geq n \), the performance index in (2.10) may take the value of zero, implying that the linear, closed-loop, state responses of (2.9) are achieved. The preceding state feedback is denoted in the compact form:

\[ u = \Psi(x, x_{ss}) \]

(2.12)

**Example: Linear Unstable, Non-Minimum-Phase Process**

Consider the linear process:

\[
\begin{align*}
\frac{dx_1}{dt} &= x_2 \\
\frac{dx_2}{dt} &= 10x_1 + 9x_2 + u \\
y &= 2x_1 - x_2
\end{align*}
\]

This process is unstable (has poles at -1 and 10), and non-minimum-phase (has a transmission zero at 2). The relative orders of the states, \( x_1 \) and \( x_2 \), are: \( r_1 = 2 \) and \( r_2 = 1 \).

For \( \varepsilon_1 = 0.8 \) and \( \varepsilon_2 = 0.01 \), the closed-loop eigenvalues of this process under the state feedback in (2.11) are given in Table 2.1. The state feedback of (2.11) is capable of operating the process at any steady state when \( p_1 \) and \( p_2 \) are chosen such that \( p_1 \geq 2 \) and \( p_2 \geq 1 \).
Table 2.1  Closed-loop eigenvalues of the linear example for several $p_1$ and $p_2$ values.

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>-100.00</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-47.89</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>-30.59</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>-21.99</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>-16.86</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-1.44</td>
<td>-1.090</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>-1.52</td>
<td>-1.020</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>-1.60</td>
<td>-0.970</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>-1.67</td>
<td>-0.930</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>-1.73</td>
<td>-0.900</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>-1.00</td>
<td>-0.150</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>-1.00</td>
<td>-0.153</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-1.00</td>
<td>-0.153</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>-1.00</td>
<td>-0.153</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>-1.00</td>
<td>-0.153</td>
</tr>
<tr>
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<td>1</td>
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<td>-0.017</td>
</tr>
<tr>
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<td>2</td>
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<td>-0.017</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>-1.00</td>
<td>-0.017</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>-1.00</td>
<td>-0.017</td>
</tr>
</tbody>
</table>

2.3.2 Reduced-Order State Observer

In general, measurements of all state variables are not available. In such cases, estimates of the unmeasured state variables can be obtained from the output measurements. Here, we use the ‘closed-loop’, reduced-order, nonlinear, state observer of (2.8) to reconstruct
the unmeasured state variables. The state observer is applicable to both stable and unstable processes.

2.3.3 Integral Action

To ensure offset-free response of the closed-loop system in the presence of constant disturbances and model errors, the control system should have integral action. An estimate of disturbance-free process outputs is first calculated by using the closed-loop process model:

\[
\dot{w} = f(w, \Psi(w, x_{ssy})) \\
\xi = h(w)
\]  

(2.13)

where \( \xi \) is the estimate of the disturbance-free controlled output. The difference between this estimate and the measurement of the controlled outputs, \( y \), is then added to the output set-point, leading to:

\[
x_{ssy} = F \left( y_{sp} + (h(w) - y) \right)
\]

(2.14)

An interesting feature of this approach to adding integral action is that the addition of the dynamic system of (2.13) to the state feedback of (2.12) [calculation of \( x_{ssy} \) according to (2.14)] adds no additional conditions for closed-loop asymptotic-stability. In other words, if the closed-loop system is asymptotically stable under the state feedback of (2.12) alone, it is asymptotically stable under (2.12), (2.13) and (2.14).

2.3.4 Controller System

Combining the state feedback of (2.12), the reduced-order observer of (2.8), and the dynamic sub-system of (2.13) and (2.14), leads to the following controller system that has integral action:
\[
\dot{z} = F_\eta(\hat{\eta}, y, Y, u) - L_1 F_y(\hat{y}, y, Y, u) - L_2 F_y(\hat{y}, y, Y, u)
\]
\[
\hat{\eta} = z + L_1 y + L_2 y
\]
\[
\hat{x} = T^{-1}(\hat{\eta}, y, Y)
\]
\[
\dot{\hat{w}} = f(\hat{w}, \Psi(w, v))
\]
\[
v = F\left(y_{sp} - y + h(w)\right)
\]
\[
u = \Psi(\hat{x}, v)
\]

(2.15)

The control system parameters, \(\varepsilon_1, \cdots, \varepsilon_n\), set the speed of the closed-loop state responses; the smaller the value of \(\varepsilon_i\), the faster the \(x_i\) response. When the process is to operate at a minimum-phase steady state, choosing \(p_1 = r_1, \ldots, p_n = r_n\) is adequate to ensure the asymptotic stability of the closed-loop system. When the process is to operate at a non-minimum-phase steady state, one should choose \(p_1 > r_1, \ldots, p_n > r_n\). A block diagram of the controller system is shown in Figure 2.1.

![Diagram](image)

**Figure 2.1** Parameterized controller system.
2.4 Illustrative Examples

2.4.1 Single-Input Single-Output Chemical Reactor

Consider the constant-volume, non-isothermal, continuous, stirred-tank reactor shown in Figure 2.2, in which the reaction $A \rightarrow B$ takes place in the liquid phase. The reactor model has the form:

$$\frac{dC_A}{dt} = -s \exp \left( -\frac{E_a}{RT} \right) C_A + (C_A - C_{A_f}) \frac{F}{V}$$

$$\frac{dT}{dt} = \gamma_s \exp \left( -\frac{E_a}{RT} \right) C_A + (T_i - T) \frac{F}{V} + Q$$

where $F$ is the volumetric flow rate of the reactor feed and product streams, $V$ is the reactor volume, and $C_{A_f}$ is the concentration of $A$ in the feed stream. The reactor parameter values are given in Table 2.2. This reactor has multiple steady states.

![Figure 2.2](image_url)  
**Figure 2.2** Schematic of the non-isothermal continuous stirred tank reactor.
The reactor temperature, $T$, is the controlled output; it is desired to operate the reactor at its middle (unstable) steady state by manipulating the feed flow rate, $F$. The operating range of the feed flow rate is $0.2 \leq F \leq 1.5 \ m^3 / h$. Here, $x = [C_A \ T]^T$, $u = [F]$, and $y = [T]$.

The steady-state pair corresponding to $y_{sp} = 302$ is $(x_{1ss} = 6.319, x_{2ss} = 302, u_{ss} = 0.45)$ which is unstable (eigenvalues of the process Jacobian evaluated at the steady-state pair are -4.5 and 0.309). The relative orders of the state variables are both unity ($r_1 = r_2 = 1$). The zero dynamics of this process are governed by:

$$\frac{d\zeta}{dt} = \left\{-1 - \gamma \frac{C_A - \zeta}{T_i - y_{sp}}\right\} s \exp\left(\frac{-E_a}{R y_{sp}}\right) \zeta - \frac{C_A - \zeta}{T_i - y_{sp}} Q$$

whose Jacobian evaluated at the desired steady state has an eigenvalue in the right half of the complex plane at 36.27. Thus, the middle steady state is unstable and non-minimum-phase.
Controller System

For this process, the controller system of (2.15) with \( \theta_1 = \varepsilon_1^2, \theta_2 = \varepsilon_2^2, p_1 = 2, \) and \( p_2 = 2, \)
takes the form:

\[
u = \Psi(\hat{x}, v) = \arg \left\{ \min_u \left[ -(v_1 + A_{10} + A_{11}u + A_{12}u^2)^2 + (-v_2 + A_{20} + A_{21}u + A_{22}u^2)^2 \right] \right\}
\]
subject to:

\[0.2 \leq u \leq 1.5\]

where

\[
A_{10} = \hat{x}_1 + \frac{\varepsilon_1^2 s}{R\hat{x}_2} \left\{ -\left( \varepsilon_1 Q E_a + 2R\hat{x}_2^2 \right) \exp \left( -\frac{E_a}{R\hat{x}_2} \right) + \varepsilon_1 s \left( -E_a \gamma \hat{x}_1 + R\hat{x}_2^2 \right) \exp \left( -\frac{2E_a}{R\hat{x}_2} \right) \right\}
\]

\[
A_{11} = \frac{\varepsilon_1 (2C_A - 2\hat{x}_1)}{V} + \frac{\varepsilon_1^2 s}{VR\hat{x}_2} \exp \left( -\frac{E_a}{R\hat{x}_2} \right) \left\{ -R(C_A - 2\hat{x}_1)\hat{x}_2^2 + E_a \hat{x}_1 (\hat{x}_2 - T_i) \right\}
\]

\[
A_{12} = -\frac{\varepsilon_1^2 (C_A - \hat{x}_1)}{V^2}
\]

\[
A_{20} = x_2 + 2\varepsilon_2 \left\{ Q + \gamma \hat{x}_1 s \exp \left( -\frac{E_a}{R\hat{x}_2} \right) + \frac{\varepsilon_1^2 \gamma s}{R\hat{x}_2} \exp \left( -\frac{2E_a}{R\hat{x}_2} \right) \left\{ Q E_a \exp \left( \frac{E_a}{R\hat{x}_2} \right) + \hat{x}_1 s \gamma E_a - \hat{x}_2^2 R s \right\} \right\}
\]

\[
A_{21} = \frac{\varepsilon_2 (2T_i - 2\hat{x}_2 - \varepsilon_2 Q)}{V} + \frac{\varepsilon_2^2 \gamma s}{VR\hat{x}_2} \exp \left( -\frac{E_a}{R\hat{x}_2} \right) \left\{ (T_i - \hat{x}_2) \hat{x}_1 E_a + (C_A - 2\hat{x}_1) R\hat{x}_2^2 \right\}
\]

\[
A_{22} = -\frac{\varepsilon_2^2 (T_i - \hat{x}_2)}{V^2}
\]

\[
\hat{x}_1 = z + Ly
\]

\[
\hat{x}_2 = y
\]

\[
\dot{z} = \left\{ -s \exp \left( -\frac{E_a}{Ry} \right) (z + Ly) + (C_A - z - Ly) \frac{u}{V} \right\} - L \left\{ \gamma s \exp \left( -\frac{E_a}{Ry} \right) \left( z + Ly \right) + (T_i - y) \frac{u}{V} + Q \right\}
\]
\[ \dot{w}_1 = -s \exp \left( -\frac{E_a}{Rw_2} \right) x_1 + (C_A - w_1) \frac{\Psi(w, v)}{V} \]

\[ \dot{w}_2 = \gamma s \exp \left( -\frac{E_a}{Rw_2} \right) x_1 + (T_i - w_2) \frac{\Psi(w, v)}{V} + Q \]

\[ v_i = \frac{\exp \left( \frac{E_a}{Rv_2} \right) QV - sV(C_A \gamma + T_i - v_2) + \sqrt{V^2 \left( \exp \left( \frac{E_a}{Sv_2} \right) Q + s(C_A \gamma + T_i - v_2)^2 \right) + 4 \exp \left( \frac{E_a}{Sv_2} \right) sQ(v_2 - T_i)}}{2 \gamma sV} \]

\[ v_2 = y_{sp} - \hat{x}_2 + w_2 \]

The following controller parameter values are used: \( \varepsilon_1 = 468 s \), \( \varepsilon_2 = 468 s \), and \( L = 20 \).

The closed-loop eigenvalues of this process under the state feedback of (2.11), for several \( p_1 \) and \( p_2 \) values, are given in Table 2.3. MATLAB optimization toolbox is used to solve the constrained optimization problem of the controller system. In this optimization, the number of optimizing variables is equal to the number of manipulated inputs. Therefore, the computational cost of this controller is much less than that of a typical model-predictive controller.

**Controller Performance**

Servo and regulatory responses of the controller are shown in Figures 2.3 and 2.4. Figure 2.3 depicts the evolution of the process state variables for two initial conditions, \([x_1(0), x_2(0)] = [9.162, 290] \) in the minimum-phase region, and \([x_1(0), x_2(0)] = [2.923, 320] \) in non-minimum-phase region, with \( C_A = 12 \text{ kmol/m}^3 \). The set-point, \( y_{sp} = 302 \), corresponds to \((x_{1ss} = 6.319, x_{2ss} = 302) \). In addition, when the process is at the desired set-point, the regulatory performance of the controller is studied. A step change from 12 to 10 \text{ kmol/m}^3 in \( C_A \) is made at \( t= 2 \text{ hr} \) (unmeasured disturbance). The simulation results show that the controller successfully operates the reactor at the desired
steady state, which is non-minimum-phase and open-loop unstable. The controller is capable of operating the process at the desired steady state, regardless of the initial conditions of the process. The integral action of the controller ensures offset-free response in the presence of the unmeasured disturbance.

Table 2.3  Closed-loop eigenvalues of the CSTR example for several $p_1$ and $p_2$ values.

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7.1628</td>
<td>-3.3332</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-3.1375 - 0.5928i</td>
<td>-3.1375 + 0.5928i</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>-4.264</td>
<td>-1.4298</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>16.118</td>
<td>-3.024</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>-3.4071 + 0.7204i</td>
<td>-3.4071 - 0.7204i</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>-4.3881</td>
<td>-1.5140</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.0003</td>
<td>-4.9778</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>-3.4752 + 0.3415i</td>
<td>-3.4752 - 0.3415i</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-4.3489</td>
<td>-1.5317</td>
</tr>
</tbody>
</table>
Figure 2.3  Closed-loop response of the state variables of the chemical reactor
Figure 2.4  Manipulated input and unmeasured disturbance profiles corresponding to Figure 2.3
2.4.2 Single-Input Single-Output Bioreactor

Consider the constant-volume, continuous bioreactor described in [7]. The process dynamics are described by:

\[ \dot{x}_1 = -x_1 u + x_1 (1 - x_2) \exp\left(\frac{x_2}{\gamma_1}\right) \]
\[ \dot{x}_2 = -x_2 u + x_1 (1 - x_2) \exp\left(\frac{x_2}{\gamma_1}\right) \frac{1 + \beta_1}{1 + \beta_1 - x_2} \]
\[ y = x_1 \]

where \( x_1 \) is the dimensionless cell-mass concentration in the bioreactor, \( x_2 \) is the dimensionless substrate concentration, and \( u \) is the dilution rate. The model parameters are \( \beta_1 = 0.02 \) and \( \gamma_1 = 0.48 \). It is desired to maintain the cell-mass concentration, \( x_1 \), at a set-point by manipulating the dilution rate, \( u \), within the operating range, \( 0.4 \leq u \leq 1.0 \).

The process steady-state pair corresponding to \( y_{sp} = 0.1448 \) (\( x_{1ss} = 0.1448, x_{2ss} = 0.8455, u_{ss} = 0.9 \)) is unstable (eigenvalues of the open-loop Jacobian evaluated at the steady state are \( 0.061 + 1.731i \) and \( 0.061 - 1.731i \)). The relative orders of the state variables are: \( r_1 = 1 \) and \( r_2 = 1 \). The zero dynamics of the process is governed by:

\[ \frac{d\zeta}{dt} = -y_{sp} \zeta (1 - \zeta) \exp\left(\frac{\zeta}{\gamma_1}\right) + y_{sp} (1 - \zeta) \exp\left(\frac{\zeta}{\gamma_1}\right) \frac{1 + \beta_1}{1 + \beta_1 - \zeta} \]

whose Jacobian evaluated at the steady state has an eigenvalue in the right half of the complex plane at 1.374. Thus, the desired steady state is unstable and non-minimum-phase.

Controller System

For this process, the controller system of (2.14), with \( \theta_1 = \epsilon_1, \theta_2 = \epsilon_2, p_1 = 1 \) and \( p_2 = 1 \), takes the form:
\[ u = \Psi(\hat{x}, \nu) = \arg \left\{ \min_u \left[ (-v_1 + A_{10} + A_{11}u)^2 + (-v_2 + A_{20} + A_{21}u)^2 \right] \right\} \]

subject to:

\[ 0.4 \leq u \leq 1.0 \]

where

\[
\begin{align*}
A_{00}(\hat{x}) &= \hat{x}_1 + \varepsilon_1 \hat{x}_1 (1 - \hat{x}_2) \exp(\frac{\hat{x}_2}{\gamma_1}) \\
A_{11}(\hat{x}) &= -\hat{x}_1 \varepsilon_1 \\
A_{20}(\hat{x}) &= \hat{x}_2 + \varepsilon_2 \hat{x}_1 (1 - \hat{x}_2) \exp(\frac{\hat{x}_2}{\gamma_1}) \frac{1 + \beta_1}{1 + \beta_1 - \hat{x}_2} \\
A_{21}(\hat{x}) &= -\hat{x}_2 \varepsilon_2
\end{align*}
\]

\[
\begin{align*}
\hat{x}_1 &= y \\
\hat{x}_2 &= z + Ly
\end{align*}
\]

\[
\begin{align*}
\dot{z} &= \left\{ -(z + Ly)u + x_1 (1 - z - Ly) \exp(\frac{z + Ly}{\gamma_1}) \frac{1 + \beta_1}{1 + \beta_1 - z - Ly} \right\} \\
&\quad - L \left\{ -x_1u + x_1 (1 - z - Ly) \exp(\frac{z + Ly}{\gamma_1}) \right\}
\end{align*}
\]

\[
\begin{align*}
\dot{w}_1 &= -w_1q + w_1 (1 - w_2) \exp(\frac{w_2}{\gamma_1}) \\
\dot{w}_2 &= -w_2q + w_1 (1 - w_2) \exp(\frac{w_2}{\gamma_1}) \frac{1 + \beta_1}{1 + \beta_1 - w_2}
\end{align*}
\]

\[
\begin{align*}
v_1 &= y_{sp} - \hat{x}_1 + w_1 \\
v_2 &= F(v_1) \\
q &= \Psi(w, \nu)
\end{align*}
\]

The following controller parameter values are used: \( \varepsilon_1 = 0.001, \varepsilon_2 = 0.15, \) and \( L = 0.01. \)

The closed-loop eigenvalues of this process under the state feedback of (2.11), for several \( p_1 \) and \( p_2 \) values, are given in Table 2.4.
Table 2.4 Closed-loop eigenvalues of the bioreactor example for several $p_1$ and $p_2$ values.

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-10</td>
<td>3.4557</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-2.91</td>
<td>-0.85</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-2.91</td>
<td>-0.83</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-2.91</td>
<td>-0.83</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>-2.91</td>
<td>-0.83</td>
</tr>
</tbody>
</table>

Controller Performance

Figures 2.5 and 2.6 show the servo and regulatory responses of the controller. For two initial conditions, $[x_1(0), x_2(0)]=[0.1, 0.75]$ and $[0.25, 0.2]$, the process state profiles are depicted in Figure 2.5. The set-point, $y_{sp} = 0.1448$, corresponds to the steady state ($x_{1ss} = 0.1448$, $x_{2ss} = 0.8453$). Figure 2.5 shows that the controller successfully operates the bioreactor at the desired steady state, irrespective of the initial conditions of the process. To evaluate the regulatory performance of the controller, a step change from 0.02 to 0.022 in $\beta_1$ of the process (unmeasured disturbance) was made at $t=8$ hr when the process was at the desired steady state. As Figure 2.5 shows, the controller rejects the unmeasured disturbance asymptotically.
Figure 2.5  Closed-loop response of the bioreactor state variables.
Figure 2.6  Manipulated input and unmeasured disturbance profiles corresponding to Figure 2.5.
2.4.3 Multi-Input, Multi-Output Chemical Reactor

Consider the constant-volume, non-isothermal, continuous, chemical reactor shown in Figure 2.7, in which the series reactions, \( A \rightarrow B \rightarrow C \), take place in the liquid phase. The process dynamics are represented by the following model:

\[
\begin{align*}
\frac{dC_A}{dt} &= \frac{F}{V} (C_A - C_i) - s_1 \exp\left(-\frac{E_1}{RT}\right)C_A^2 \\
\frac{dC_B}{dt} &= -\frac{F}{V} C_B + s_1 \exp\left(-\frac{E_1}{RT}\right)C_A^2 - s_2 \exp\left(-\frac{E_2}{RT}\right)C_B \\
\frac{dT}{dt} &= \frac{F}{V} (T_i - T) + \frac{(-\Delta H_1)}{\rho c_p} s_1 \exp\left(-\frac{E_1}{RT}\right)C_A^2 + \frac{(-\Delta H_2)}{\rho c_p} s_2 \exp\left(-\frac{E_2}{RT}\right)C_B + \frac{US}{\rho c_p V} (T_j - T) \\
\frac{dT_j}{dt} &= \frac{F_j}{V} (T_{ji} - T_j) + \frac{US}{\rho_{j} c_{pj} V_j} (T_j - T) \\
y_1 &= C_B \\
y_2 &= T
\end{align*}
\]

The reactor parameter values are given in Table 2.5. It is desired to maintain \( C_b \) and \( T \) at set-point values by manipulating \( F \) and \( F_j \) within the operating ranges, \( 10 \leq F \leq 150 \ l/h \) and \( 10 \leq F_j \leq 150 \ l/h \). In this process, only the state variable, \( C_A \), is not measured.

![Figure 2.7](image-url)  
Figure 2.7  Schematic of the multivariable non-isothermal chemical reactor.
Table 2.5 Process parameters of the non-isothermal reactor with cooling jacket.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>80.95</td>
<td>$l\ hr^{-1}$</td>
</tr>
<tr>
<td>$F_j$</td>
<td>100</td>
<td>$l\ hr^{-1}$</td>
</tr>
<tr>
<td>$C_A$</td>
<td>12</td>
<td>mol $l^{-1}$</td>
</tr>
<tr>
<td>$s_1$</td>
<td>$2.5 \times 10^{10}$</td>
<td>$lmol^{-1}hr^{-1}$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>$1.5 \times 10^{10}$</td>
<td>$hr^{-1}$</td>
</tr>
<tr>
<td>$-E_1/R$</td>
<td>8000</td>
<td>K</td>
</tr>
<tr>
<td>$-E_2/R$</td>
<td>9100</td>
<td>K</td>
</tr>
<tr>
<td>$T_i$</td>
<td>320</td>
<td>K</td>
</tr>
<tr>
<td>$T_{ji}$</td>
<td>298.15</td>
<td>K</td>
</tr>
<tr>
<td>$-\Delta H_1$</td>
<td>-20</td>
<td>$kJ\ mol^{-1}$</td>
</tr>
<tr>
<td>$-\Delta H_2$</td>
<td>-80</td>
<td>$kJ\ mol^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1</td>
<td>kg $l^{-1}$</td>
</tr>
<tr>
<td>$\rho_j$</td>
<td>1.1</td>
<td>kg $l^{-1}$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>2.25</td>
<td>$kJ\ kg^{-1}K^{-1}$</td>
</tr>
<tr>
<td>$c_{p_j}$</td>
<td>3</td>
<td>$kJ\ kg^{-1}K^{-1}$</td>
</tr>
<tr>
<td>$U$</td>
<td>3825</td>
<td>$kJ\ m^{-2}K^{-1}s^{-1}$</td>
</tr>
<tr>
<td>$S$</td>
<td>0.225</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$V$</td>
<td>5</td>
<td>$l$</td>
</tr>
<tr>
<td>$V_j$</td>
<td>5</td>
<td>$l$</td>
</tr>
</tbody>
</table>
Here, \( x = [C_A C_B T T_j]^T \), \( u = [F F_j]^T \), \( y = [C_B T]^T \) and \( Y = [T_j] \). The steady state corresponding to the set-points, \( y_{sp1} = 5.233 \) and \( y_{sp2} = 443.92 \), is \( (x_{1ss} = 0.701, \ x_{2ss} = 5.233, \ x_{3ss} = 443.92, \ x_{4ss} = 403.24) \), which is unstable (eigenvalues of the Jacobian of the process evaluated at the steady state are \( -491.65, \ -90.71, \ 86.29, \) and \( -15.36 \)). The relative orders of the state variables are: \( r_1 = 1, \ r_2 = 1, \ r_3 = 1, \) and \( r_4 = 1 \).

The zero dynamics of this process is governed by:

\[
\frac{d \zeta_1}{dt} = \frac{1}{y_{1sp}} \left[ s_1 \exp\left(-\frac{E_1}{Ry_{2sp}}\right) \zeta_1^2 - s_2 \exp\left(-\frac{E_2}{Ry_{2sp}}\right) y_{1sp} \right] (C_{A0} - \zeta_1) - s_1 \exp\left(-\frac{E_1}{Ry_{2sp}}\right) \zeta_1^2 
\]

whose Jacobian evaluated at the steady state has eigenvalues at \( 589.59 \). Thus, the desired steady state is unstable and non-minimum-phase. For this process, the controller system (2.15), with \( \theta_1 = \varepsilon_1^2, \ \theta_2 = \varepsilon_2^2, \ \theta_3 = \varepsilon_3^2, \ \theta_4 = \varepsilon_4, \ p_1 = 2, \ p_2 = 2, \ p_3 = 2 \) and \( p_4 = 1 \), is implemented, with \( 10 \leq u_1 \leq 150 \) and \( 10 \leq u_2 \leq 150 \). The following controller parameter values are used: \( \varepsilon_1 = 0.17, \ \varepsilon_2 = 0.17, \ \varepsilon_3 = 0.17, \ \varepsilon_4 = 0.17, \ L_{11} = -0.03, \ L_{12} = -0.03, \) and \( L_{21} = 0 \).

**Controller Performance**

It is desired to maintain the controlled outputs at \( y_{sp1} = 5.233 \) and \( y_{sp2} = 443.92 \), which correspond to the steady state \( (x_{1ss} = 0.701, \ x_{2ss} = 5.233, \ x_{3ss} = 443.92, \ x_{4ss} = 403.24) \).

Figures 2.8, 2.9 and 2.10 show the servo and regulatory responses of the controller. The process state variable profiles for two initial conditions, \( [x_1(0), x_2(0), x_3(0), x_4(0)] = [1.814, 9.446, 387.69, 380.67] \) and \( [0.519, 3.5, 455, 410] \), are shown in Figures 2.8
and 2.9; the controller is capable of operating the process at the unstable non-minimum-phase steady state. After the process reached the desired steady state, a step change from 12 $\text{kmol/m}^3$ to 11.8 $\text{kmol/m}^3$ in $C_A$ (unmeasured disturbance) was made at $t=8$ hr. From Figures 2.8 and 2.9, it can be seen that the controller provides offset-free responses in the presence of the unmeasured disturbance. The corresponding manipulated input profiles are shown in Figure 2.10.

### 2.5 Conclusions

A control law is presented that is applicable to general multivariable processes, whether minimum- or non-minimum-phase. The controller forces process state variables to follow their corresponding reference trajectories. The state feedback is obtained by requesting state responses closest to a set of desired linear state responses. The integral action ensures offset-free, closed-loop, output response in the presence of constant disturbances and process-model mismatch. A closed-loop, reduced-order observer is used to estimate the unmeasured state variables. Compared to a general, multivariable, model-predictive controller, this control system has less tuning parameters to achieve closed-loop, asymptotically stability. The control system does not have the limitation of the control method presented by Kanter et al. [1]. However, because of the optimization (numerical) and shortest-prediction-horizon forms of the controller system, analytical proof of asymptotic, closed-loop stability of the closed-loop system remains an open problem.
Figure 2.8  Closed-loop response of the first two state variables of the chemical reactor.
Figure 2.9  Closed-loop response of the last two state variables of the chemical reactor.
Figure 2.10  Manipulated input profiles corresponding to Figures 2.8 and 2.9.
### 2.6 Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A, B, C$</td>
<td>Chemical species</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Heat capacity of feed and product, $kJ \ kg^{-1} \ K^{-1}$</td>
</tr>
<tr>
<td>$c_{p_j}$</td>
<td>Heat capacity of jacket fluid, $kJ \ kg^{-1} \ K^{-1}$</td>
</tr>
<tr>
<td>$C_{A_i}$</td>
<td>Inlet concentration of the reactant, $kmol \ m^{-3}$</td>
</tr>
<tr>
<td>$C_A$</td>
<td>Outlet concentration of the reactant, $kmol \ m^{-3}$</td>
</tr>
<tr>
<td>$C_B$</td>
<td>Outlet concentration of $B$, $mol \ l^{-1}$</td>
</tr>
<tr>
<td>$D$</td>
<td>Differential operator, $D = \frac{d}{dt}$.</td>
</tr>
<tr>
<td>$E_a, E_1, E_2$</td>
<td>Activation energy, $kJ \ mol^{-1}$</td>
</tr>
<tr>
<td>$F$</td>
<td>Feed flow rate to reactor</td>
</tr>
<tr>
<td>$F_j$</td>
<td>Jacket coolant flow rate, $l \ h^{-1}$</td>
</tr>
<tr>
<td>$J_{ol}$</td>
<td>Open-loop Jacobian</td>
</tr>
<tr>
<td>$J_{cl}$</td>
<td>Closed-loop Jacobian</td>
</tr>
<tr>
<td>$L_1, L_2$</td>
<td>Observer gains</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of manipulated inputs and controlled outputs</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of state variables</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Relative order of state variable $x_i$</td>
</tr>
<tr>
<td>$R$</td>
<td>Universal gas constant, $kJ \ kmol^{-1} K^{-1}$</td>
</tr>
<tr>
<td>$s, s_1, s_2$</td>
<td>Pre-exponential factor, $s^{-1}, l \ mol^{-1} \ hr^{-1}, hr^{-1}$</td>
</tr>
<tr>
<td>$S$</td>
<td>Heat transfer surface area, $m^2$</td>
</tr>
<tr>
<td>$t$</td>
<td>Time, $s$</td>
</tr>
<tr>
<td>$T$</td>
<td>Reactor outlet temperature, $K$</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Reactor inlet temperature, $K$</td>
</tr>
<tr>
<td>$T_j$</td>
<td>Jacket temperature, $K$</td>
</tr>
<tr>
<td>$T_{ji}$</td>
<td>Jacket inlet temperature, $K$</td>
</tr>
<tr>
<td>$u$</td>
<td>Vector of manipulated inputs</td>
</tr>
</tbody>
</table>
$U$ Overall heat-transfer coefficient, $kJ \ m^{-2} K^{-1} s^{-1}$

$V$ Reactor volume, $m^3$

$V_j$ Jacket volume, $m^3$

$x$ Vector of state variables

$y$ Vector of controlled outputs

$y_{sp}$ Vector of set-points

$-\Delta H_1, -\Delta H_2$ Heat of reaction, $kJ \ mol^{-1}$

**Greek**

$\varepsilon_1, \ldots, \varepsilon_n$ Adjustable parameters of controller

$w$ Vector of the controlled state variables

$\gamma$ Reactor model parameter, $K \ m^3 kmol^{-1}$

$\zeta$ Vector of the state variable of zero dynamics

$\rho$ Density of mixture in reactor, $kg \ l^{-1}$

$\rho_j$ Density of jacket fluid, $kg \ l^{-1}$

**Superscripts**

$\hat{}$ Estimate

**Subscripts**

$A, B$ Chemical species

$ss$ Steady State

$sp$ Set-point
Chapter 3: Input-Output Linearization with Stability Constraint

3.1 Introduction

Output tracking has been studied very extensively. The control problem is to design a controller that forces the controlled output(s) to asymptotically track reference trajectory(ies). Since the early 1990’s, many research efforts have been made to solve the problem of output tracking of non-minimum-phase (NMP) nonlinear systems. These efforts have been mainly within the frameworks of model-predictive control and differential geometric control.

In model-predictive control (MPC), controller action is the solution to a constrained optimization problem that is solved repeatedly on-line. In MPC, global optimality may not imply closed-loop stability. To ensure closed-loop stability in MPC, it has been proposed to add Lyapunov stability constraints or penalty terms to the optimization problem. For example, terminal state constraints in [34, 36, 38], terminal state penalty in [35], terminal inequality constraint in [35], and contractive constraint in [40] have been used. In non-minimum-phase systems, output tracking through MPC requires longer prediction horizon(s) than in minimum-phase systems.

Differential geometric control is a direct synthesis method in which the controller is derived by requesting a desired closed-loop output response in the absence of input constraints. A widely used differential geometric control method is input-output linearization, which cannot be used to operate a process at a NMP steady state. Efforts to make input-output linearization applicable to processes with a NMP steady state include the use of equivalent output(s) for the controller design [82-84], coordinated control [30],
controller design by inverting the minimum-phase part [2, 25], and approximate input-output linearization [1, 85, 86]. Other methods that are applicable to nonlinear systems with a non-minimum-phase steady state include stable inversion [23, 24, 26], $H_\infty$ control [27, 33], and total linearization [48].

Another popular control framework is Lyapunov-based control, in which the central focus in the controller design is on stability through Lyapunov’s direct method. The asymptotic decay of a norm of the state variables is ensured by the use of a proper Lyapunov function in the controller design.

This chapter presents a new method that addresses a major limitation of input-output linearization. The proposed method is applicable to stable and unstable processes, whether non-minimum- or minimum-phase. A novel Lyapunov stability constraint that does not suffer from the singularity (at the desired steady state) problem of the standard Lyapunov constraints is presented. The control method is obtained by further exploiting the connections between input-output linearization and model-predictive control [46, 80, 87]. It performs optimal output tracking via input-output linearization and guarantees closed-loop asymptotic stability within an assessable domain of attraction by satisfying a hard Lyapunov stability constraint. This hard constraint requires all state variables to evolve within a shrinking state tube. Whenever output tracking alone is unable to ensure closed-loop asymptotic stability, the closed-loop system evolves while being at the hard constraint. Upon the arrival of the closed-loop system in a state-space region in which output tracking alone can ensure asymptotic stability, the hard constraint becomes inactive.
This organization of this chapter is as follows. The scope of the study and some mathematical preliminaries are given in Section 3.2. Section 3.3 presents the nonlinear control method. The application and performance of the control method are illustrated by numerical simulation of two examples in Section 3.4.

3.2 Scope and Mathematical Preliminaries

Consider the general class of multivariable processes with a mathematical model in the form:

\[
\begin{align*}
\dot{x} &= f(x,u), \\
y &= h(x) \\
x(0) &= x_0
\end{align*}
\]  

(3.1)

with the input constraints

\[
u_i \leq u_i \leq u_{hi}, \quad i = 1, \ldots, m
\]

where \(x \in X \subset \mathbb{R}^n\) is the vector of state variables, \(u \in U \subset \mathbb{R}^m\) is the vector of manipulated inputs, \(y \in \mathbb{R}^m\) is the vector of controlled outputs, and \(f(x,u)\) and \(h(x)\) are smooth functions on \(X \times U\) and \(X\), respectively. \(X\) is an open connected set that contains the nominal steady state value of \(x\). \(U\) is a closed connected set that contains the nominal steady state value of \(u\): \(U = \{u | u_i \leq u_i \leq u_{hi}, \quad i = 1, \ldots, m\}\). The relative orders (degrees) of the controlled outputs \(y_1, \ldots, y_m\) are denoted by \(R_1, \ldots, R_m\), respectively, where \(R_i\) is the smallest integer for which \(\partial \left[ d^{R_i} y_i / dt^{R_i} \right] / \partial u \neq 0\). The following notation will be used:
\[ h_i^0(x) = h_i(x) = y_i \]
\[ h_i^1(x) = \frac{dy_i}{dt} \]
\[ \vdots \]
\[ h_i^{R-1}(x) = \frac{d^{R-1}y_i}{dt^{R-1}} \]
\[ h_i^R(x,u) = \frac{d^R y_i}{dt^R} \]  \hspace{1cm} (3.2)

Assumptions:

A1. The relative orders (degrees), \( R_1 \ldots R_m \), are finite.

A2. The characteristic (decoupling) matrix of the process is nonsingular on \( X \times U \).

A3. The process is controllable and observable on \( X \times U \).

The steady state pair(s) \((x_{ss}, u_{ss})\) corresponding to a given output setpoint, \( y_{sp} \), satisfy:

\[ 0 = f(x_{ss}, u_{ss}) \]
\[ y_{sp} = h(x_{ss}) \]

These relations are used to describe the dependence of a nominal steady state, \( x_{ss}^N \), on the set point: \( x_{ss}^N = F(y_{sp}) \).

### 3.2.1 MPC Formulation of Input-Output Linearization

This section presents a very short review of the MPC formulation of input-output linearization that can be found in detail in [46, 80, 87]. This formulation will be used in the main body of this paper.

A linear response of the following form:

\[ (\varepsilon_i D + 1)^{R_i} y_i = y_{sp_i} \]
\[ \vdots \]
\[ (\varepsilon_m D + 1)^{R_m} y_m = y_{sp_m} \]  \hspace{1cm} (3.3)
where $D = d/dt$, and $\varepsilon_1, \ldots, \varepsilon_m$ are positive constants that set the speed of the closed-loop output responses, can be induced to an \textit{unconstrained} process of the form (3.1) by implementing the solution to the following unconstrained optimization problem at each time instant [46, 80, 87]:

$$\min_{u(t)} \sum_{i=1}^{m} \| \hat{y}_i(\tau) - y_{d_i}(\tau) \|^2$$

(3.4)

where $\|q(\tau)\|$ denotes the 2-norm of a scalar function $q(\tau)$, given by:

$$\|q(\tau)\| \triangleq \sqrt{\int_{\tau}^{\tau+T_b} (q(\tau))^2 d\tau}$$

$\hat{y}_i(\tau)$ is model-predicted future value of $y_i$ at time $\tau$, given by a Taylor series expansion of $y_i$ around the present time, $t$:

$$\hat{y}_i(\tau) = h_i(x(t)) + \sum_{l=1}^{R_i-1} h_i^l(x(t)) \frac{[\tau-t]^l}{l!} + h_i^{R_i}(x(t),u(t)) \frac{[\tau-t]^{R_i}}{R_i!} + \text{h.o.t.}$$

and $y_{d_i}(\tau)$ is the predicted future value of the reference trajectory of $y_i$ at time $\tau$, given by:

$$(\varepsilon_i D + 1)^{R_i} y_{d_i}(\tau) = y_{sp_i}$$

$$(\varepsilon_m D + 1)^{R_m} y_{d_m}(\tau) = y_{sp_m}$$

(3.5)

initialized at:

$$\frac{d^{l}y_{d_i}(t)}{dt^l} = h_i^l(x(t)), \quad l = 0, \cdots, R_i - 1, \quad i = 1, \cdots, m$$

Taylor series expansion of $y_{d_i}$ around the present time, $t$, yields:
After substituting for $\hat{y}_i(t)$ and $y_{d_i}$, for a very small prediction horizon $T_h$, the minimization problem of (3.4) takes the simplified form:

$$\min \sum_{i=1}^{m} \left[ y_{sp_i} - h_i(x) - \sum_{l=1}^{R_i-1} \varepsilon_i^l \left( R_i \right) h_i^l(x) - \varepsilon_i^R h_i^R(x,u) \right]^2$$

(3.6)

Equation (3.6) represents a state feedback, which is denoted by:

$$u = \bar{\Psi}(x,y_{sp})$$

If this state feedback makes the performance index in (3.4) zero at each time instant, the linear closed-loop response of (3.3) is induced; that is, the state feedback of (3.6) is input-output linearizing. Needless to mention that the closed-loop system under the input-output linearizing state feedback of (3.6) is stable when the desired steady state is minimum-phase.

### 3.2.2 Reduced-Order State Observer Design

This subsection presents a brief review of the nonlinear, reduced-order, state observer described in [81]. The state observer will be used later in this paper to reconstruct unmeasured state variables of the process under consideration. Consider a nonlinear process in the form of (3.1) with additional output measurements $Y_1, \cdots, Y_q$; that is,
\[
\frac{dx}{dt} = f(x, u) \quad x(0) = x_0 \\
\begin{align*}
y &= h(x) \\
Y &= K(x)
\end{align*}
\] 

The non-redundancy of the measured outputs ensures the existence of a locally-invertible state transformation of the form

\[
\begin{bmatrix}
\eta \\
y \\
Y
\end{bmatrix} = T(x) = \begin{bmatrix} Qx \\
h(x) \\
K(x)
\end{bmatrix}
\]

where \( \eta = \begin{bmatrix} \eta_1, \cdots, \eta_{n-m-q} \end{bmatrix}^T \), and \( Q \) is a constant \((n - m - q) \times n\) matrix which for the sake of simplicity, is chosen such that:

\begin{itemize}
  \item[c)] Each row of \( Q \) has only one nonzero term equal to one; and
  \item[d)] Locally
\end{itemize}

\[
\text{rank} \left\{ \frac{\partial}{\partial u} \begin{bmatrix} Qx \\
h(x) \\
K(x)
\end{bmatrix} \right\} = n
\]

Thus, the state transformation \( \begin{bmatrix} \eta & y & Y \end{bmatrix}^T = T(x) \) is locally invertible. The system of (3.1), in terms of the new state variables, \( \eta, \cdots, \eta_{n-m-q}, y, Y \), takes the form

\[
\begin{align*}
\dot{\eta} &= F_\eta(\eta, y, Y, u) \\
\dot{y} &= F_y(\eta, y, Y, u) \\
\dot{Y} &= F_Y(\eta, y, Y, u)
\end{align*}
\]

(3.7)

where

\[
F_\eta(\eta, y, Y, u) = Qf^T [T^{-1}(\eta, y, Y), u]
\]

\[
F_y(\eta, y, Y, u) = \frac{\partial h(x)}{\partial x} \bigg|_{x = T^{-1}(\eta, y, Y)} f^T [T^{-1}(\eta, y, Y), u]
\]
\[ F_y(\eta, y, Y, u) = \frac{\partial K(x)}{\partial x} \bigg|_{x=\Gamma^{-1}(\eta, y, Y)} f[\Gamma^{-1}(\eta, y, Y), u] \]

A closed-loop, reduced-order state observer is then designed in the form:

\[
\begin{aligned}
\dot{z} &= F_\eta(z + L_1 y + L_2 y, y, y, u) - L_1 F_y(z + L_1 y + L_2 y, y, y, u) - L_2 F_y(z + L_1 y + L_2 y, y, y, u) \\
\dot{x} &= \Gamma^{-1}(z + L_1 y + L_2 y, y, y) \\
\end{aligned}
\]

(3.8)

where the constant \([n-m-q] \times m\) and \([n-m-q] \times q\] matrices, \(L_1\) and \(L_2\), are the observer gains. The observer gains should be set such that the observer error dynamics are asymptotically stable, which requires all eigenvalues of the \([n-m-q] \times (n-m-q)\] matrix

\[
\begin{aligned}
\frac{\partial F_\eta(\eta, y, Y, u)}{\partial \eta} - L_1 \frac{\partial F_y(\eta, y, Y, u)}{\partial \eta} - L_2 \frac{\partial F_y(\eta, y, Y, u)}{\partial \eta}
\end{aligned}
\]

evaluated at the nominal steady-state pair, to be in the left half of the complex plane.

### 3.3 Nonlinear Control Method

This section presents the nonlinear control method and discusses practical issues in the implementation of the method.

#### 3.3.1 Lyapunov Stability Constraint

The idea of designing an input-output linearizing controller with a stability constraint has been inspired by contractive (stability) constraints used in model-predictive control to ensure closed-loop stability [40] and that input-output linearization is a shortest prediction horizon continuous-time model-predictive controller [46, 80, 87]. A widely used stability constraint in model-predictive control is:

\[
\| \overline{x}(k+1) \| \leq \alpha \| \overline{x}(k) \|, \quad 0 < \alpha < 1
\]

(3.9)
where \( \bar{x} = x - x^N_x \). The preceding stability constraint has the general form:

\[
V(\bar{x}(k+1)) - \alpha V(\bar{x}(k)) \leq 0
\]

where \( V(\bar{x}) \) is a positive definite function. The preceding inequality has the continuous-time form:

\[
\frac{dV(\bar{x})}{dt} + \gamma V(\bar{x}) \leq 0
\]

(3.10)

where \( \gamma = -\frac{1}{\Delta t} \ln \alpha \). The manipulated input that makes \( V(\bar{x}) \) satisfy (3.10) is the solution for \( u \) of:

\[
\frac{\partial V(\bar{x})}{\partial \bar{x}} f(x,u) + \gamma V(\bar{x}) \leq 0
\]

Since

\[
\left. \frac{\partial V(\bar{x})}{\partial \bar{x}} \right|_{\bar{x} = x^N_x} = 0
\]

as \( x \to x^N_x \), (3.10) becomes ill-conditioned/singular and thus cannot be solved for \( u \).

Several approaches/approximations have been proposed to address this singularity problem in Lyapunov control [53, 88-90].

Consider the specific Lyapunov function:

\[
V(\bar{x}) = \bar{x}^T P \bar{x}
\]

(3.11)

where \( P \) is the positive-definite symmetric matrix that satisfies the Riccati equation

\[
A^T P + PA - PB^T BP = -Q
\]

\( Q \) is a positive definite matrix,

\[
A = \left. \frac{\partial f(x,u)}{\partial x} \right|_{(x^N_x, u^N_u)} \quad B = \left. \frac{\partial f(x,u)}{\partial u} \right|_{(x^N_x, u^N_u)}
\]
If the Lyapunov function, \( V(\bar{x}) \), satisfies
\[
\beta^2 \frac{d^2 V}{dt^2} + 2\beta \frac{dV}{dt} + V \leq 0
\]
(3.12)
in closed-loop over \( X \), where \( \beta \) is a positive constant that set the rate of decay of \( V(\bar{x}) \), then the closed-loop system is asymptotically stable over \( X \). If \( V(\bar{x}) \) is required to be governed by (3.12), then
\[
\beta^2 \left[ f^T P f + \frac{\partial V}{\partial X} \left\{ \frac{\partial f}{\partial X} f + \frac{\partial f}{\partial u} du \right\} \right] + 2\beta \frac{\partial V}{\partial X} f + V \leq 0
\]
(3.13)
which is not singular at \( x = x^N_{sx} \), allowing one to solve for \( u \) at every \( x \in X \).

### 3.3.2 Nonlinear State Feedback Design

To derive a state feedback that can achieve output tracking with guaranteed closed-loop stability, we solve the following constrained optimization problem at each time instant, \( t \):
\[
\min_{u(t)} \sum_{i=1}^{m} \left\| \hat{y}_i(\tau) - y_{di}(\tau) \right\|^2
\]
subject to the process model of (3.1); that is,
\[
\begin{align*}
\dot{x} &= f(x,u), \quad x(0) = x_0 \\
y &= h(x)
\end{align*}
\]
with \( u_l \leq u \leq u_u \), \( i = 1, \ldots, m \) and the stability constraint:
\[
\beta^2 \hat{V} + 2\beta \dot{V} + V \leq 0
\]
where
\[
V = (x - x^N_{sx})^T P(x - x^N_{sx}),
\]
\[
x^N_{sx} = F(y_{sp})
\]
Assuming this optimization is feasible (has a solution $u \in U$), let us denote the solution by the state feedback:

$$u = \Psi(x, y_{sp})$$  \hspace{1cm} (3.15)

The next theorem summarizes the stability properties of the preceding state feedback.

**Theorem.** For a process in the form of (3.1), the closed-loop system under the state feedback (3.15) is asymptotically stable in the region in which the state feedback is feasible.

The tunable parameter $\beta$ is suggested to be chosen such that $\beta > \max(\varepsilon_1, \ldots, \varepsilon_m)$. This choice of $\beta$ can prevent unnecessary activation of the stability constraint when output tracking can ensure closed-loop stability.

**3.3.2.1 Implementation of Nonlinear State Feedback**

The state feedback of (3.15) is the solution of the constrained optimization problem:

$$\begin{align*}
\min_u \sum_{i=1}^{m} \left[ y_{sp_i} - h_i(x) - \sum_{l=1}^{R-1} \varepsilon_i^l \left( \frac{R_i}{l} \right) h_i^l(x) - \varepsilon_i^{R_i} h_i^{R_i}(x, u) \right]^2 \\
\text{subject to} \quad u_{h_i} \leq u_i \leq u_{h_i}, \quad i = 1, \ldots, m
\end{align*}$$  \hspace{1cm} (3.16)

subject to

$$u_{h_i} \leq u_i \leq u_{h_i}, \quad i = 1, \ldots, m$$

$$\beta^2 \dot{V} + 2\beta \dot{V} + V \leq 0$$

When the process is away from the nominal steady state $x_{ss}^N$; that is, $V(x - x_{ss}^N) \geq \sigma$, 

where \( \sigma \) is a small positive constant that is set by the controller designer, the stability constraint of (3.12) takes the form:

\[
\beta^2 \left[ f^T P f + \frac{\partial V}{\partial X} \left\{ \frac{\partial f}{\partial X} f + \frac{\partial f}{\partial u} \frac{du}{dt} \right\} \right] + 2 \beta \frac{\partial V}{\partial X} f + V \leq 0
\]

which is denoted by the compact form:

\[
S(x, x_{ss}, u, \dot{u}) \leq 0
\]

When the process is close to the nominal steady state \( x_{ss}^N \), that is, \( V(x - x_{ss}^N) < \sigma \), the stability constraint of (3.12) takes the form:

\[
\beta^2 f^T P f + V \leq 0
\]

which is denoted by:

\[
S_c(x, u) \leq 0
\]

The time derivative of the manipulated input vector, \( \dot{u} \), can be approximated by \( \dot{u}(t) = [u(t) - u(t - \Delta t)]/\Delta t \), where \( \Delta t \) is time-discretization interval. Thus, the state feedback of (3.15) is the solution of the constrained optimization problem:

\[
\min_{u(t)} \sum_{i=1}^{m} \left[ y_{sp_i} - h_i(x(t)) - \sum_{l=1}^{R-1} \varepsilon_i^l \left( \frac{R_i}{l} \right) h_i^l(x(t)) - \varepsilon_i^R h_i^R(x(t), u(t)) \right]^2
\]

subject to

\[
u_{i_l} \leq u_i \leq u_{i_h}, i = 1, \ldots, m
\]

\[
S(x(t), x_{ss}^N, u(t), [u(t) - u(t - \Delta t)]/\Delta t) \leq 0, \quad V(x(t) - x_{ss}^N) \geq \sigma
\]

\[
S_\sigma(x(t), x_{ss}^N, u(t)) \leq 0, \quad V(x(t) - x_{ss}^N) < \sigma
\]
**Remark 1:** In the case that \( u \in \mathcal{R} \), the numerical solution to the constrained optimization problem of (3.21) is obtained by using the following algorithm:

1. Solve

\[
\min_{u(t)} \sum_{i=1}^{m} \left[ y_{e_i} - h_i(x(t)) - \sum_{l=1}^{R-1} \varepsilon_i \left( R_i \right) h'_i(x(t)) - \varepsilon_i^R h'^R_i(x(t),u(t)) \right]^2
\]

subject to

\[
u_i \leq u_i(t) \leq u_i, \quad i = 1, \ldots, m
\]

2. If the \( u \) calculated in step 1 satisfies

\[
\beta^2 \left[ f^T P f + \frac{\partial V}{\partial X} \left\{ \frac{\partial f}{\partial X} f + \frac{\partial f}{\partial u} du \right\} \right] + 2\beta \frac{\partial V}{\partial X} f + V \leq 0
\]

then implement the \( u \).

3. If the \( u \) calculated in step 1 does not satisfy the inequality of (3.23), calculate \( u \) from

\[
S(x(t),x^N_{ss},u(t),[u(t) - u(t - \Delta t)]/\Delta t) \leq 0, \quad V(x(t) - x^N_{ss}) \geq \sigma
\]

\[
S^T (x(t),x^N_{ss},u(t)) \leq 0, \quad V(x(t) - x^N_{ss}) < \sigma
\]

and then implement the \( u \).

The algorithm indicates that the state feedback of (3.15) is witches hybrid of an input-output linearizing state feedback and a Lyapunov-based state feedback. Indeed, at any time instant, it is an input-output linearizing or a Lyapunov-based state feedback. It may switch from one state feedback to the other. Figure 3.1 illustrates a typical closed-loop output response.
Example: Unstable Non-Minimum-Phase Linear System

Consider the linear system:

\[
\begin{align*}
\frac{dx_1}{dt} &= x_2, \\
\frac{dx_2}{dt} &= 10x_1 + 9x_2 + u, \\
y &= 2x_1 - x_2
\end{align*}
\]

which is unstable and non-minimum-phase. The relative order of this system is \( R = 1 \).

For this system, the constraint optimization problem in (3.16) takes the form:

\[
\min_u \left[ \frac{2x_1 - x_2 - \varepsilon_1(u + 10x_1 + 7x_2) - y_{sp}}{\varepsilon_1} \right]^2
\]

subject to
Here $y_{sp}=1$ ($x_{1ss}=1$, $x_{2ss}=0$, $u_{ss}=-10$). The following controller parameter values are chosen: $\varepsilon_i=0.2$, $\beta=0.7$ and $P=\begin{bmatrix} 20 & 0 \\ 0 & 1 \end{bmatrix}$.

For the two sets of initial conditions, $[x_1(0), x_2(0)]=[0.8, 0.2]$ and $[0.2, 0.8]$, the performance of the state feedback of (3.16) is shown in Figures 3.2a and 3.2b. As can be seen, the controller successfully drives both state variables to their desired steady state values.

### 3.3.3 Reduced-Order State Observer

In general, measurements of all state variables are not available. In such cases, estimates of the unmeasured state variables can be obtained from the output measurements. Here, we use the ‘closed-loop’, reduced-order, nonlinear, state observer of (3.8) to reconstruct the unmeasured state variables. The state observer is applicable to processes operating at both stable and unstable steady states.

### 3.3.4 Integral Action

To ensure offset-free response of the closed-loop system in the presence of constant disturbances and model errors, the control system should have integral action. An estimate of disturbance-free process outputs is first calculated by using the *closed-loop* process model:

\[
\dot{w} = f(w, \Psi(w, y_{sp})) \\
\dot{\xi} = h(w) 
\]

(3.24)
Figure 3.2a  State responses under the controller system and the controller flag.
Figure 3.2b  Manipulated input response and the Lyapunov function under the controller system.
where $\xi$ is the estimate of the disturbance-free controlled output. The difference between this estimate and the measurement of the controlled outputs, $y$, is then added to the output set-point, as in internal model control, leading to:

$$
\begin{align*}
\dot{w} &= f(w, \Psi(w, v)) \\
v &= y_{sp} - y + h(w) \\
u &= \Psi(\hat{x}, v)
\end{align*}
$$

(3.25)

An interesting feature of this approach to adding integral action is that the addition of the dynamic system of (3.24) to the state feedback of (3.14) [calculation of $x_{ss,v} = v$ according to (3.25)] adds no additional conditions for closed-loop asymptotic stability. In other words, if the closed-loop system is asymptotically stable under the state feedback of (3.14) alone, it is also asymptotically stable under (3.25).

### 3.3.5 Controller System

Combining the state feedback of (3.14), the reduced-order observer of (3.8), and the dynamic sub-system of (3.24), leads to the following controller system that has integral action:

$$
\begin{align*}
\dot{z} &= F_\eta(\hat{\eta}, y, Y, u) - L_4 F_y(\hat{\eta}, y, Y, u) - L_2 F_y(\hat{\eta}, y, Y, u) \\
\dot{\hat{\eta}} &= z + L_1 y + L_2 Y \\
\hat{x} &= T^{-1}(\hat{\eta}, y, Y) \\
\dot{\hat{x}} &= f(w, \Psi(w, v)) \\
v &= F(y_{sp} - y + h(w)) \\
u &= \Psi(\hat{x}, v)
\end{align*}
$$

(3.26)

A parameterization of the controller system is shown in Figure 3.3.
Figure 3.3  Parameterization of the controller system.

Figure 3.4  Schematic of non-isothermal continuous stirred tank reactor (CSTR).
3.4 Illustrative Examples

3.4.1 Single-Input Single-Output Chemical Reactor

Consider the constant-volume, non-isothermal, continuous stirred-tank reactor shown in Figure 3.4, in which the reaction $A \rightarrow B$ takes place in the liquid phase. The reactor dynamics are represented by the model:

$$\frac{dC_A}{dt} = -Z \exp\left(-\frac{E_a}{RT}\right)C_A + (C_A - C_i) \frac{F}{V_0}$$

$$\frac{dT}{dt} = \gamma Z \exp\left(-\frac{E_a}{RT}\right)C_A + (T_i - T) \frac{F}{V_0} + q$$

where $F$ is the volumetric flow rate of the inlet and outlet streams, $V_0$ is the reactor volume, and $C_{Ai}$ is the concentration of $A$ in the inlet stream. The reactor parameter values are given in Table 3.1. This reactor has multiple steady states. It is desired to control the reactor temperature by manipulating feed flow rate, $F$. All state variables are assumed to be measured.

Here $x = [C_A \ T]^T$, $u = [F]$, and $y = [T]$. The steady state corresponding to $y_{sp} = 293.91$ ($x_{1ss} = 8.39$, $x_{2ss} = 293.91$, $u_{ss} = 0.45$) is stable (eigenvalues of the Jacobian evaluated at the steady state are $-4.5$ and $-0.5$, and to $y_{sp} = 302$ ($x_{1ss} = 6.319$, $x_{2ss} = 302$, $u_{ss} = 0.45$) is unstable (eigenvalues of the Jacobian evaluated at the steady state are $-4.5$ and $0.309$). The zero dynamics of this process are governed by:

$$\frac{d\xi}{dt} = \left\{-1 - \gamma \frac{C_A - \xi}{T_i - y_{sp}}\right\}Z \exp\left(-\frac{E_a}{Ry_{sp}}\right)\xi - \frac{C_A - \xi}{T_i - y_{sp}}q$$

whose Jacobian evaluated (a) at $\xi_{ss} = 8.39$ has an eigenvalue at $-10.91$ and (b) at $\xi_{ss} = 6.319$ has an eigenvalue at $36.27$. Thus, the steady state corresponding to the lower
Table 3.1  Parameter values of the non-isothermal reactor.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z$</td>
<td>$5.0\times 10^8$</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$E_a / R$</td>
<td>8100</td>
<td>$K$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.9</td>
<td>$m^3 K \text{ kmol}^{-1}$</td>
</tr>
<tr>
<td>$q$</td>
<td>$-2.519\times 10^{-2}$</td>
<td>$K s^{-1}$</td>
</tr>
<tr>
<td>$C_{A_i}$</td>
<td>12</td>
<td>$\text{kmol m}^{-3}$</td>
</tr>
<tr>
<td>$T_i$</td>
<td>300</td>
<td>$K$</td>
</tr>
<tr>
<td>$V_0$</td>
<td>0.1</td>
<td>$m^3$</td>
</tr>
</tbody>
</table>

The temperature set-point is stable and minimum-phase, and the one corresponding to the higher temperature is unstable and non-minimum-phase.

For this process, the state feedback of (3.14) takes the form:

$$
\min_u \left[ \gamma Z \exp \left( \frac{-E_a}{Rx_2} \right) x_1 + q + \frac{x_2 - y_{sp}}{\epsilon_1} \right] + \frac{T_i - x_2}{V_0} u^2
$$

subject to

$$
\beta^2 \left[ f^T P f + \frac{\partial V}{\partial x} \left( \frac{\partial f}{\partial x} f + \frac{\partial f}{\partial u} u \right) \right] + 2 \beta \frac{\partial V}{\partial x} f + V \leq 0
$$

The process is initially at $[x_1(0), x_2(0)] = [10, 290]$, which located in the minimum-phase region. Initially $y_{sp} = 293.91$, and it then increases to $y_{sp} = 302$. The following controller parameter values are used: $\epsilon_1 = 0.2$, $\beta = 0.4$, and $P = \begin{bmatrix} 0.0755 & 0.116 \\ 0.116 & 0.1524 \end{bmatrix}$. 
Figure 3.5a  State responses of the non-isothermal CSTR under the controller and the controller flag.
Figure 3.5b  Manipulated input response and the Lyapunov function, corresponding to Figure 3.5a
Figure 3.5a shows the closed-loop responses of the state variables under the control system of \((3.26)\); the control system successfully operates the reactor at the desired steady state, whether stable minimum- or unstable non-minimum-phase. The figure also shows the controller flag. Controller flag of one indicates that the stability constraint is active. With \(y_{sp} = 293.91\), the Lyapunov stability constraint is active until the input-output linearizing state feedback satisfies the stability constraint. When the set point changes to \(y_{sp} = 302\), the constraint remains continuously active because the output tracking cannot stabilize the closed-loop system at the non-minimum-phase steady state. The corresponding manipulated input and Lyapunov function profiles are shown in Figure 3.5b.

3.4.2 Two-Input Two-Output Chemical Reactor

Consider the same reactor of the previous example but with feed flow rate, \(F\), and the rate of heating/cooling, \(q\), as manipulated inputs to maintain the reactor temperature, \(T\), and the concentration of A in the reactor, \(C_A\), at three different set points. All state measurements are assumed to be available. Here, \(x = [C_A \ T]^T\), \(u = [F \ q]^T\), and \(y = [C_A \ T]^T\).

The stability of the steady states corresponding to the set points is as follows. The one corresponding to \([y_{sp_1} = 8.39, y_{sp_2} = 293.91]\) is stable (eigenvalues of Jacobian evaluated at the steady state \(\lambda_1(J_{ol}) = -4.5\) and \(\lambda_2(J_{ol}) = -0.5\)), the one corresponding to \([y_{sp_1} = 6.319, y_{sp_2} = 302]\) is unstable (\(\lambda_1(J_{ol}) = -4.5\) and \(\lambda_2(J_{ol}) = 0.309\)), and the one
corresponding to \( y_{sp_1} = 4.57, y_{sp_2} = 308.82 \) is stable ( \( \lambda_1(J_a) = -4.5 \), and \( \lambda_2(J_a) = -0.742 \)). All of the steady states are minimum-phase.

For this process, the constraint optimization problem in (3.14) with \( R_1 = R_2 = 1 \) takes the form:

\[
\min_u \left\{ \left[ a_1(x) + b_1(x,u) \right]^2 + \left[ a_2(x) + b_2(x,u) \right]^2 \right\}
\]

subject to

\[
\beta^2 \left[ f^T P f + \frac{\partial V}{\partial x} \left\{ \frac{\partial f}{\partial x} + \frac{\partial f}{\partial u} \right\} \right] + 2\beta \frac{\partial V}{\partial x} f + V \leq 0
\]

where

\[
a_1(x) = -Z \exp \left( \frac{-E_a}{R x_2} \right) x_1 + \frac{x_1 - y_{sp_1}}{\epsilon_1}, \quad b_1(x,u) = \frac{(C - x_1) u_1}{V_0}
\]

\[
a_2(x) = \gamma Z \exp \left( \frac{-E_a}{R x_2} \right) x_1 + \frac{x_2 - y_{sp_2}}{\epsilon_2}, \quad b_2(x,u) = \frac{(T - x_2) u_1 + u_2}{V_0}
\]

The initial conditions \( [x_1(0), x_2(0)] = [10, 290] \) are in the minimum-phase region. At first, \( y_{sp_1} = 8.39 \) and \( y_{sp_2} = 293.91 \). They are then changed to \( y_{sp_1} = 6.319 \) and \( y_{sp_2} = 302 \) and finally to \( y_{sp_1} = 4.57 \) and \( y_{sp_2} = 308.82 \). The following controller parameter values are used: \( \epsilon_1 = 0.3 \), \( \epsilon_2 = 0.3 \), \( \beta = 0.6 \), and \( P = \begin{bmatrix} 0.0069 & 0 \\ 0 & 0.0016 \end{bmatrix} \).

As Figures 3.6a and 3.6b show, the control system of (3.26) successfully operates the reactor at the three steady states. In Figure 3.6a shows that the Lyapunov constraint is inactive when the state variables are close to their steady state values. The decay of the
Figure 3.6a State responses of the non-isothermal MIMO CSTR under the control system and the controller flag.
Figure 3.6b Manipulated input responses and the Lyapunov function under the control system, corresponding to Figure 3.6b.
Lyapunov function with time confirms the asymptotic approach of the state variables to their desired steady-state values.

3.5 Conclusions

A control method that can be used to operate nonlinear processes at stable and unstable steady states, whether non-minimum- or minimum-phase, was presented. The control method has advantages of both input-output linearization and Lyapunov control. Input-output linearization performs output tracking while the Lyapunov stability constraint ensures asymptotic closed-loop stability when the tracking is incapable of ensuring the stability. The feasibility of the control system of (3.14) implies asymptotically stability of the closed-loop system, and the feasibility region depends on the choice of matrix $P$. 
3.6 Notation

\( A \) Reactant
\( B \) Product
\( C_A \) Inlet concentration of the reactant, \( kmol \ m^3 \)
\( C_A \) Outlet concentration of the reactant, \( kmol \ m^3 \)
\( D \) Differential operator, \( D = d / dt \)
\( Z \) Reaction rate constant, \( s^{-1} \)
\( m \) Number of manipulated inputs
\( n \) Number of state variables
\( R_i \) Relative order (degree) of output \( y_i \)
\( t \) Time, \( s \)
\( T \) Reactor outlet temperature, \( K \)
\( T_i \) Reactor inlet temperature, \( K \)
\( u \) Vector of manipulated inputs
\( V \) Reactor volume, \( m^3 \)
\( x \) Vector of state variables
\( y \) Vector of controlled outputs
\( y_{sp} \) Vector of set-points
\( F \) Reactor feed flow rate \( m^3 \ h^{-1} \)
\( E_a \) Activation energy of the reaction \( kJ \ kmol^{-1} \)
\( R \) Universal gas constant \( kJ \ kmol^{-1} K^{-1} \)

Greek

\( \epsilon_1, \ldots, \epsilon_n \) Adjustable controlled parameters
\( \gamma \) Reactor model parameter, \( K \ m^3 \ kmol^{-1} \)
\( \beta \) Adjustable controlled parameter
\( \sigma \) Adjustable controlled parameter
Subscripts

\( A, B \) \hspace{1cm} \text{Chemical species}
\( ss \) \hspace{1cm} \text{Steady State}
\( sp \) \hspace{1cm} \text{Set-point}
\( 0 \) \hspace{1cm} \text{Initial value}

Math Symbols

\[
\binom{a}{b} = \frac{a!}{b!(a-b)!}
\]
Chapter 4: Software for Analytical Model-Based Controller Design

4.1 Introduction

The differential-geometric control has not been widely implemented in the process industries due to the difficulties and complexities inherent in the controller design. It requires taking analytical partial derivatives and performing symbolic manipulations, which become cumbersome as the relative order and/or the level of complexity of the model increases. Furthermore, many steps are involved in the design and implementation of differential controllers including process modeling, process analysis, controller design, controller verification, and code implementation. The design tasks are very elaborate and error prone. Today, computers play a significant role in solving complex scientific and engineering problems. Development of software simplifying the design of the model-based controller is of great interest in the field of process control [65-74].

This chapter presents the development of a software package motivated by the deficiencies of the existing analytical model-based controller design software. A new software package that has a user-friendly interface and fully automates the design of differential-geometric controllers is introduced herein for general nonlinear processes. The software package generates controller equations in C, FORTRAN, and MATLAB formats and carries out closed-loop simulations. The MATHEMATICA program is selected as the symbolic computational engine, because it has built-in functions that provide adequate precision and accuracy in computation, allowing the creation of specific programming packages. In the package, the users can develop their own functions that support their individual calculating needs based on the available
functions in MATHEMATICA. Furthermore, it does not require the declaration of symbolic variables in an expression, and it supports the communication between an external program (front-end interface) and its kernel. The Visual Basic language is used for creating a stand-alone application that communicates with the MATHEMATICA kernel to perform symbolic manipulations and calculations using the MathLink program. The differential-geometric control methods presented in [91-95] are programmed as MATHEMATICA packages, to design controllers for general nonlinear processes whether stable, unstable, minimum-phase, and/or non-minimum-phase. The software automates the design of analytical model-based controllers for both continuous and batch processes, avoiding laborious analytical calculations and manipulations when the process model is complex or has high relative order(s).

The organization of this chapter is as follows. The scope of the study and some mathematical preliminaries on differential-geometric, model-based control are given in Section 4.2. Section 4.3 presents the software environment and its details. Finally, the application and implementation of the software are illustrated in Section 4.4. Concluding remarks are given in Section 4.5.

4.2 Mathematical Preliminaries

The control package introduced herein synthesizes the differential-geometric control methods presented in [91-94, 96], always including a state feedback. It may also include a state observer to reconstruct unmeasured state variables and/or another dynamic system (compensator) to add integral action. In this section, the state feedback design is reviewed briefly. For the observer design, the reader can refer to [95].
Consider the general class of multivariable processes having mathematical models in the form:

\[
\frac{dx}{dt} = f(x, d, u) \quad x(0) = x_0 \quad y = h(x)
\]

where \( x = [x_1 \cdots x_n]^T \in \mathbb{R}^n \) is the vector of state variables, \( u = [u_1 \cdots u_m]^T \in \mathbb{R}^m \) is the vector of manipulated inputs, \( d = [d_1 \cdots d_q]^T \in \mathbb{R}^q \) is the vector of measured disturbances, \( y = [y_1 \cdots y_m]^T \in \mathbb{R}^m \) is the vector of controlled outputs, and \( f(x, u) = [f_1(x, u) \cdots f_n(x, u)]^T \) and \( h(x) = [h_1(x) \cdots h_n(x)]^T \) are smooth functions.

A) **Minimum-phase processes.** For this sub-class of processes, one can implement the input-output linearization method described in [91, 96] for processes with measured disturbances. For a process in the form of (4.1), closed-loop output responses of the following form are requested:

\[
(\varepsilon_1 D + 1)^{R_1} y_1 = y_{sp_1} \\
\vdots \\
(\varepsilon_m D + 1)^{R_m} y_m = y_{sp_m}
\]

where \( D = d/dt \), \( R_1, \ldots, R_m \) are relative orders of the process outputs, \( y_1, \ldots, y_m \), and \( \varepsilon_1, \ldots, \varepsilon_m \) are positive constants that set the speed of the process outputs, \( y_1, \ldots, y_m \), respectively. If equation (4.2) is solvable for \( u \) (after substituting for the time derivatives of the outputs from the process model), a state feedback of the following form can be calculated when the process has no measured disturbances:

\[
u = \psi_p(x, y_{sp})
\]
B) Stable Non-minimum-phase processes. Kanter et al. [92] extended the method in [96] to stable, non-minimum-phase, nonlinear processes, leading to an approximate input-output linearizing control method. For the processes in the form of (4.1), with no measured disturbances, linear responses of the closed-loop process outputs are requested, having the form:

\[(\varepsilon_i D + 1)^{P_i} y_1 = y_{sp_1} \]
\[\vdots \]
\[(\varepsilon_m D + 1)^{P_m} y_m = y_{sp_m} \]  
(4.4)

where \( P_1 \geq R_1, \ldots, P_m \geq R_m \). This leads to a dynamic state feedback in the compact form:

\[\Phi_p(x, u, u^{(1)}, u^{(2)}, \ldots) = y_{sp} \]  
(4.5)

After setting all the time derivatives of \( u \) to zero, if (4.5) is solvable for \( u \), a static state feedback in the general form:

\[u = \psi_p(x, y_{sp}) \]  
(4.6)

is calculated. The tunable parameters \( \varepsilon_1, \ldots, \varepsilon_m \) are chosen such that for every \( l = 1, \ldots, m \), all of the eigenvalues of \([I + \varepsilon_l J_{ol}(x_m, u_{ss})]\) lie inside the unit circle.

C) General processes. The application of the method described in [92] is limited to stable processes. To design controllers for unstable, non-minimum-phase processes, Panjapornpon et al. [93] and Panjapornpon and Soroush [94] propose two controller design methods.

Approximate input-state linearizing control method. This approach is an extension of [92]; it involves requesting higher-order linear responses for the state variables [93]. The following constrained optimization problem is solved at each time instant:
\[
\min_u \sum_{i=1}^n w_i \left[ \frac{(\varepsilon_i D + 1)^{\beta_i} x_i - x_{ss_i}}{\varepsilon_i^{\beta_i}} \right]^2
\]

subject to:

\[u^{(l)} = 0, \quad l \geq 1\]

where \(x_{ss_i}\) is the steady-state value of state, \(x_i\), corresponding to a given output set-point, \(y_{sp}\), \(w_1, \ldots, w_n\), are adjustable, positive, scalar weights, whose values are set according to the relative importance of the state variables; the higher the value of \(w_i\), the smaller the \(x_i\) response time.

**Input-output linearizing control method with stability constraint.** This approach involves tracking linear output trajectories subject to a hard stability constraint at each time instant [94]:

\[
\min_u \sum_{i=1}^m \left[ \frac{(\varepsilon_i D + 1)^{\beta_i} y_i - y_{sp_i}}{\varepsilon_i^{\beta_i}} \right]^2
\]

subject to:

\[\beta^2 \dot{V} + 2\beta \dot{V} + V \leq 0\]

where \(V\) is a Lyapunov function given by:

\[V = (x - x_{ss})^T P(x - x_{ss})\]

\(\varepsilon_1, \ldots, \varepsilon_m\) are positive constants that set the response speed of the process outputs, \(y_1, \ldots, y_m\), \(P\) is a positive-definite matrix, and a tuning parameter, \(\beta\), is chosen such that \(\beta > \max(\varepsilon_1, \ldots, \varepsilon_m)\).
4.3 Software Environment

Given a process model, the software allows one to synthesize easily differential-geometric, model-based controllers and perform closed-loop numerical simulations. The software has three principal features: a user interface, MATHEMATICA routines, and a simulation algorithm. A flow diagram of the software, showing the interactions among these features, is shown in Figure 4.1. The software receives the process model and parameter values through the front-end window, and then sends a set of commands to the MathLink program. The latter invokes the MATHEMATICA kernel to execute the

Figure 4.1  Flow diagram of the major components of the model-based controller design software.
control packages. The kernel handles the controller design task based upon the process information and the control method chosen by the user. The designed controller equations and the tuning parameter values are used to carry out numerical simulations. The performance of the closed-loop control system is presented in the form of tables and graphs. Each feature is described next.

4.3.1 User Interface

The front-end window interacts with the user in five steps: process information acceptance, process analyses, controller equation generation, closed-loop simulation, and graph selection. The user enters the requested information in each step before proceeding to the next step. A flow diagram linking the steps and the tasks involved is shown in Figure 4.2. In addition, the front-end window has an input command step that shows all of the commands sent to the MATHEMATICA kernel. The user enters the process model equations and identifies the controlled outputs, manipulated inputs, process parameters, and available measurements in the process information step shown in Figure 4.3. The process model is entered as a set of ordinary differential and algebraic equations in the MATHEMATICA format. The developed software has been tested successfully with the process models that contain state variables less than or equal to 10. It also aids the user to design both continuous and batch processes controllers. However, the controller design for the batch processes is limited to stable minimum-phase processes and cannot perform a closed-loop simulation. The following steps after the process information acceptance are only valid for the controller design of the continuous processes. In the step of process analyses, the software calculates all feasible steady-state pairs and allows the user to select a desired pair to perform local stability analysis, as illustrated in Figure 4.4.
Figure 4.2  Flow diagram of the tasks in the model-based controller design software (numbers correspond to the steps in the user interface).
Figure 4.3 The system information window (step 1) of model-based controller design software.

Depending on the analysis results (stability and minimum-phase-ness), the software allows the user to select an appropriate controller method. According to the logic shown in Figure 4.5, the software directs the user to the applicable control methods. Figure 4.6 shows the code viewer window in the controller equations step, which presents the model-based controller equations either in C, FORTRAN, or MATLAB formats. Subsequently, a file containing the code for the control (closed-loop) system is loaded in
the simulation setup step. The user enters initial values for the state variables, controller-tuning parameter values, and the simulation time, as shown in Figure 4.7. The simulation method in the dynamic library link (DLL) is selected according to the type of the control method. The dynamic library link is a library of executable functions that can be used by a Windows application. By creating a simulation function as DLLs, the new software package can be installed on any Windows-based computer without requiring the installation of MATLAB. The closed-loop simulation results are presented in the form of tables and graphs, as depicted in Figure 4.8. The user selects the plot variables in the plot selection step.

Figure 4.4 The process model window (step 2) of model-based controller design software.
Figure 4.5  Diagram showing how the software directs the user to the controller method that are applicable to the continuous process under consideration.
Figure 4.6  The code viewer window showing controller equations in the FORTRAN language format.
Figure 4.7  The simulation setup (step 4) and the plot selection (step 5) windows of the software.
Figure 4.8 A simulation plot generated by plot selection step of the software.

4.3.2 MATHEMATICA Packages

Two types of MATHEMATICA packages, that are controller design and process analysis packages, were developed. The differential-geometric control methods in [91-94, 96] are programmed into the controller design packages for application to stable and unstable processes, whether non-minimum- or minimum-phase. Each controller package designs
the state feedback, compensator, state observer, and generates controller code in a programming language, such as C, FORTRAN, or MATLAB. The input expressions for the five controller design methods are shown in Table 4.1.

The process analysis packages include two main support functions; the stability analysis function and the state equilibrium function. The stability analysis function determines the stability of the process, the zero dynamics, and the zero dynamics stability. The state equilibrium function computes all feasible steady-state pairs within given operating ranges of the state variables. The input expressions for both functions are described in Table 4.2.

Table 4.1  Controller design packages.

<table>
<thead>
<tr>
<th>Package Name</th>
<th>Input Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>StableNonMinimum.m [92]</td>
<td>StableNonminimum[InputEq, OutputEq, StateEq, UEq, OrderP, ProcPar, SSsp, Measured]</td>
</tr>
<tr>
<td>Minimum.m [96]</td>
<td>Minimum[InputEq, OutputEq, StateEq, UEq, ProcPar, SSsp, Measured]</td>
</tr>
<tr>
<td>MinimumDm.m [91]</td>
<td>MinimumDm[InputEq, OutputEq, StateEq, UEq, DmEq, ProcPar, SSsp, Measured]</td>
</tr>
<tr>
<td>UnstableNonminimum.m [93]</td>
<td>MIMOunstableNonmin[InputEq, OutputEq, StateEq, UEq, OrderP, ProcPar, SSsp, Measured]</td>
</tr>
<tr>
<td>MIMOLyapunov.m [94]</td>
<td>MIMOLyapunov[InputEq, OutputEq, StateEq, UEq, ProcPar, SSsp, Measured]</td>
</tr>
<tr>
<td>BatchMinimum.m [96]</td>
<td>BatchMinimum[InputEq, OutputEq, StateEq, UEq, ProcPar, Measured]</td>
</tr>
</tbody>
</table>
Table 4.2  Design analysis packages.

<table>
<thead>
<tr>
<th>Package Name</th>
<th>Input expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>analyse.m</td>
<td>analyse[InputEq, OutputEq, UEq, dmEq, StateEq, SSsp, ProcPar]</td>
</tr>
<tr>
<td></td>
<td>StateEquilibrium[InputEq,StateEq, OutputEq, OutputSp, ProcPar, IpRange, IpSpan ]</td>
</tr>
</tbody>
</table>

4.3.3 Simulation Algorithm

While MATHEMATICA has symbolic manipulation capabilities, its numerical algorithms are weak. For this purpose, the simulation algorithms of the model-based controller design software are created as Dynamic Library Link (DLL) functions using the COM Builder of MATLAB. The DLL functions provide a flexible environment for a stand-alone application. The controller equations in the MATLAB format, with given process parameters, are integrated by appropriate functions that are developed specifically for the control methods in [91-94, 96]. These simulation functions are OdeAna, OdeFminunc and OdeFmincon. The user adjusts the tuning parameters and simulation time to evaluate the controller performance.

4.4 Examples

Next, for several chemical and biochemical reactors, the software is applied to design differential-geometric, model-based controllers.
4.4.1 Continuous Bioreactor

Consider a constant-volume, continuous bioreactor described by:

\[
\dot{x}_1 = -x_1 u + x_1 (1 - x_2) \exp\frac{x_2}{\gamma_1} \\
\dot{x}_2 = -x_2 u + x_1 (1 - x_2) \exp\frac{x_2}{\gamma_1} \frac{1 + \alpha_1}{1 + \alpha_1 - x_2} \\
y = x_1
\]

where \( x_1 \) is the dimensionless outlet cell mass concentration, \( x_2 \) is the dimensionless outlet substrate concentration, and \( u \) is the dimensionless substrate feed rate. The parameters \( \alpha_1 = 0.02 \) and \( \gamma_1 = 0.48 \). It is desired to maintain the cell mass concentration, \( x_1 \), at 0.1448, by manipulating \( u \). It is assumed that only the cell mass concentration is measured.

With \( x = [x_1 \quad x_2]^T \), \( u = [u] \), and \( y = [x_1] \), the inputs to the software in step 1 are as follows:

**Step 1**

**Input 1:**

\[ x1'[t] = -x1[t] * u[t] + x1[t] *(1-x2[t]) * \text{Exp}[x2[t]/gamma1], \]
\[ x2'[t] = -x2[t] * u[t] + x1[t] *(1-x2[t]) * \text{Exp}[x2[t]/gamma1] *(1+alpha1)/(1+alpha1-x2[t]), \]
\[ y1 = x1[t] \]

**Input 2:** alpha1=0.02, gamma1=0.48

**Input 3:** x1[t], x2[t]  \hspace{1cm} **Input 4:** u[t]

**Input 5:**

**Input 6:** y1  \hspace{1cm} **Input 7:** 0.1448

**Input 8:** \{x1[t],0,1\}, \{x2[t],0,1\}, \{u[t], 0.2, 1.5\}

**Input 9:** 5

**Input 10:** Observer design  \hspace{1cm} **Input 11:** Continuous process
The software calculates all feasible steady state pairs within given operating ranges. The user chooses the pair \((x_{ss} = 0.1448, \ x_{2ss} = 0.8455, \ u_{ss} = 0.9)\) to perform stability analysis. At this steady state pair, the process is unstable and non-minimum-phase; the eigenvalues of the Jacobians of the process and its zero dynamics are in the right-half-plane (RHP) \((\lambda_1(J_{zd}) = 0.061 + 1.731i, \ \lambda_2(J_{zd}) = 0.061 - 1.731i \text{ and } \lambda_3(J_{zd}) = 1.374)\). When the approximate input-state linearizing control method [93] is selected as the controller design method and the orders of the requested state responses are \(p_1 = 1\) and \(p_2 = 1\), the inputs for step 2 are given below. The controller equations are generated in Step 3.

**Step 2**

*Input 1*: 0.1448, 0.8455, 0.9  \hspace{1cm}  *Input 2*: 1, 1  

*Input 3*: Approximate Input-State Linearization

Initial conditions are \([x_1(0), x_2(0)] = [0.1448, 0.8455]\), and it is desired to maintain the outputs at \(y_{sp} = 0.1448\), which is in the non-minimum-phase region. The following tuning parameter values are selected: \(\varepsilon_1 = 0.1, \ \varepsilon_2 = 0.35, \text{ and } L_2 = 0.7\), and the simulation time is 4 hours. Details of inputs in Step 4 are shown below, with the closed-loop responses in Figure 4.9.

**Step 4**

*Input 1*: [0, 4]  

*Input 2*: \{Epsilon1, Epsilon2, L11\}  \hspace{1cm}  *Input 3*: 0.1, 0.35, 0.7  

*Input 4*: x1, x2, Zeta1, Xi11, Xi21  \hspace{1cm}  *Input 5*: 0.1, 0.75, 0.05, 0.1, 0.75  

*Input 6*: u  \hspace{1cm}  *Input 7*: 0.45
4.4.2 Single-Input Single-Output Chemical Reactor

Consider a constant-volume, non-isothermal, continuous stirred-tank reactor, in which the exothermic reaction, $A \rightarrow B$, takes place in the liquid phase. The reactor dynamics are represented by the model:
where \( F_0 \) is the inlet volumetric flow rate of pure \( A \), \( V \) is the reactor volume, and \( C_{A0} \) is the concentration of \( A \) in the feed stream. The process parameter values are given in Table 4.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_0 )</td>
<td>( 5.0 \times 10^4 )</td>
<td>( s^{-1} )</td>
</tr>
<tr>
<td>( E_{a}/R )</td>
<td>8100</td>
<td>( K )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>3.9</td>
<td>( m^3 K \text{ kmol}^{-1} )</td>
</tr>
<tr>
<td>( q )</td>
<td>( -2.519 \times 10^{-2} )</td>
<td>( K s^{-1} )</td>
</tr>
<tr>
<td>( C_{A0} )</td>
<td>12</td>
<td>( \text{kmol m}^{-3} )</td>
</tr>
<tr>
<td>( T_i )</td>
<td>300</td>
<td>( K )</td>
</tr>
<tr>
<td>( V )</td>
<td>0.1</td>
<td>( m^3 )</td>
</tr>
</tbody>
</table>

The reactor temperature, \( T \), is measured and it is desired to maintain it at \( y_{sp} = 302 \) by manipulating the feed rate, \( F \). Here, \( x = [C_A \ T]^T \), \( u = [F] \), and \( y = [T] \). The inputs to the software in step 1 are given below.
Step 1

**Input 1:**
\[
x_1'[t] = u_1[t](C_0 - x_1[t])/V - k_1x_1[t]E^{(-E_1R/x_2[t])},
\]
\[
x_2'[t] = u_1[t](T_1 - x_2[t])/V + \gamma k_1x_1[t]E^{(-E_1R/x_2[t])} + Q,
\]
\[
y_1 = x_2[t]
\]

**Input 2:** \(C_0 = 12, \ T_1 = 300, \ k_1 = 1.8 \times 10^{12}, \ E_1R = 8100, \ \gamma = 3.9, \ Q = -90.7, \ V = 0.1\)

**Input 3:** \(x_1[t], x_2[t]\)  
**Input 4:** \(u_1[t]\)

**Input 5:**

**Input 6:** \(y_1\)  
**Input 7:** 302

**Input 8:** \(\{x_1[t], 0, 12\}, \{x_2[t], 290, 325\}, \{u_1[t], 0.1, 3.5\}\)

**Input 9:** 5

**Input 10:** Observer design  
**Input 11:** Continuous process

The steady state pair \((x_{1ss} = 6.319, \ x_{2ss} = 302, \text{ and } u_{ss} = 0.45)\) is selected to perform stability analysis. Eigenvalues of the Jacobians of the process and its zero dynamics are in the RHP \((\lambda_x(J_{zd}) = -0.45, \ \lambda_z(J_{zd}) = 0.309, \ \lambda_j(J_{zd}) = 36.271)\). Thus, the desired steady state is unstable and non-minimum-phase. The method of input-output linearizing control with the stability constraint [94] is selected as the controller design method. The software inputs in step 2 are given below.

Step 2

**Input 1:** 6.319, 302, 0.45  
**Input 2:** 2, 2

**Input 3:** Input-Output Linearization with Stability Constraint

The desired set point is \(y_{sp} = 302\), and the process is initially at \([x_1(0), x_2(0)] = [9.394, 293.906]\), which is in the non-minimum-phase region. The tuning parameters are \(\varepsilon_1 = 0.1, \ \beta_1 = 0.25, \ a_{11} = 0.0755, \ a_{12} = 0.105, \ a_{21} = 0.105, \text{ and } a_{22} = 0.1524\), and the
simulation time is two hours. The inputs in step 4 are given below. Finally, the performance of the controller is shown in Figure 4.10.

**Step 4**

*Input 1:* $[[0,2]]$

*Input 2:* $\{\text{Epsilon1, Beta1, a11, a12, a21, a22}\}$

*Input 3:* 0.1, 0.25, 0.0755, 0.105, 0.105, 0.1524

*Input 4:* $x_1, x_2$ \hspace{5cm} *Input 5:* 8.394, 293.906

*Input 6:* $u_1$ \hspace{5cm} *Input 7:* 0.45

![Figure 4.10](image)  
**Figure 4.10** Simulated closed-loop response of the controlled output and manipulated input of the non-isothermal CSTR.
4.4.3 Multi-Input Multi-Output Jacketed Chemical Reactor

Consider a non-isothermal, continuous chemical reactor in which the reactions $A \rightarrow B \rightarrow C$ take place in the liquid phase. The process dynamics are represented by the following model:

$$\frac{d C_A}{d t} = \frac{F}{V} (C_{A0} - C_A) - k_1 \exp\left(-\frac{E_1}{RT}\right) C_A^2$$

$$\frac{d C_B}{d t} = -\frac{F}{V} C_B + k_1 \exp\left(-\frac{E_1}{RT}\right) C_A^2 - k_2 \exp\left(-\frac{E_2}{RT}\right) C_B$$

$$\frac{d T}{d t} = \frac{F}{V} (T_j - T) + \frac{(-\Delta H_1)}{\rho c_p} k_1 \exp\left(-\frac{E_1}{RT}\right) C_A^2 + \frac{(-\Delta H_2)}{\rho c_p} k_2 \exp\left(-\frac{E_2}{RT}\right) C_B + \frac{US}{\rho c_p V} (T_j - T)$$

$$\frac{dT_j}{dt} = \frac{F}{V} (T_{j0} - T_j) - \frac{US}{\rho c_p V_j} (T_j - T_j)$$

$$y_1 = C_B$$

$$y_2 = T$$

It is desired to maintain $C_B$ at 5.233 mol/l and $T$ at 443.92 K by manipulating $F$ and $F_j$. Only measurements of $C_B$ and $T$ are available. The process parameter values are given in Table 4.4. Let $x = [C_A \ C_B \ T \ T_j]^T$, $u = [F \ F_j]^T$ and $y = [C_B \ T]^T$. The process dynamics can be expressed as:

$$\dot{x}_1 = \frac{u_1}{V} (C_{A0} - x_1) - k_1 \exp\left(-\frac{E_1}{R x_3}\right) x_1^2$$

$$\dot{x}_2 = -\frac{u_1}{V} x_2 + k_1 \exp\left(-\frac{E_1}{R x_3}\right) x_1^2 - k_2 \exp\left(-\frac{E_2}{R x_3}\right) x_2$$

$$\dot{x}_3 = \frac{u_1}{V} (T_i - x_3) + \frac{(-\Delta H_1)}{\rho c_p} k_1 \exp\left(-\frac{E_1}{R x_3}\right) x_1^2 + \frac{(-\Delta H_2)}{\rho c_p} k_2 \exp\left(-\frac{E_2}{R x_3}\right) x_2 + \frac{US}{\rho c_p V} (T_j - x_3)$$

$$\dot{x}_4 = \frac{u_2}{V} (T_{j0} - x_4) - \frac{US}{\rho c_p V_j} (x_4 - x_3)$$

$$y_1 = x_2$$

$$y_2 = x_3$$
Table 4.4  Process parameters of the non-isothermal jacketed reactor.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{A0}$</td>
<td>12</td>
<td>$mol \ l^{-1}$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>$2.5 \times 10^{10}$</td>
<td>$lmol^{-1} hr^{-1}$</td>
</tr>
<tr>
<td>$k_2$</td>
<td>$1.5 \times 10^{10}$</td>
<td>$hr^{-1}$</td>
</tr>
<tr>
<td>$E_1 / R$</td>
<td>8000</td>
<td>$K$</td>
</tr>
<tr>
<td>$E_2 / R$</td>
<td>9100</td>
<td>$K$</td>
</tr>
<tr>
<td>$T_i$</td>
<td>320</td>
<td>$K$</td>
</tr>
<tr>
<td>$T_{ji}$</td>
<td>298.15</td>
<td>$K$</td>
</tr>
<tr>
<td>$\Delta H_1$</td>
<td>$-20$</td>
<td>$kJ \ mol^{-1}$</td>
</tr>
<tr>
<td>$\Delta H_2$</td>
<td>$-80$</td>
<td>$kJ \ mol^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1</td>
<td>$kg \ l^{-1}$</td>
</tr>
<tr>
<td>$\rho_j$</td>
<td>1.1</td>
<td>$kg \ l^{-1}$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>2.25</td>
<td>$kJ \ kg^{-1} K^{-1}$</td>
</tr>
<tr>
<td>$c_{p_j}$</td>
<td>3</td>
<td>$kJ \ kg^{-1} K^{-1}$</td>
</tr>
<tr>
<td>$U$</td>
<td>3825</td>
<td>$kJ \ m^{-2} K^{-1} s^{-1}$</td>
</tr>
<tr>
<td>$S$</td>
<td>0.225</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$V$</td>
<td>5</td>
<td>$l$</td>
</tr>
<tr>
<td>$V_j$</td>
<td>5</td>
<td>$l$</td>
</tr>
</tbody>
</table>
The inputs to the software in step 1 are given below.

**Step 1**

**Input 1:**

\[
x_{1}[t] = -((k_{1} x_{1}[t]^{2})/E^{(E1R/x_{3}[t]))} + ((Ca0 - x_{1}[t])u_{1}[t])/V,
\]

\[
x_{2}[t] = (k_{1} x_{1}[t]^{2})/E^{(E1R/x_{3}[t])} - (k_{2} x_{2}[t])/E^{(E2R/x_{3}[t])} - (x_{2}[t]u_{1}[t])/V,
\]

\[
x_{3}[t] = -((k_{1} H1 x_{1}[t]^{2})/(CpE^{(E1R/x_{3}[t])}*Rho)) -
\]

\[
(k_{2} H2 x_{2}[t])/(CpE^{(E2R/x_{3}[t])}*Rho) + (S*U*(-x_{3}[t] + x_{4}[t]))/(CpRho*V) + ((Ti - x_{3}[t])u_{1}[t])/V,
\]

\[
x_{4}[t] = -((S*U*(-x_{3}[t] + x_{4}[t]))/(CpjRhoj*Vj)) + ((TJi - x_{4}[t])u_{2}[t])/Vj,
\]

\[
y_{1} = x_{2}[t],
\]

\[
y_{2} = x_{3}[t]
\]

**Input 2:**

\[
k_{1} = 2.5*10^{10}, k_{2} = 1.5*10^{10}, H1 = -20, H2 = -80, Ca0 = 12, V = 5, Vj = 5,
\]

\[
E1R = 8000, E2R = 9100, Rho = 1, Rhoj = 1.1, Cp = 2.25, Cpj = 3, Ti = 320, TJi = 298.15,
\]

\[
U = 3825, S = 0.225
\]

**Input 3:**

\[
x_{1}[t], x_{2}[t], x_{3}[t], x_{4}[t]
\]

**Input 4:**

\[
u_{1}[t], u_{2}[t]
\]

**Input 5:**

**Input 6:**

\[
y_{1}, y_{2}
\]

**Input 7:**

\[
5.233, 443.92
\]

**Input 8:**

\[
\{x_{1}[t], 0, 1.5\}, \{x_{2}[t], 0, 10\}, \{x_{3}[t], 400, 450\}, \{x_{4}[t], 400, 440\}, \{u_{1}[t], 50, 120\},
\]

\[
\{u_{2}[t], 50, 120\}
\]

**Input 9:**

\[
5
\]

**Input 10:**

Observer design

**Input 11:**

Continuous process

The steady-state pair corresponding to the desired set-point, \( y_{sp} = 5.233 \), and \( y_{sp2} = 443.92 \) (\( x_{1ss} = 0.7, x_{2ss} = 5.233, x_{3ss} = 443.92, x_{4ss} = 403.24 \), \( u_{1ss} = 80.95 \), and \( u_{2ss} = 100.94 \)), is selected. The process is unstable \( \lambda_{1}(J_{ol}) = -491.38, \lambda_{2}(J_{ol}) = -90.71 \), \( \lambda_{3}(J_{ol}) = 86.28, \lambda_{4}(J_{ol}) = -15.36 \) and non-minimum-phase \( \lambda_{5}(J_{zd}) = 589.59 \). The approximate input-state linearizing control method [93] is selected as the controller.
design method. The orders of the linear responses of state variables, \( x_1, x_2, x_3, \) and \( x_4, \) are requested to be \( p_1 = 2, p_2 = 2, p_3 = 2, \) and \( p_4 = 1, \) respectively. The inputs to the software in step 2 are given below.

**Step 2**

*Input 1:* 0.700, 5.233, 443.92, 403.24, 80.95, 100.94  \hspace{1cm} *Input 2:* 2, 2, 2, 1

*Input 3:* Approximate Input-State Linearization

The process is initially at \([x_1(0), x_2(0), x_3(0), x_4(0)] = [0.7, 5.233, 443.92, 403.24], \) \( y_{sp_1} = 5.233, \) and \( y_{sp_2} = 443.92. \) The set-point is in the non-minimum-phase region. The following values are selected: \( \epsilon_1 = 0.09, \) \( \epsilon_2 = 0.09, \) \( \epsilon_3 = 0.08, \) \( \epsilon_4 = 0.08, \) \( L_1 = 0.5, \) \( L_2 = 0.5, \) with a simulation time of three hours. The inputs to step 4 are given below. The simulation results showing the controller performance are shown in Figure 4.11.

**Step 4**

*Input 1:* [0,3]

*Input 2:* Epsilon1, Epsilon2, Epsilon3, Epsilon4, L11, L22

*Input 3:* 0.09, 0.09, 0.08, 0.08, 0.5, 0.5

*Input 4:* x1, x2, x3, x4, Zeta1, Zeta2, Xi11, Xi12, Xi21, Xi22, Xi31, Xi32, Xi41


*Input 6:* u1, u2

*Input 7:* 40, 10
Figure 4.11 Simulated closed-loop response of the controlled outputs and manipulated input of the MIMO chemical reactor.
### 4.4.4 Continuous Chemical Reactors in Series

Consider two isothermal, CSTRs in series, in which the reactions \( A \rightarrow B \rightarrow C \) take place in the liquid phase. The process model is:

\[
\begin{align*}
\frac{dC_{A1}}{dt} &= \frac{F_1}{V_1} (C_{A0} - C_{A1}) - k_A C_{A1}^2 \\
\frac{dC_{B1}}{dt} &= -\frac{F_1}{V_1} C_{B1} + k_A C_{A1}^2 - k_B C_{B1} \\
\frac{dC_{A2}}{dt} &= \frac{F_1}{V_2} (C_{A1} - C_{A2}) + \frac{F_2}{V_2} (C_{A0} - C_{A2}) - k_A C_{A2}^2 \\
\frac{dC_{B2}}{dt} &= \frac{F_1}{V_2} (C_{B1} - C_{B2}) - \frac{F_2}{V_2} C_{B2} + k_A C_{A2}^2 - k_B C_{B2}
\end{align*}
\]

where \( F_1 \) and \( F_2 \) are the inlet volumetric flow rates of pure \( A \), \( V_1=0.01 \), \( V_2=0.01 \), \( C_{A0}=7 \), \( k_A=6 \), and \( k_B=1 \). The concentrations, \( C_{B1} \) and \( C_{B2} \), are measured and it is desired to maintain them at \( y_{sp1}=2 \), and \( y_{sp2}=3 \) by manipulating the feed rates, \( F_1 \) and \( F_2 \).

With \( x=[C_{A1} \ C_{B1} \ C_{A2} \ C_{B2}]^T \), \( u=[F_1 \ F_2]^T \), and \( y=[C_{B1} \ C_{B2}]^T \), the inputs to the software in step 1 are given below.

**Step 1**

**Input 1:**

\[
\begin{align*}
x1'[t] &= -(kA*x1[t]^2) + ((Ca0 - x1[t])*u1[t])/V1, \\
x2'[t] &= kA*x1[t]^2 - kb*x2[t] - (x2[t]*u1[t])/V1, \\
x3'[t] &= -(kA*x3[t]^2) + ((x1[t] - x3[t])*u1[t])/V2 + ((Ca0 - x3[t])*u2[t])/V2, \\
x4'[t] &= kA*x3[t]^2 - kb*x4[t] + ((x2[t] - x4[t])*u1[t])/V2 - (x4[t]*u2[t])/V2, \\
y1 &= x2[t],
\end{align*}
\]
The steady state pair \((x_{1ss} = 0.699, x_{2ss} = 2, x_{3ss} = 1.101, x_{4ss} = 3, u_{1ss} = 0.005, u_{2ss} = 0.0013)\) that corresponds \(y_{sp1} = 2\) and \(y_{sp2} = 3\) is selected. The pair is stable \([\hat{\lambda}_1(J_{ol}) = -14.90, \hat{\lambda}_2(J_{ol}) = -8.85, \hat{\lambda}_3(J_{ol}) = -2.72, \hat{\lambda}_4(J_{ol}) = -1.46]\) and non-minimum-phase \([\hat{\lambda}_1(J_{ol}) = 17.57, \hat{\lambda}_2(J_{ol}) = 11.03]\). The approximate input-output linearizing control method [9], with \(P_1 = 2\) and \(P_2 = 2\), is used. The inputs to step 2 are detailed below.

**Step 2**

**Input 1:** 0.699, 2.0, 1.101, 3.0, 0.005, 0.013    **Input 2:** 2, 2

**Input 3:** Approximate Input-Output Linearization

With \([x_1(0), x_2(0), x_3(0), x_4(0)] = [0.75, 2.192, 0.75, 2.192], \varepsilon_1 = 0.4, \varepsilon_2 = 0.3\), and six hours of simulation time, the inputs to step 4 are given below. Figure 4.12 shows the closed-loop responses.
Step 4

**Input1**: [0, 6]

**Input2**: \{Epsilon1, Epsilon2\}  \hspace{1cm}  **Input3**: 0.4, 0.3

**Input4**: \{x1, x2, x3, x4, Eta11, Eta12, Eta21, Eta22\}

**Input5**: 0.75, 2.192, 0.75, 2.192, 0.75, 2.192, 0.75, 2.192

**Input6**: \{u1, u2\}  \hspace{1cm}  **Input7**: 0.1, 0.1

4.5 Conclusions

A user-friendly, integrated, software package for the design of nonlinear model-based controllers is presented. Given a process model in the form of ordinary differential and algebraic equations, the software derives an analytical model-based controller, generates the controller equations in C, FORTRAN, or MATLAB format, and carries out closed-loop simulations. A MathLink program provides an interface for communication with the MATHEMATICA kernel. The software is applied for the control of chemical and biochemical reactors. The software allows control engineers to design differential-geometric, model-based controllers with ease. With the software, differential-geometric, model-based controllers can be designed for processes involving complex models, as implemented in industry.
Figure 4.12  Simulated closed-loop response of the controlled outputs and manipulated input of the isothermal chemical reactors in series.
4.6 Notation

$A, B, C$ 
Chemical species

c\text{p} \quad \text{Heat capacity of feed and product, } kJ \text{ kg}^{-1} \text{ K}^{-1}$

c\text{p} \_i \quad \text{Heat capacity of jacket fluid, } kJ \text{ kg}^{-1} \text{ K}^{-1}$

C\text{A0} \quad \text{Inlet concentration of the reactant } A, \text{ mol l}^{-1}$

C\text{A}, C\text{A1}, C\text{A2} \quad \text{Outlet concentration of the reactant } A, \text{ mol l}^{-1}$

C\text{B}, C\text{B1}, C\text{B2} \quad \text{Concentration of } B, \text{ mol l}^{-1}$

D \quad \text{Differential operator, } D = d / dt$

E\text{a}, E\text{1}, E\text{2} \quad \text{Activation Energy, } kJ \text{ mol}^{-1}$

F \quad \text{Reactor flow rate } m^3 \text{ h}^{-1} \text{ or } l \text{ h}^{-1}$

F\text{j} \quad \text{Jacket coolant flow rate, } l \text{ h}^{-1}$

J\text{ol} \quad \text{Open loop Jacobian}$

J\text{cl} \quad \text{Closed-loop Jacobian}$

J\text{zd} \quad \text{Jacobian of the zero dynamics}$

L \quad \text{Matrix of observer gain}$

m \quad \text{Number of manipulated inputs}$

n \quad \text{Number of controlled outputs}$

p \quad \text{Vector of the orders of the requested output responses}$

\text{P} \quad \text{Vector of the orders of the requested state responses}$

q \quad \text{Cooling/heating rate, } kJ \text{ s}^{-1}$

r\text{i} \quad \text{Relative order of state variable } x\text{i}$

R\text{i} \quad \text{Relative order of output variable } y\text{j}$

R \quad \text{Universal gas constant, } kJ \text{ kmol}^{-1} \text{ K}^{-1}$

S \quad \text{Heat transfer surface area, } m^2$

t \quad \text{Time, } s$
\( T \)  
Reactor outlet temperature, \( K \)

\( T_i \)  
Reactor inlet temperature, \( K \)

\( T_j \)  
Jacket temperature, \( K \)

\( T_{ji} \)  
Jacket inlet temperature, \( K \)

\( u \)  
Vector of manipulated inputs

\( U \)  
Overall heat-transfer coefficient, \( kJ m^{-2}K^{-1}s^{-1} \)

\( V, V_1, V_2 \)  
Reactor volume, \( m^3 \)

\( V_j \)  
Jacket Volume, \( l \)

\( x \)  
Vector of state variables

\( y \)  
Vector of controlled outputs

\( y_{sp} \)  
Vector of set-points

\( k_0, k_1, k_2 \)  
Reaction rate constant, \( h^{-1}, l\ mol^{-1}h^{-1}, h^{-1} \)

\( -\Delta H_1, -\Delta H_2 \)  
Heat of reaction, \( kJ mol^{-1} \)

InputEq  
Process model equation

OutputEq  
Output variables

StateEq  
State variables

UEq  
Output variables

DmEq  
Measured disturbances

OrderP  
Request output order

SSsp  
Select steady state pair

ProcPar  
Process parameters

Measured  
Complete or incomplete state measurements

OutputSp  
Output set point

IpRange  
Input ranges of state variables

IpSpan  
Input span ranges of state variables
Greek

$\varepsilon_1, \ldots, \varepsilon_n$  Adjustable parameters of controller

$\beta$  Adjustable parameter of stability constraint

$\gamma$  Reactor model parameter,

$\eta$  State for integral action

$\lambda_1, \lambda_2, \ldots$  Eigenvalues of Jacobian

$\rho$  Density of mixture in reactor,

$\rho_j$  Density of jacket fluid,

Subscripts

$A, B$  Chemical species

$ss$  Steady State

$sp$  Set-point

$0$  Initial value, process
Chapter 5: Conclusions and Future Research Directions

5.1 Conclusions

This research project was motivated by the inadequacy of the existing controller design methods for general nonlinear processes and the difficulty of designing and implementing differential-geometric model-based controller design. The specific contributions of this research are as follows:

- Two new control methods that are applicable to input-constrained general multivariable processes, whether minimum- or non-minimum-phase, were developed. Both control methods address limitations of I-O linearization.

- The nonlinear state feedback of the first control method was obtained by requesting state response as close as possible to a set of desired linear state responses. The integral action ensures the offset-free of closed-loop response in the presence of process-model mismatch. A closed-loop, reduced order observer was used to estimate the unmeasured state variables. Compared to a general, multivariable model-predictive controller, this control system has less tuning parameters to achieve closed-loop, asymptotically stability. The advantages of this control system over a general multivariable model-predictive controller will be explained further at the end of this subsection.

- The control system of the first control method does not have the limitations of the control method presented by Kanter et al. [1]. However, because of the optimization form (numerical nature) and shortest prediction horizon form of the
control system, an analytical proof of the closed-loop stability for the proposed control system is not available now.

- The second control method has a hybrid control structure that is a combination of I-O linearization and Lyapunov control. The I-O linearization performs optimal tracking while the Lyapunov stability constraint ensures asymptotic closed-loop stability within an assessable domain of attraction. The stability is ensured by requiring state variables to evolve within a shrinking state-boundary. A feasible solution to optimization problem in (3.15) implies asymptotically stability. The size of the feasibility region depends on the adjustable $P$ and $\beta$.

- A new stability Lyapunov constraint was presented. It does not suffer from the singularity problem that the existing one does. In addition, the proposed method has a larger successive domain of attraction than one designed based on only Lyapunov-based control.

- A new software package for analytical model-based controller design and closed-loop simulation was developed. With its user-friendly environment, the software package designs the controller on the basis of a process model in a set of differential and algebraic equations and generates the analytical model-based controller equations in multiple formats. The model-based software integrates the simulation tools to simulate the closed-loop response and display the results. This prototype software package was developed to simplify industrial implementation and testing of the model-based controllers. The new design software for nonlinear model-based control would allow control engineers to more conveniently design nonlinear controllers. This will lead to an increase in industrial implementations.
of the nonlinear model-based controllers. The developed software has already been tested successfully on the process models with the number of state variables less than or equal to 10.

The new control methods have less number of tuning parameters in comparison with a typical MPC. For example, consider the general class of multivariable processes in the form of (2.1) under the following MPC [97]:

\[
\min_{\Delta U(k)} J = (Y_r(k + 1) - Y(k + 1))^T Q (Y_r(k + 1) - Y(k + 1)) + \Delta U(k)^T R \Delta U(k) \tag{5.1}
\]

where

\[
Y_r(k + 1) \triangleq [y_r(k + 1), y_r(k + 2), \ldots, y_r(k + P)]^T
\]

\[
Y(k + 1) \triangleq [y(k + 1), y(k + 2), \ldots, y(k + P)]^T
\]

\[
\Delta U(k) \triangleq [(u(k) - u(k - 1)), (u(k + 1) - u(k)), \ldots, (u(k + M - 1) - u(k + M))]^T
\]

\(Y_r(k + 1)\) is a vector of reference trajectory outputs over prediction horizon \(P\). \(Y(k + 1)\) is a vector of outputs over the prediction horizon \(P\). \(\Delta U(k)\) is a vector of control action changes over control horizon \(M\). \(Q\) is an \(mP \times mP\) matrix and \(R\) is an \(mM \times mM\) matrix weighting on the predicted error vectors of outputs and inputs respectively. Figure 5.1 illustrates the MPC concept and the control horizon. The control action is obtained by carrying out open-loop optimization locally. The model-predictive control is seldom applied to the unstable processes because the controller performance quickly deteriorates in the presence of disturbance or process-model mismatch. Therefore, the unstable process needs to be stabilized before implementing model-predictive control. The MPC needs a large prediction horizon to handle the non-minimum-phase processes. To achieve asymptotic stability, the MPC in (5.1) has \((mM \times mM + mP \times mP)\) parameters to tune.
Figure 5.1 Illustration of the output prediction and control action in model-predictive control with prediction horizon $P$ and control horizon $M$.

while the feedback controllers in both approximate input-state linearization and I-O linearization with stability constraint have $n$ and $m + n \times n$ parameters, respectively. For example, consider the process with 4 state equations, 2 manipulated inputs, 2 outputs, the prediction horizon $P=4$, and control horizon $M=4$. The optimization in (5.1) requires 32 tuning parameters. In contrast, the approximate input-state linearization and the I-O linearization with stability constraint require only 4 and 18 tuning parameters, respectively.

To further illustrate the advantage of proposed control methods, consider the application to the continuous bioreactor in section 2.4.2, where two different model-predictive controllers have also been applied to this example (Brengel and Seider [7] and Ramaswamy et al. [98]). The optimization problem in [7] was formulated by using two outputs ($x_1, x_2$) and one manipulated input ($u$). It requires five tuning parameters (two weights on the outputs, the prediction horizon, the control horizon, and the relinearization instants per sampling interval). Alternatively, the optimization problem in [98]
formulated by using only one output \((x_i)\) and one manipulated input \((u)\). It requires six tuning parameters \((M, P, Q, R, S\) and sampling periods \(\Delta t\)). The state feedback of approximate input-state linearization has only two tuning parameters. Even though the state feedback of input-output linearization with stability constraint requires five parameters, it is applicable to a wide operating range. In the developed control methods, the values of tuning parameters are initially determined by selecting the tuning parameters of each state variable equal to its time constants. The tuning parameters are adjusted based on the input and output response: the smaller the values of tuning parameters, the faster the state responses.

5.2 Future Research Directions

As a consequence of carrying out this research project, the open problems listed below were encountered.

The control method with guaranteed closed-loop stability

The first proposed control method is capable of guaranteeing the asymptotic stability of the closed-loop system. However, the analytical proof of closed-loop stability under this control method is difficult and still open due to the ‘short prediction horizons’, optimization form of the control method. The shortness of the prediction horizons can be easily seen, when one derives the controller by using the model-predictive approach described in 3.2.1.

Feasibility of control action

In the design of stability constraint, the second derivative of the Lyapunov function was used to avoid the singularity problem at the equilibrium point, which led to a more
complex stability constraint. A simpler stability constraint without the singularity problem is of interest. In addition, the existence of a control action satisfying the constraint is a major problem in the design of the domain of attraction around the reference steady state. The trajectories that are both optimal and feasible are complicated by the fact that the space of possible control action is large and possibly non-convex. The domain of attraction should not contain the singular point in the state trajectory because it hinders achieving the requested response. To improve the feasibility of the control action, some techniques such as the multiple Lyapunov functions can be used to enlarge the domain of attraction. To guarantee feasibility, an efficient algorithm for computing time variant subsets of piecewise quadratic Lyapunov function is required.

**Deadtime in the input and output**

Deadtimes in outputs and/or manipulated inputs can deteriorate the closed-loop stability. Controllers that are capable of handling deadtimes are needed. Controllers that can handle deadtimes in the outputs can be designed by including the time delays in the desired closed-loop response. Controllers that can handle deadtimes in the manipulated inputs are more difficult to design.

**Robustness and optimality**

In this work, the controllers were developed under the assumption that unmeasured disturbances are piece-wise constant. However, many processes present different classes of uncertainty, such as time-varying uncertainty. If the uncertainties are not accounted for, they can lead to serious degradation of closed-loop performance. To achieve robust stability, techniques such as the dual mode or hybrid controllers can be deployed. The I-O linearization is generally used as an inner loop controller to maintain stability. The
Lyapunov direct method is used as an ‘outer’ controller to handle the uncertainty and force the state variables to the desired steady state to guarantee the stability of the ‘inner’ controller.

Software for nonlinear model-based control

The prototype software for analytical model-based controller design can be improved further to make it more convenient for engineering practice. Suggestions for the software improvement are as follows:

- The software should be flexible to add, remove or update the control methods into the software environment.
- When the process has a highly nonlinear complex model, an efficient algorithm that can speed up the calculation of the steady state pairs is required.
- The algorithm of zero dynamics calculation should not only be applicable to non-redundant equations.
- The software should support controller design for broader classes of processes.
- Currently, the software needs Mathematica as the symbolic calculation engine. Future controller design software should be completely stand-alone applications.
- The software that integrates the controller design into real-time implementation is also needed. Software such as LABVIEW is generally used to acquire real-time data and supports the real-time implementation of PID controllers on the process. Controller design software that can interface with LABVIEW will be of interest to control engineers.
List of References


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