Optomechanical Behavior of Embedded Fiber Bragg Grating Strain Sensors

A Thesis
Submitted to the Faculty
of
Drexel University
by
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in partial fulfillment of the requirements for the degree of
Doctor of Philosophy
in
Materials Engineering
July 2005
Dedications

For Ryan, Cole, and baby Brooke Mastro.

The collective apple of my eye
Acknowledgements

First and foremost, I would like to thank my advisor and friend Dr. Mahmoud El-Sherif for all of his support during my graduate work at Drexel University. I thank Dr. El-Sherif for his unparalleled patience, guidance, and generosity. I would also like to thank the special members of my research group, FOPMEC, for their support and creativity, including Lalit Bonsar, Bulent Kose, Dianne Rothstein, Rachid Gafsi, and Jianming Yuan. Special thanks are given to Dr. Alan Lau for his guidance, experience and advice.

I thank the rest my thesis committee, Dr. Christopher Li, Dr. Roger Doherty, and Dr. Michelle Marcolongo for their invaluable guidance, experience and time.

I would also like to thank the faculty, staff, and students of the Materials Science & Engineering Department of Drexel University who offered so much of their technical help, encouragement, camaraderie and friendship. Special thanks are offered to Antonios Zavaliangos, Josh Houskamp, Adam Procopio, and Jing Jhang for their technical assistance. Special thanks are also offered to Roger Doherty, Rick Knight, Michel Barsoum, Surya Kalidindi, Alan Lawley and Judy Trachtman for their friendship. And very special thanks also to Jason Lyons, my consummate “bar raiser” for his friendship and encouragement.

I am grateful to those at the Naval Surface Warfare Center, Carderock Division, Philadelphia, whose support made this work possible. Very special thanks go to Mr. Greg “Doug” Anderson for his invaluable technical assistance and help with the “hard math”. Thanks to Jack Overby and Henry Whitesel for their helpful expertise in fiber
optics over the past decade. Very special thanks to Jim “MacGyver” Valentine for making the experiments possible through technical assistance and procuring the endless needed components. Thanks also to my encouraging cheering section and souls willing to hear my constant rants and complaining: Greg Anderson, Mark Zerby, Chris Dafis, Sam Doughty, Mike Zink, and John Roach. I would also like to thank a long but very significant list of supportive members of my management teams over the past years who provided unflinching financial and moral support including (in safely alphabetical order) Dan Devine, Dr. Michael Golda, Al Ortiz, John Sofia, Lisa Teodoro, Fred Twardowski, Bill Valentine, and Charlie Zimmerman. I would like to thank John Barkyoubmb and Bruce Douglas, and the past and present Carderock Division Training Advisory Board members, Technical Directors, and Commanding Officers for their approval and use of the In-House Laboratory Independent Research (ILIR) and Extended Term Training (ETT) programs, which supported this work.

Last, and most significantly I would like to thank my family for their years of support, understanding, and love. These thanks go to my “home team MVPs”, my wife Donna, and my children Ryan, Cole, and baby Brooke, and to my parents, Amedeo and Loretta.

And to the late, great Rose Consalvi, I say finally “Yes, NOW I am done!”
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Fiber Bragg gratings (FBGs) can provide extremely sensitive strain measurements for various materials and structures. The main functionality of the Bragg grating is along the fiber’s main axis, where changes in the grating’s spacing can be converted into strain measurements. Previous work from a number of researchers has identified bifurcation and broadening of the Bragg signal under transverse loading. The work presented in this thesis highlights efforts to relate transverse loading to changes in index of refraction in the fiber core cross section, and then ultimately to predicted changes in Bragg signals. The background of FBGs, their application, manufacturing, and operation is outlined. In addition, background on the general concept of photoelasticity, the relationship of stress and index of refraction, in glass materials is presented. A theoretical analysis was performed for uncoated silica fiber to calculate the stresses within an optical fiber core under transverse loading. The transverse loading profile ranged from pure diametric point loading to a more distributed profile. The stresses calculated were translated into changes of index of refraction and FBG signal values. The analysis was then simulated utilizing a numerical model, calculating stress, change of index of refraction, and change in FBG signal with various transverse loading profiles. In addition to an uncoated fiber, a polymer coated fiber system was analyzed. The model was verified by performing a laboratory experiment where FBGs were loaded transversely and their signal monitored.
A special loading rig was designed and fabricated to impart transverse loading to the fiber while monitoring the compression load and deflection of the loading plates. The laboratory experienced showed reasonable agreement with the numerical model. The data show that side loading of the FBG caused a bifurcation of the signal, and that this effect can be predicted by the theoretical model. The modeling work completed provides a useful tool in predicting effects on FBGs of potential transverse loading scenarios, whether these effects are undesirable, or sought after.
Chapter 1 Introduction

Optical fiber is presently being applied for a number of niche sensor applications. These applications include mechanical, electrical, biomedical, and chemical sensing technologies. The broad use of optical fiber as a communications media has, through economies of scale, enabled sensor designers to affordably address niche applications where conventional sensor technologies cannot fulfill the measurement requirement. The accurate characterization of the performance of these devices requires the observation of the optical fiber as a materials system, subject to electromagnetic and mechanical perturbations.

Many different types of fiber optic sensors have been developed that operate principally by modulating the intensity, phase, or polarization of the light passing these sensors. These sensors have been developed to measure strain, temperature, pressure, proximity, current, voltage and chemical presence. One type of sensor used for strain measurement is based on the application of fiber Bragg grating (FBG) structures. They are known as optical strain gauges. These optical strain gauges modulate a particular wavelength of the light passing through them, in a very sensitive manner, as a strain is induced in the optical fiber. It is assumed that the sensor measurements are related only to axial strain, however looking to the FBG as a material system raised a number of critical questions based on Poisson’s effect and transverse loads. The answer to these questions is the core of this thesis work. A study of the optomechanical and structural behavior of optical fiber and FBG sensors under various loading conditions is presented in this thesis.
The optomechanical behavior of optical fiber is a critical issue when considering fiber optic sensors making mechanical measurements. When the optical fiber core carrying the light undergoes mechanical perturbations that alter the material properties of the waveguide – a change in the signal is always possible. Therefore, it is critical to characterize fiber optic sensors that will encounter mechanical perturbations such as stress and strain. This is particularly important given the thrust of applications for FBG strain sensors. In both the defense and private sector, FBGs are being used for static and dynamic strain measurements by being embedded into composite materials and structures. This application inherently introduces the sensor to strains not along the fiber’s major axis. This makes characterization of the FBG under transverse strain particularly important for proper calibration of the sensor measurements.

Information was gathered to provide a background as to the fabrication and operation of a FBG sensor and FBGs in general. In addition, information on ordinary glass fiber and its materials properties were collected. Data and general information on anomalous readings from FBG sensors under off-axis loading were analyzed, and provided the baseline and impetus of this work. After analyzing in detail how mechanical loading could affect index of refraction (and resulting anomalies in FBG readings) of glass fiber, theoretical analyses were conducted of various loading conditions in glass fiber, and resulting change in index of refraction. First, in the simplest case, fibers were loaded transversely by a point load, and the resulting stress field (and resulting index of refraction fields) was analyzed (Hertz). Next, a distributed load was applied to a fiber cross-section and analyzed (Hondros). Finally, an analysis was conducted to determine a stress field within the optical fiber when embedded in a
rectangular cross section solid. These analyses were conducted analytically then solved numerically. Following these analyses, experimental work was done to verify the theoretical analysis. The resulting stress fields were converted to fields of index of refraction – and these non-homogenous index of refraction fields are then presented as to their effect on the FBG signal. In the end a relationship between applied load and FBG measurements is produced.

A literature review is presented in Chapter 2. The basics of FBGs are presented for fabrication and operation for sensors applications. In addition, the concept of anomalous data from FBGs under transverse loads is introduced. In Chapter 3 the concept of photoelasticity is presented. Photoelasticity, or the relationship between stress in an optical material and its index of refraction, is critical to understanding the implications of how external loading of optical fiber sensors effects their properties and the sensor output measurements.

In Chapter 4 a theoretical analysis of FBGs under transverse load is accomplished. Analyses are presented of optical fiber under transverse load with appropriate analytical equations, and a numerical analysis. The analysis includes concentrated diametric transverse loading, and more distributed loading case that would be closer to the case of loading the fiber with parallel plates.

The numerical analysis of the FBG under transverse load is covered in Chapter 5. The analysis is similar to that in chapter 4, but also adds an analysis for polymer coated optical fiber. Numerical analyses are performed for a diametrically loaded case, a distributed load case, and an embedded load case where the fiber is embedded in an
epoxy host. Changes in index of refraction and FBG signal are calculated from the stresses calculated in the numerical analysis.

Chapter 6 describes laboratory tests that were conducted to validate the theoretical analyses in Chapters 4 and 5. A carefully constructed set of tests was conducted using parallel plates to observe the change in FBG with applied load. In combination with results from Chapter 4, an argument can be made as to the level of side loading that would be significant to a FBG signal. This discussion is contained in Chapter 7, along with proposals for future work.
2.1 Bragg Grating Background

Fiber Bragg gratings (FBGs) are formed by constructing periodic changes in index of refraction in the core of a single mode optical fiber. This periodic change in index of refraction is typically created by exposing the fiber core to an intense interference pattern of UV energy. The formation of permanent grating structures in optical fiber was first demonstrated by Hill and Meltz in 1978 at the Canadian Communications Research Centre (CRC) in Ottawa, Ontario, Canada [1]. In groundbreaking work, they launched high intensity Argon-ion laser radiation into germanium doped fiber and observed an increase in reflected light intensity. After exposing the fiber for a period of time it was found that the reflected light had a particular frequency. After the exposure, spectral measurements were taken, and confirmed that a permanent narrowband Bragg grating filter had been created in the area of exposure. This was the beginning of a revolution in communications and sensor technology using FBG devices.

The Bragg grating is named for William Lawrence Bragg who formulated the conditions for X-ray diffraction (Bragg's Law). These concepts, which won him the Nobel prize in 1915, related energy spectra to reflection spacing. In the case of fiber Bragg gratings, the Bragg condition is satisfied by the above mentioned area of modulated index of refraction in two possible ways based on the grating’s structure. One is the Bragg reflection grating, which is used as a narrow optical filter or reflector. The
second is the Bragg diffraction grating which is used in wavelength division multiplexing and de-multiplexing of communication signals.

The gratings first written at CRC, initially referred to as “Hill gratings”, were actually a result of research on the nonlinear properties of germanium-doped silica fiber. It established, at the time, a previously unknown photosensitivity of germanium-doped optical fiber, which led to further studies resulting in the formation of gratings, Bragg reflection, and understanding of its dependence on the wavelength of the light used to form the gratings. Studies of the day suggested a two-photon process, with the grating strength increasing as a square of the light intensity [2]. At this early stage, gratings were not written from the “side” (external to the fiber) as commonly practiced now, but created by creating a standing wave of radiation (visible) interference within the fiber core introduced from the end of the fiber (Figure 2.1).

In 1989, Meltz, Morey and Glenn showed that it was possible to write gratings from outside the fiber [3]. This proved to be a significant achievement as it made possible future low cost manufacturing methods of Bragg gratings and enabled continuous writing or “writing-on-the-fly”. With this method of writing gratings, it was discovered that a grating made to reflect any wavelength of light could be created by illuminating the fiber through the side of the cladding with two beams of coherent UV light. By using this method (holography) the interference pattern (and therefore the wavelength of reflected light from the grating) could be controlled by the angle between the two beams, something not possible with the internal writing method, as seen in Figure 2.2. Figure 2.2 and Figure 2.3 show two methods of manufacturing a side-written
Figure 2.1 Grating written with standing wave inside fiber

Figure 2.2 Side-written Bragg gratings (Interference Pattern)
grating. Figure 2.2 shows the grating being formed by two interfering light beams, and in Figure 2.3 a light beam incident to the fiber with a phase mask. In Figure 2.2 the two coherent beams of light form an interference pattern which creates a standing wave with variable intensity of light. The variable radiation intensity occurs within the fiber core. This variation in radiation intensity creates a modulated index of refraction profile within the fiber core. In Figure 2.3 the modulation is created by using a single light beam and a phase mask. In areas where the mask allows light transmission, the index of refraction is changed within the core, creating the grating. This technique is particularly useful to write gratings quickly. Both of these methods allowed for “tuning” of the grating to whatever wavelength was desired. This too was an important development, as it allowed gratings to be easily written at various wavelengths to follow the communications
industry’s changing source wavelengths. In addition, it was found at the time that this method was far more efficient.

2.2 Photosensitivity

Photosensitivity of optical fibers, the phenomena that makes the writing of FBGs possible, was first observed in experiments in which continuous wave blue light (~488nm) from an Argon laser source was launched into an optical fiber. After exposure, an increased reflectance of the light in the fiber was noted with time. This effect was termed as “photosensitivity”, and this is the effect that creates the FBG. It is assumed that an interference pattern arose from the propagating light and the reflected light and that at the radiation maxima, photosensitivity allowed for the change in index of refraction (Figure 2.1). The gratings at this early stage could be as long as 1m. The limitation of the early Hill gratings was that they could only operate in the visible range near the wavelength of the light that created the grating.

When the UV radiation creates the grating, the effect (change in index of refraction) is permanent. The effect will last indefinitely as long as temperatures remain within the stated operational temperatures. The upper temperature that gratings can survive (before the grating is effectively washed out) depends on annealing after formation, but in some fibers can operate up to 600 C. In work done on gratings designed for high temperature applications, the gratings operated at 600 C, however when operated at 700 C a decay was observed as little as 30% in 30 minutes. At 800C the gratings last a very short time before they disappear through material flow or diffusion at
that temperature [4]. While many materials have shown to exhibit photosensitivity, germanium doped silica has remained the predominant choice of FBG manufacturers.

The actual amount of the change of index of refraction experienced depends on a number of factors, such as the actual composition of the glass, processing of the fiber, and conditions of exposure to radiation including wavelength, intensity, angle of sources etc. In general the radiation causes an increase in the index of refraction ($\Delta n$). The change of index of refraction has to do with the color centers of the glassy material, and the UV radiation interaction with the Ge-SiO$_2$ bonds. This creates a quantity of GeO which when exposed to UV radiation creates oxygen deficient centers, which cause an increase in the index of refraction. In addition it is thought that some densification may occur as well, raising the index of refraction. This process can be slow and is dependent on the UV light source intensity. Hydrogen loading can be used to make the process quicker by making the fiber core more sensitive. In essence a second photosensitive effect is created as the introduction of Hydrogen in the fiber core allows the UV radiation to create Si-OH which raises the index of refraction as well.

The increase in index of refraction can vary with various methods to be 0.0001 to 0.05, and the reflectivity (the % of light reflected by the grating) of the entire grating structure can be up to 100%. Each grating or line of increased index of refraction provides a reflection point, and the grating will reflect all of the light given enough length (sufficient number of reflection points). The amount of light a given grating reflects is called its efficiency. Typically the grating spacing (if the Bragg reflection is at 1300 nm) will be on the order of 450 nm, and the grating will be 1cm+ long to provide ~100%
reflectivity. A grating 1cm long with these specifications would have on the order of 23,000 lines of modulated index of refraction in the grating.

The work by Meltz et. al. made it possible to create the grating at any wavelength desired. The holographic method of writing gratings presented big advantages to the method employed by Hill for a number of reasons. Meltz decided that the two-photon process employed by Hill could be made far more efficient with a one photon process at half the wavelength (~245nm) and that with the holographic method described above, the gratings could be made to operate at any wavelength. At this wavelength (UV), the fiber cladding is transparent (while the fiber core would absorb the radiation), so there would be no need to remove the cladding. This is due to the doping of the fiber core.

After the work of Hill, it became obvious that there were some significant potential applications for the FBG. The FBG could obviously be used as a wavelength specific filter, which could be used in a myriad of communications applications, and also as a mechanical strain gage based on the grating’s periodicity and spacing [3, 5, 6].

2.3 Fabrication Techniques

As described above, the first method of fabricating gratings was internal writing through standing waves of radiation and the second method was the holographic side writing of gratings. Today, both of these methods have been surpassed/enhanced by the use of the phase mask [7, 8] (see Figure 2.3). The phase mask is a planar slide of silica glass or similar structure which is transparent to UV light. A periodic structure with the appropriate periodicity is etched onto the glass slide to approximate a square wave using photolithography (as viewed from the side). The optical fiber is placed very close to the
phase mask while the grating is written. UV light is introduced to the fiber, and is diffracted by the periodic structure of the phase mask, creating the grating structure described above. The periodic structure created in the fiber is ½ that of the spacing of the periodic structure in the phase mask. In this manufacturing technique, the periodicity of the FBG is independent of the wavelength of the UV light source. The wavelength of the UV light source is selected based on the absorbance spectra of the doped optical fiber core – thereby maximizing the source’s efficiency in writing gratings.

Use of phase masks made lower cost, greater precision Bragg gratings possible by simplifying the manufacturing process. In addition, the phase mask technique made possible the ability to automate grating writing, and the ability to write multiple gratings on a fiber simultaneously. The phase mask procedure allowed for the efficient writing of other types of gratings such as chirped gratings which have non-constant periodicities for a wider spectral response (useful in the manufacturing of dispersion compensators) [3].

2.4 General Applications of Fiber Bragg Gratings

As mentioned earlier, FBGs and their characteristics make them useful devices for sensors, telecommunications, and other applications. As an “on fiber” device, they are ideal to integrate into optical fiber systems. Applications vary from laser systems, optical switching and signal processing (multiplexing and demultiplexing) to integrated optics, and optical computing. Meltz [3] listed a number of existing and potential applications:

*Telecommunication Applications*

- Dispersion Compensator
- Wavelength Selective Device
• Band-Rejection Filter
• Long Period Grating
• Fiber Taps
• Fiber Erbium Amplifiers
• Network Monitoring
• Cascaded Raman Amplification
• Fiber Lasers
• Semiconductor Lasers with External Bragg Grating Reflectors

Other Applications

• Optical Fiber Mode Converter
• Spatial Mode Converter
• Polarization Mode Converter
• Grating Based Sensors
• Optical Signal Processing
• Delay Line for Phased Array Antennae
• Fiber Grating Compressor
• Electro-optic Devices
• Wavelength Conversion Devices
• Optical Storage, Holographic Storage

This long list shows the wide application possibilities for the FBG. The applications listed have had varying levels of success over the past 10 years. A classic and illustrative example of another application, more communications oriented, is that of the FBG wave division multiplexer. As a pulse of light travels through optical fiber it experiences
dispersion. This means that the pulse of light broadens, as the longer and shorter wavelengths of light in the pulse travel at different speeds. With huge numbers of light pulses being sent through optical fiber, over long lengths, this dispersion will cause the light pulses to overlap causing errors in the overall signal, acting as a limit on the system. A so-called “chirped” grating, one with a periodic structure that has both long and short periods (varied linearly) will change the speeds of wavelengths, essentially “squeezing” the signal together again, and increasing the available data rate and length.

The application of a FBG as a strain sensor is the focus of this thesis. The use of a FBG for measuring strain is detailed below. Applications of FBGs as strain sensors range from use a slow frequency strain sensors in structures and materials to high speed accelerometers. FBGs have been applied to composites manufacturing, bridges, roads, tunnels, Navy ships, towed arrays, and many other applications where strain, acceleration or shape data is needed. Especially attractive is the ability to embed optical fiber into these systems for long term or real time strain measurements – something nearly impossible to accomplish with foil-type strain gages.

2.3 Fiber Bragg Grating Operation

Of principal interest in this thesis is the operation of the FBG strain sensor. As described above, the FBG strain sensor senses strain by extension or compression of the optical fiber along its major axis, and resulting in a change in the wavelength of the reflected Bragg signal.

As described above, when a light signal encounters a material of higher index of refraction (one of the periodic gratings), some of the light signal is reflected, some
refracted, and some is transmitted. The Bragg grating allows a portion of a light signal propagating through an optical fiber to be reflected. Light is reflected in a Bragg grating when the light satisfies the Bragg condition. This reflection is due to an induced 180° phase change in the propagating signal by each Bragg period. This occurs when the wavelength (known as the Bragg wavelength) satisfies:

\[ \lambda_b = 2n_{\text{eff}} \Lambda \]  

(2.1)

where \( n_{\text{eff}} \) is the effective index of refraction and \( \Lambda \) is the grating spacing. The amount of light reflected at that wavelength is dependent on the efficiency, which is based on index of refraction change or \( \Delta n \), of the gratings, and also the total grating length, as each grating line serves to reflect a portion of the light signal \([5, 9, 10]\). This effect is seen in Figure 2.4. Light from an LED is launched into the fiber, a broadband source whose center wavelength is close to the Bragg wavelength. The light propagates through the grating, and a portion of the signal is reflected at the Bragg wavelength. The complimentary portion of the process shows a small sliver of signal removed from the total transmitted signal. This clearly shows the Bragg grating to be an effective optical filter. With today’s manufacturing techniques, 100% reflection is commonly found at the Bragg wavelength. A small percentage broadband absorption and instrumentation tolerances are also present. The reflected signals have a wavelength spread of up to a few tenths of a nanometer, or about 0.1nm in most cases. This operation is seen in Figure 2.4. An LED light source is used as a light input, and a narrow band reflection is seen
corresponding to the Bragg condition. As seen in (2.1), this reflection is centered at a wavelength based on the grating spacing and the index of refraction of the grating.

As defined in Equation (2.1), the Bragg wavelength is directly proportional to the spacing of the grating lines $\Lambda$. Therefore, any change in the grating spacing will cause a direct change in the Bragg wavelength $\lambda_b$. This is the basis for the use of a Bragg grating as a strain sensor [11-13]. A change in dimension of the fiber core along its major axis causes changes in the spacing of the grating lines (Figure 2.5). This change is principally due to thermal expansion or mechanically induced strain. As the fiber core is stretched or

**Figure 2.4 Bragg grating response to broad spectrum LED**

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compressed, there is a corresponding increase or decrease in the value of $\Lambda$, and therefore, a corresponding change in the value of $\lambda_b$.

By tracking the changes of the Bragg wavelength, the elongation or compression of the fiber core can be measured. Since a Bragg grating has a well-defined $\lambda_b$ for each localized area of the Bragg structure, a number of sensors can be put on the same fiber core (Figure 2.6). The gratings can therefore be uniquely identified regardless of the distance between them along an optical fiber. This would allow for a number of strain measurements along one fiber, with one light source and detector. This arrangement maximizes the multiplexing potential of Bragg gratings, and keeps system cost to a
minimum as opto-electronics are orders of magnitude more expensive than FBG fiber. The reflecting wavelength of a Bragg grating will shift linearly with temperature (expansion and contraction) and with strain. For simplicity, most stresses are considered just in direction of the fiber’s major axis, as early experiments showed that response in this direction was by far the most sensitive.

![Figure 2.6 Bragg grating sensors multiplexed through unique wavelength coding](image)

It is important to note that a broad LED is not the only light source that can be used with a FBG. A scanning laser diode could also be utilized, as can a broad white light source. The choice of light source and detection scheme will be made based on how many FBGs need to be employed, and how fast the data are needed. The dynamic response of the FBG is very high, and is limited only by the optoelectronics of the sensor.
The more gratings that need to be read, the lower the speed of the sensor system will be.

FBG signals can be detected in a number of ways. One is to read a signal in both light amplitude and wavelength. This can be accomplished with an optical spectrum analyzer, which analyzes the light across a range of wavelengths. In addition, fiber etalons, interferometers, and other devices can be used to examine signals in FBGs in both the wavelength and time domains. In every case, the object is to address each sensor individually, measure its Bragg wavelength, and in the case of multiple sensors, scan through them. In a system utilizing 1-5 gratings, systems have been developed that can read into the MHz range [14, 15]. For larger systems of gratings, speeds are slower, but can routinely be in the kHz range. This makes the FBG an ideal candidate for a highly sensitive distributed strain/velocity/acceleration sensor.

The tracking of strain with the changing of \( \lambda_b \) as described above is made simple through basic assumptions about the measurement system. In looking at the strain condition purely, it is first necessary to remove temperature effects (thermal expansion and contraction). Any analysis of complex strain effects on the signal necessitates this step. The other major assumption made is that axial strain, and perhaps a small Poisson effect, is the only strain reflected in the shift of \( \lambda_b \). In addition, it is assumed that the fiber core is perfectly cylindrical, and that the material properties in all directions are homogenous. In some applications, these assumptions are acceptable. With the proposed use of fiber optic Bragg gratings as an embedded sensor used to characterize materials, however, it is important to observe the sensor’s response to mechanical perturbations in all directions.
While experiencing transverse load, orthogonal to the fiber core’s axis, anomalies have been observed in the Bragg grating signal, as seen in Figure 2.7 [16, 17]. The conceptual Bragg transmitted signal at the top is depicted in actual laboratory output signal traces, below, before and after transverse loading. A portion of the total transmitted signal, as seen at the top of the figure, is examined with an optical spectrum analyzer. The trace on the left is the transmitted unperturbed Bragg grating signal, and the trace to the right is the signal with a transverse load applied to the sensor. The Bragg grating signal has broadened, shifted to a higher wavelength, and has started to form multiple distinct peaks.
Figure 2.7 Effect of transverse loading on Bragg grating

Poisson effects can easily explain the general shift to the right: as the fiber is compressed from the side, it expands in the direction along its major axis, causing the same effect as elongating the fiber. In addition to shifting, the signal has broadened, and split into two well-defined components. It is evident that the optical properties of the waveguide have changed, as have the material properties of the Bragg structure to cause this unusual signal. It is also evident that additional explanation is needed to properly infer strain measurement from such a signal.
2.4 Embedding of FBGs and Transverse Load Effects

The analysis of transverse loading and strain in FBGs is application driven. The principal value of FBGs is the ability to integrate them into or onto structures to monitor strain [18-20]. This provides advantages to the well-known mature technology of foil strain gages. Traditional strain gages are used to measure changes in strain after a circuit and electronics are attached to the system in a Wheatstone bridge. If the electronics are removed, the system has to be “re-zeroed” after the circuit is reconnected. This makes the use of foil strain gages less than optimal for long term monitoring. In addition, the wiring of traditional foil strain gages can be cumbersome if multiple readings are desired in a structure. Foil strain gages can also be susceptible to electromagnetic interference, and are difficult to embed within a material or structure. FBGs can be multiplexed, and can easily be integrated into composite materials [21, 22]. These strands of FBGs can provide virtual “nerves” to structures, allowing for both sensing strain and for initiating actuators.

Embedding FBG sensors, however, introduces loads and strain in all directions, not just in the axial direction. This concept is further complicated by issues of embedment into fibrous composites, where FBG fiber size and alignment are important issues. In addition, the introduction of embedded fiber sensors can cause residual stresses and concentrated stresses that need to be eliminated or accounted for when analyzing the structure as a whole.

With respect to optical fibers effect on the host material in a strain sensing application, the effects are comprised of the fiber size, fiber jacket properties and size and the Young’s modulus of the jacket, as well as the material properties of the host [23-29].
By tuning these parameters to the greatest extent possible the detrimental effect of the fiber sensor can be minimized, while maintaining an appropriate sensor sensitivity and accuracy. In general, matching as close as possible the fiber jacket parameters to the host parameters will minimize the stress concentration caused by the fiber. In some applications special coatings and jackets are used to minimize concentrated stresses at the fiber surface, and improve the interface adhesion with the host material. It's important to note that the stress experienced by the embedded FBG is that of the localized stress/strain and not that of the far field strain.

A number of researchers have examined the effect of transverse loading, or multidimensional strain fields on FBG signals. In the mid-1990’s Sirkis et. al. at the University of Maryland studied the effects of transverse loads on both Bragg gratings and interferometric fiber optic sensors [13, 30, 31]. Along with other researchers including Eric Udd at Blue Road Research, and Mahmoud El-Sherif at Drexel University work was done on examining the effect of putting a Bragg grating into a strain field that included off-axis strain [16, 17, 32-44].

In Udd’s work, it is stated that multi-axis strain effects are minimized when FBGs are utilized as surface mounts sensors vs. embedded sensors. Side loads cause a bifurcation of the classic Bragg response. Udd and others propose to use the bifurcation as a tool to measure strain in two directions (axial and orthogonal to axial) by utilizing the bifurcation effect along with polarization maintaining (PM) fiber. Polarization maintaining fiber is optical fiber with a manufactured-in birefringence. Birefringence refers to the condition where 2 indices of refraction exist in a material. A grating written into such a fiber will have a dual-peaked Bragg signal even in the unloaded state.
Relative motion of the peaks can be attributed to the changes in $\Delta n$ along the axes of the PM fiber, while axial strain can be detected from shift in both peaks together. By writing gratings at two different wavelengths, Udd was able to separate the effects for a sensor. The experiments demonstrating these concepts show response from various loading profiles [34, 35]. In addition to shifting peaks, transverse loading also commonly causes spreading of the Bragg peak, and also noise on the signal. This can prove problematic for automatic tracking of the Bragg signal, as the location of the true center wavelength cannot be identified.

Most of the work in the field is empirical measurement of the split peak feature of transversely loaded FBGs, with some work detailing the principles of the bifurcation analytically, and some general FEA work focused principally on stress concentrations caused by embedded fibers [13, 30, 31, 33, 36, 37] in a host material. Most of the work is focused on response within a high-birefringence fiber as this shows the most promise for application and use of the transverse load effect on FBG signals.

2.5 Transverse Loading and Bragg Grating Signal Anomalies

The signal anomaly seen in Figure 2.7 has been observed in many instances [13, 16-18, 34, 40, 42, 43, 45]. It is obvious that in this case a bifurcation is taking place, which is splitting the optical signal in two. Examining the signal on the right in Figure 2.7, it is clear that the signal shows two predominant waveforms with two predominant wavelengths. In some fashion, it can be assumed that the material properties of the silica waveguide are being altered as to allow reflection at more than one dominant wavelength.
Common in descriptions of the above effect is the concept of birefringence. By definition, birefringence is the condition where two orthogonal components of the optical fiber cross section have different indices of refraction. In practice, this phenomena in relation to transverse load is explained next.

In looking at the Bragg grating condition highlighted in Equation 2.2, a temperature change of $\Delta T$ and an axial strain of $\varepsilon$ would result in a wavelength shift of the Bragg condition ($\Delta \lambda_b/\lambda_b$) given by [1]:

$$\frac{\Delta \lambda_b}{\lambda_b} = \varepsilon_z - \frac{n_{\text{eff}}^2}{2} [\varepsilon_{xy}(p_{11} + p_{12}) + p_{12}\varepsilon_z] + (\alpha + \zeta)\Delta T \tag{2.2}$$

Where $\alpha$ is the thermal expansion coefficient for the fiber, $\zeta$ is the thermoptic coefficient or $dn/dT$ of the doped silica core material, $\varepsilon_z$ is strain in the axial direction, $\varepsilon_{xy}$ is strain in the radial direction and $p$ is the photoelastic constant which relates changes in $n$ with strain. This is sometimes called the strain-optic coefficient, and it relates the concepts of photoelasticity to the change in Bragg wavelength (see Chapter 3).

By relating the basic equations of three-dimensional mechanics a number of important equations can be devised to describe generically the mechanics of the system. Ignoring temperature changes to concentrate on stress related concepts, the change in Bragg wavelength can be described as \[46-48\]:
where $F$ is the load on the fiber. Introducing $n$ as a function of direction, both parallel and perpendicular to a side load to the fiber cross-section, birefringence, $B$ can be defined as:

\[
B = \left| \frac{n_{\text{parallel}} - n_{\text{perp}}}{n_{\text{eff},0}} \right| = B_0 + \frac{\Delta n_y - \Delta n_z}{n_{\text{eff},0}}
\]  

(2.4)

where $B_0$ is any inherent birefringence present before loading, and $n_{\text{eff},0}$ is the initial effective index of refraction before loading. These equations lead to important optomechanical relations that can provide the change in index of refraction and the resultant change in Bragg wavelength, due to loading in all three axes. These relations are described generically as [1, 10, 40]:

\[
\Delta n_{\text{eff}}(x,y,z) = f(n_{\text{eff},0}, n_{x,y,z}, p_{x,y,z}, \sigma_{x,y,z}, E, \nu)_{x,y,z}
\]  

(2.5)

\[
\Delta \Lambda_{b}(x,y,z) = f(\Lambda_{b,0}, n_{\text{eff},0}, p_{x,y,z}, \sigma_{x,y,z}, E, \nu)_{x,y,z}
\]  

(2.6)

where $\sigma$ is stress, $E$ is Young’s modulus, and $\nu$ is Poisson’s ratio. These relations are expanded upon in Chapter 4. The key feature is the ability to relate mechanically induced changes in index of refraction to changes in the Bragg condition. The ultimate goal is to
mathematically predict the changing Bragg signal under transverse loading. In the compressed direction, the index of refraction will increase and in the orthogonal direction the index of refraction will decrease (See Chapter 3). The compressed direction is commonly called the “slow” axis, referring to the decrease in light speed along the path with higher n, and vice versa. Under these concepts the bifurcation of the Bragg grating signal is due to these two indices of refraction (the maximum and minimum found in any particular situation) that can satisfy the Bragg condition.

Figure 2.8 Birefringence effect on Bragg grating

There are two common ways to create birefringence in Bragg gratings. One is the application of transverse load. Another is polarization maintaining optical fiber (useful in telecommunications applications), which creates a condition of birefringence in the fiber core through permanent residual stresses during the manufacturing process [30, 49-52].
In both cases the data show two distinct peaks forming from the single peak predicted by the models, as seen in Figure 2.8. Where the cross section of the FBG has a distribution of values of index of refraction, the maximum and minimum values are observed to satisfy the Bragg condition and result in a double peak signal. In some cases, bending of the fiber, which can cause unequal grating spacing or other mechanical effects, has been seen to create multiple peaks (2+) from a single Bragg grating signal.

As described above, the Bragg condition, when satisfied, is what causes a reflection of the light signal at a particular wavelength (Equation 2.1). This condition can theoretically be satisfied for cases where either \( n \), \( \Lambda \) or both are not constant. It has been established that in most cases, any reasonably large splitting of the Bragg grating signal results in two peaks – ostensibly one for each birefringent index of refraction found in the core [13, 17, 40]. Focus is normally given to the fact the index of refraction change is made in two extremes, the maximum and the minimum as defined by birefringence, and that there are usually two orthogonal components of electromagnetic fields propagating through a single mode fiber core. This idea limits discussion of the presence of more than one Bragg condition to a split into two. The observation or discussion of multiple peaks (2+) is usually explained away by considering chirping effects or bending which would cause \( \Lambda \) to change within a single grating. Theoretically, however, wherever the Bragg condition is satisfied, through \( n_{\text{eff}} \) or \( \Lambda \), a reflection will occur.

In describing the theoretical operation of a Bragg grating, a very specific set of material conditions is assumed within the fiber core. This ideal case description will provide the basis for analysis. Described is a perfect cylinder of silica, doped with Ge. There are no residual stresses from manufacturing, and the density, molecular
composition, and index of refraction are all homogenous throughout. After manufacturing the grating, there would be a periodic slight change in the refractive index in the z-direction. However, it is assumed that the refractive index is uniform across the core cross-section at any location in the z-direction.

![Figure 2.9 Side loading effects on index of refraction](image)

In the case of transverse loading, the core cross section would be deformed into an elliptical shape, with a compressive force on the short axis, and vice versa. While there is indeed a condition of birefringence, different indices of refraction along two orthogonal components, it is important to note that there is a gradual change in values of $n$ between the maximum and the minimum (Figure 2.9). In this case, if linearly polarized light were launched through the grating along the direction of the birefringent axes, then only an analysis along these two axes would be necessary – but this is not the case in the general condition, with non-polarized light used in the experiment. The index of
refraction, as a function of photoelastic effects of the fiber being loaded transversely, causes a continuous change of values of $n$ to be present throughout the cross section of the Bragg structure. This means, theoretically, that a number of solutions to the Bragg condition might exist.

If, as explained above, there is a continuous change of indices of refraction in the FBG cross-section, and Poisson effects in the $z$-direction, it is possible that a number of different solutions to the Bragg condition would exist, causing numbers of peaks in the Bragg signal. In most cases these peaks may not be observed. Why? The answer lies in the level or amplitude of the physical and material parameters discussed above, as well as the resolution of the devices used to observe the Bragg signal. In the manufacturing of a Bragg grating, an increase of index of refraction of only a few percent in each line of the grating is enough to satisfy the Bragg condition at a particular wavelength. When introducing a light transverse load the index of refraction is changed though photoelastic effects, and the grating spacing is changed through Poisson’s effect. This initially causes weak birefringence and a broadening of the reflected spectrum may exist. As the level of loading increases and the birefringence increases as well two peaks may appear, depending on the resolution of the spectrum analyzer. As the loading and birefringence is increased further, two solutions to the Bragg condition become much clearer, corresponding to the maximum and minimum index of refraction.
Chapter 3 Photoelasticity and the Behavior of FBGs

3.1 Introduction

During the development of advanced fiber optic sensors, issues frequently arise that necessitate the study of the basic properties of glass. Currently, work is ongoing in the development and analysis of fiber optic sensors that measure various properties within a mechanical system. As with any sensor, the performance and properties of the fiber optic sensor (a mechanical and material system itself) must be fully understood before any application of the technology can be made. The following information serves to help fulfill the need to complete the understanding of the FBG fiber optic sensor system by examining a basic property of glass directly related to its behavior in transmitting coherent light. The optical behavior of light in glass under stress directly affects the performance of FBGs.

In many cases, fiber optic sensors operate by correlating some physical perturbation with the light being transmitted within the optical fiber itself. As heat and electromagnetic fields can affect current in a wire, certain external phenomena can effect light transmission within an optical fiber. While the precise cause and effect relationship between external perturbations and light signal are the focus of some detailed research work, it becomes useful to first examine the properties of glass fiber. This analysis is a look at the phenomenon of photoelasticity.

Photoelasticity is a rather important concept with respect to the analysis of fiber optic sensors, and transmission in glass as a whole. It fundamentally describes the interaction of light with the glass matter and describes the propagation of electromagnetic radiation in glass submitted to stresses. With respect to fiber optic sensors, we are
frequently interested in the effect of stresses and strains anywhere on the optical fiber on the sensor signal. For some fiber optic sensors (such as Bragg gratings), the effect of photoelasticity can be significant as optical anisotropy through the fiber core can directly effect performance of the system.

Glass, in its various forms, is one of the most common materials in everyday life. Glass is known to have been made as early as 2500 B.C. The first experiments demonstrating the “double refraction of glass” (essentially the fact that index of refraction could be anisotropic within glass) under stress were carried out by Brewster in 1816 [53]. In these experiments it was shown that glass in compression acted as a uniaxial negative crystal, and as a positive one under tension, with the optical axis aligning with the direction of loading/stress. Wertheim first measured the stress-optical coefficient of glasses and other various materials and laid the foundation for the concept of photoelasticity in 1864 [54, 55]. In these experiments the optical effect of pure tension and pure compression were studied. The above mentioned experiments and analysis disclosed the discovery of photoelasticity as an effect present in glass under stress. The first quantitative determination of the absolute stress-optical coefficients of a piece of glass was made in the late 1800’s [56]. In 1925 these experiments were repeated (up to stresses of 10 MPa), and the law connecting birefringence and stress was found to be linear and the same for compression and tension. This behavior is generally observed in all glasses of various compositions. A detailed analysis of the efforts of the pioneers in photoelasticity can be found in the text by Coker and Filon [57].
3.2 Effect of Glass Composition on Photoelasticity

The well-known scientist Pockels conducted the first study on how the composition of a glass affects its photoelastic coefficient. The photoelastic coefficient scales the difference in stress with the difference in index of refraction. This is defined by the stress-optic law as stated by Maxwell in 1852 [58]. For a given material, in the most general case, the optical properties at any point can be represented by an index ellipsoid as shown in Figure 3.1. The principal indices of refraction and principal stresses coincide in direction. If \( n_1, n_2 \) and \( n_3 \) represent the principal refractive indices, \( n_0 \) represents the index of refraction of the unstressed material, and \( \sigma_1, \sigma_2, \sigma_3 \) the principal stresses then according to Maxwell the following relations can be made:

\[
\begin{align*}
  n_1 &= n_0 + C_1 \sigma_1 + C_2 (\sigma_2 + \sigma_3) \\
  n_2 &= n_0 + C_1 \sigma_2 + C_2 (\sigma_3 + \sigma_1) \\
  n_3 &= n_0 + C_1 \sigma_3 + C_2 (\sigma_1 + \sigma_2)
\end{align*}
\]

(3.1)  
(3.2)  
(3.3)

where \( C_1 \) and \( C_2 \) are constants depending on material properties. By combining these equations the following relations relate change in stress with change in index of refraction:

\[
\begin{align*}
  n_1 - n_2 &= C(\sigma_1 - \sigma_2) \\
  n_1 - n_3 &= C(\sigma_1 - \sigma_3) \\
  n_2 - n_3 &= C(\sigma_2 - \sigma_3)
\end{align*}
\]

(3.4)  
(3.5)  
(3.6)

where \( C = C_1 - C_2 \) and \( C \) is the stress-optic or photoelastic constant.
Pockel’s showed the effect of the composition of a glass material on the photoelastic constant in test results shown in Figure 3.2. These results show how the photoelastic constant varies with the lead oxide content in flint glass, along with the results of similar tests by Waxler [59] and Filon [59, 60]. The curve in this figure indicates that $C$ vanishes when the percentage of lead oxide is approximately 75% and shows that $C$ becomes negative at values higher than 75%. This means that at this point, the law stated by Brewster that the glass acts as a negative crystal no longer applies.

![Figure 3.1 Index of refraction ellipsoid](image)

Filon studied photoelastic behavior of borosilicate glasses and found that $C$ was increased with the addition of $\text{B}_2\text{O}_3$, and was decreased by $\text{K}_2\text{O}$. This effect, however, was found to be very small compared to that found when lead oxide content was changed. In 1927, studies had shown the photoelastic constant of silica to be $3.45\ \text{TPa}^{-1}$. In 1945, Balmforth and Holland reported results of experiments in which $\text{CaO}$ was progressively
replacing \( \text{Na}_2\text{O} \) on a molar basis with a parent glass whose general formula was \( 6\text{SiO}_2(2-x)\text{Na}_2\text{O}_x\text{CaO} \) [61]. This replacement resulted in a small but significant increase in the photoelastic constant as seen in Figure 3.3. Schweiker similarly studied the influence of \( \text{SiO}_2 \) on the photoelastic constant of various glasses [55]. The literature is full of such examples. The key feature is the observance of the altering of the photoelastic constant \( C \) with glass composition. This is usually explained by the polarizability of the components involved, the change in modulus from the changing chemical composition, and the changing of the physical structure of the glass with changing concentration of components.

![Figure 3.2 Change in photoelastic constant with lead oxide content](image-url)
3.3 General Theory of Photoelasticity

The electron density distribution determines the optical properties of a deformable solid. As light interacts with the matter in a deformable solid, the density of electrons determines to a large degree the amount of retardation the light encounters. It is assumed that the photoelastic birefringence in the visible part of the electromagnetic spectrum is due to the anisotropy in the electron density distribution caused by elastic strain.

The earliest theory to explain this phenomenon was developed by Banerjee and Herzfeld [62, 63]. They theorized that the presence of birefringence was entirely due to the change in the arrangement of the atoms in the deformable solid. If a solid with an isotropic distribution of atoms is uniaxially strained in tension, the average distance between neighboring atoms becomes larger in the direction of strain than in a direction perpendicular to the applied strain – consequently, the propagating lightwave component

![Figure 3.3 Change in photoelastic constant with CaO concentration](image)
parallel to the strain axis meets a lower concentration of electrons that the lightwave component perpendicular to the strain axis. In 1938, Mueller calculated this effect known as the lattice effect according to the Lorentz-Lorenz theory of light refraction. Mueller showed that the lattice effect could be quantitized by purely as a function of index of refraction [64]:

$$L = p - q = - (n^2 - 1)^2 / 5n^3$$

(3.7)

Where $p$ and $q$ are Neumann’s strain-optic coefficients and $n$ is the refractive index. The quantity $(p-q)$ can alternatively be computed with experimental data of the photoelastic constant $C$ and the shear modulus $G$ as

$$L = p - q = 2CG / n^2$$

(3.8)

Equation 3.7 shows that the lattice effect produces negative birefringence while the photoelastic constant is generally positive. Therefore, another effect must contribute to the photoelastic birefringence. This effect, known as the atomic effect, is inferred from the difference between the value of $p-q$ in equations 3.7 and 3.8.

Mueller in his studies noted the difference between the natural and forced birefringence. In a naturally birefringent crystal (birefringence in this case is defined as the refraction of light in an anisotropic material in two slightly different directions) every atom is in its natural surroundings, or natural state. In an elastically deformed/strained body the arrangement of the atoms to their nearest neighbors is distorted, as seen in
This leads to the concept that the electronic density anisotropy of a material depends on the deformation of the neighboring atoms (via strain) as much as on their positions. Taking both of these effects into account on the photoelastic birefringence, Mueller defined the overall “atomic effect”, $A$, defined as:

$$A = \frac{2CG}{n^2} + \frac{[(n^2-1)^2/5n^3]}$$

This relation gives a qualitative explanation of why the photoelastic constant can be positive (for low index glasses) or negative (for high index glasses). In glasses where the refractive index is low, the lattice effect is small and the atomic effect dominates the

Figure 3.4 Lattice Effect with a) no loading  b) uniaxial strain state
value of the photoelastic constant and frequently leads to a positive value. The opposite
is true for glasses with a high index of refraction, where the lattice effect dominates the
value of the photoelastic constant, and can cause it to be negative.

![Figure 3.5 Polarizable cations and anions](image)

In 1956, Weyl and Tashiro theorized a different qualitative interpretation [65, 66].
It was pointed out at this time that the deformation of the atomic structure of strained
glass doesn’t follow a simple expansion in the direction of loading. They used concepts,
which had previously been used to explain the role of ionic polarization in determining
the mechanical and optical properties of glass, to show that ionic deformation depends on
the ion charge and the ion polarizability. As a consequence, the sign and magnitude of
the photoelastic birefringence of glass would be the result of two different effects,
described in Figure 3.5. The first effect depends on the deformation of the polarizable
anions, such as $\text{O}^{2-}$, which would deform in a direction perpendicular to the strain. The
second effect would be a counter-effect resulting from the deformation of polarizable
cations such as $\text{Pb}^{2+}$, which would occur in the direction of the strain. The first effect
would cause a positive birefringence and the second effect would cause a negative birefringence. In addition to PbO, other oxides such as BaO, SrO and Tl₂O, which also contain polarizable cations, will change the value of birefringence of a glass in the negative direction. Similarly, Li₂O, Na₂O, K₂O and other alkaline oxides with cations that are not easily polarizable, make the birefringence of glasses with these compounds positive.

**Figure 3.6 Atomic/Lattice Effect of Bridging Oxygen and Non-Bridging Oxygen for a) Unstressed State and b) Uniaxial Tension**

In the mid 1980’s Matusita et al. argued against Weyl’s theories and against the idea that anions such as O²⁻ would deform preferentially in a direction perpendicular to the strain [6]. Experimental results at the time show a decrease in the photoelastic coefficient when modifier oxide is added – a result that directly contradicts the results of Tashiro. Matusita’s group found that when a strain is applied to a silicate glass, the bridging oxygens with covalent bonds will deform. This deformation will vary with the change in the Si-O-Si bond angle as shown in Figure 14. In this fashion, the deformation
of the electron clouds due to the alternating electric field of the light in the direction parallel to the stress would be much larger than in the perpendicular direction. This leads to the indication of a strong atomic effect. As a modifier oxide is added to SiO₂ glass, non-bridging oxygens are introduced. The electrons of the non-bridging oxygens are bonded less strongly than those in bridging oxygens. Matusita’s group assumed that electrons in non-bridging oxygens respond to the electrical field of light similarly in both the parallel and perpendicular directions to the strain. Therefore, the atomic effect decreases with increasing amounts of modifier oxides.

The same group also studied fluoride glasses based on the ZrF₄ – BaF₂ system, and found them to have stress and strain-optical coefficients which were very small, nearing zero. The conclusion was that the highly ionic nature of the bonds in that glass gave rise to very small atomic effects and to very small photoelastic effects. This differs from the small photoelastic effect in lead glasses, which is due to the highly negative nature of the lattice effect, and high values of n.

### 3.4 Influence of Temperature and Thermal History

There is not a large amount of experimental data available on the effect of temperature and thermal history on the photoelastic coefficient of glass. Since temperature has a significant effect on optical fiber, this concept is of particular interest here.
A classical study on the effect of temperature and thermal history on glass was conducted by Van Zee and Noritake [67]. In these experiments, the optical retardation in glass was measured while the specimen was subjected to a constant bending moment, and in an oven for a range of temperatures. They found that the photoelastic coefficient, $C$, increased gradually with temperature. These findings have been substantiated by additional experiments, usually with container and plate glasses. An investigation to find experiments of this type on optical fiber is ongoing.

Manns and Bruckner conducted experiments in the 1980’s, which studied the behavior of glass by varying the temperature and loading rates [68]. As seen in Figure 3.7, the value of $C$ increases gradually with temperature, and then increases at a greater rate above $T_g$. $T_g$ is the glass transition temperature which defines a change in material properties from supercooled liquids to a glass state. It relates to the ability of molecules
to move past each other within the material, and is marked by a discontinuity of density with temperature (second order transition). In these experiments, the samples were loaded in uniaxial compression, and the loading times varied from 50 ms to 500 s. To measure the dynamic birefringence, an automatic polariscope was used. The photoelastic constant of all the glasses increased similarly and rather linearly from room temperature up to $T_g$. The actual coefficients varied from $1 \times 10^{-4}$ TPa$^{-1}$ for silica to $3.5 \times 10^{-4}$ TPa$^{-1}$ for one of the more common container glasses. Within the transformation range the temperature dependence increased, and the value of $C$ increased with time under load. This is shown in Figure 3.7 for a container glass under various loading rates.

Manns and Bruckner interpreted their results as the superposition of an effect of instantaneous elasticity and an effect of delayed elasticity. This refers to the phenomena where the characteristic time decreases (as with viscosity) when the temperature increases.

Van Zee and Noritake in one case and Manns and Bruckner in the other case found conflicting results when taking loading time into account when determining the photoelastic constant. Part of the difference in their results may be due to the method of bending the first experiments were done in bending, the latter with uniaxial compression. As seen in Figure 3.8, the instantaneous elasticity would be additive when the load is compressive and subtractive when the load is tensile. The theory of photoelasticity would then dictate that the bridging oxygen concentration is the key factor in the atomic effect. Consequently, applying a prolonged tension to the glass specimen in the transformation range would reduce the photoelastic effect by breaking the Si-O bonds. In
the case of prolonged compression, the opposite would occur, increasing the bridging oxygen to non-bridging oxygen ratio. The temperature effect can be directly attributed to

![Figure 3.8 Delayed Retardation in Plate Glass](image)

Figure 3.8 Delayed Retardation in Plate Glass a) Under load b) Load just released c) 15 minutes after load release

a density change in the glass, and a commensurate change in index of refraction, and a lattice-effect on the glass. Pindera and Sinha, and Fontana obtained some results as to the effect of thermal history on the photoelastic effect [69, 70]. Thermal history’s effect on photoelasticity can be observed through tests using annealed and unannealed glass. Figure 3.9 shows the photoelastic constant for annealed and unannealed soda-lime glass.
The data show that the unannealed glass has a higher photoelastic coefficient than the annealed glass when the temperature is below the transition temperature. The increased density of annealed glass, and the related lattice effect explain this. In a related set of studies, photoelasticity was used to study the relaxation of glass by bending it at a constant rate at various constant temperatures. The experimental results showed that the birefringence of the glass at constant temperature is proportional to the stress, even in the viscoelastic range. After using the initial birefringence at any given temperature as a normalizing factor, the data of the birefringence vs. time at each constant temperature mimics the stress relaxation curves expected from the glass viscosity. This is seen in Figure 3.10.

The effect of temperature on photoelasticity on FBGs is manifested in two ways. First, high temperature operation of the FBGs can affect the photoelastic constant in a
significant manner. This means any model, analysis, or experimental work at these temperatures would have to take a varying level of photoelastic constant. The other effect of high temperature on FBGs is the effect on the Bragg structure itself. As stated earlier, the Bragg is manufactured by raising the index of refraction of the core in a modulated fashion. These areas of higher index of refraction were formed by creating GeO and Si-OH. At high temperatures (as low as 300°C) these areas of higher index of refraction diffuse, and the Bragg structure is destroyed. In addition to these effects, it’s also important to remember that polymer coating used to protect the fiber can begin to degrade at temperatures as low as 100°C.

Figure 3.10 Relaxation of (normalized) birefringence of soda-lime glass

3.5 Photoelastic Constant Dependency on Wavelength of Transmitted Light

Many measurements of the photoelastic coefficient of glass as a function of wavelength have been made. In general, the photoelastic coefficient decreases as the
wavelength of transmitted light increases. Coker and Filon expressed the dependency of the photoelastic constant $C$ on wavelength as [57]:

$$C = C_0/1-\left(\lambda_0/\lambda\right)$$

(3.10)

where $C_0$ is usually defined as 3 TPa$^{-1}$, and $\lambda_0=500$nm. Other experiments have been conducted focused on defining change in photoelastic constant with a change in index of refraction. Berezina and Golynja conducted such measurements by measuring the dispersion of the photoelastic constant for different glasses in the visible and near infrared. They determined through their experiments a relationship between photoelastic constant and index of refraction to be [71]:

$$\Delta C/\Delta \lambda = A \left(\Delta n/\Delta \lambda\right)$$

(3.11)

where $A$ is a glass material dependent constant varying between 4 and 27 TPa$^{-1}$. Pindera and Sinha did similar experiments, specifically, between 400nm and 1000nm. Between those wavelengths they detected a variation of the photoelastic constant up to 10%. In another study, similar experiments were carried out by Hoffmann, and resulted in a different empirical relationship [72]:

$$C(\lambda) = B(n^2(\lambda)-1/2n(\lambda))$$

(3.12)

Where the constant $B$ is dependent on the glass tested.
3.6 Residual and Microstructure Anisotropy Effects on Photoelasticity

Anomalous birefringence or birefringence that is uniform and not compensated by birefringence of opposite sign was first discovered by Filon and Harris [73]. Basically, birefringence was found in glass specimens when heated past transition, then cooled under a compressive load. This leads to the concept of birefringence through residual stress.

Other experiments outlined a test where a glass rod was subjected to a tensile load while being cooled to solidification [74]. After the tension was released, high birefringence was observed at room temperature. The glass in these particular experiments was 8.5% B₂O₃ and 12% alkaline earth oxides, and the birefringence reached a value of 6 E-4. This value of C had the same sign as that obtained when the rod was in axial compression at room temperature.

Residual birefringences of opposite sign, in contrast, were obtained by Stirling during tests of soda-lime and Pyrex glass samples. In these experiments, the key feature was the observation that birefringences had a direct relationship with delayed elasticity. Delayed elasticity in these type of glass samples is due to the migration of alkaline ions. When the flowing glass is unidirectionally stressed, these ions diffuse and take up sites of minimum energy in the elastically deformed vitreous network.

After the glass is cooled and unloaded, the diffused ions remain trapped, unable to move back to their normal position in the material. In this way, the glass is unable to return to its normal equilibrium state. The sign of this now permanent birefringence is the same that would occur if the specimen were under load since it is now mainly due to
the polarization of the bridging oxygens. This process is known as birefringence due to frozen-in strain.

Another interesting example of an anomalous birefringence can be observed when looking at glass compounds containing boron. Botvinkin and Ananich experiments dealing with glass containing boron were done by subjecting rods of varying composition to thermal treatment after creating an anomalous birefringence [75]. The birefringence was then monitored during heating and cooling. The data showed differing behavior of various rods based on their composition. In the case of B₂O₃ or of zinc borate glasses (35% B₂O₃ and 65% ZnO), a decrease in birefringence was noted with heating and very little change due to cooling.

In the case of glasses of sodium borosilicate, some of the birefringence lost in heating was regained during cooling. The experimenters theorized that for the zinc borate glasses, the elongation of the rod produced an anisotropy similar to that found in some organic polymers cooled down under strain. This birefringence can be eliminated by thermal treatment. The best example of this phenomenon is phosphate glass, which is comprised of naturally occurring chains of PO₄ tetrahedra.

In the case of the borosilicate glasses, something different occurs. In borosilicates, there is often, under heating, a separation of phases, one rich in silica, and another rich in B₂O₃. Researchers theorize that the stretching of the glass causes a preferential orientation containing these separated phases, causing nonisotropic microstresses during cooling. After heating again, the microstresses are relieved by the softening of the B₂O₃ phase, reducing the macroscopic birefringence, but the structural anisotropy is not completely cancelled out – meaning at least some of the birefringence
returns upon cooling. The above concepts of birefringence can be split into two categories:

a. Frozen-in strain caused birefringence (which is annealable, and directly related to photoelasticity)

and

b. Microstructural birefringence (which is not fully annealable, and not directly related to photoelasticity.)

Photoelasticity relates stress & dimensional changes in glass to the propagation of light within it. As such, it is a crucial concept if the properties of light propagation in glass are to be studied. While loading conditions, stress and strain, are very dominant factors as to the level of photoelasticity in glass, other factors, most notably the chemical makeup of the glass, have an effect on the photoelastic constant. In addition, temperature and thermal loading effects, annealing, and heat treating have an effect on the photoelastic constant, as residual stresses and the melting and freezing of the vitreous glass material have an effect on the photoelastic constant. The presence of photoelasticity in optical fiber is far from trivial in many sensor applications, and is relevant to any discussion of the effect of loads and stress to the optical properties of glass.

The concept of photoelasticity is critical to the performance of a FBG sensor, and is related in the analysis in later chapters. The key concept is the direct relationship of stress in silica glass and a changing of index of refraction. The stress within an optical fiber core, resulting form loading, changes its index of refraction. Caused by lattice
effects (spacing of molecules) and by electronic effects (biasing of electron clouds) stress causes a change in the way light propagates through optical fiber. In the case of FBGs this is significant. As stress is induced in the fiber core, the FBG response, directly related to the Bragg signal, will change. The interaction of the light with each line in the grating will vary if those lines are not homogeneous in index of refraction. Therefore, photoelastic effects are the tool used for examining changes in the FBG signal with side loading.
Chapter 4 Theoretical Analysis of Bragg Gratings under Transverse Load

4.1 Introduction

A theoretical analysis was performed to determine the effect of side loading on the signal of a FBG. The analysis is comprised of two major phases, first using appropriate descriptive equations to describe the state of stress and change in index of refraction within an optical fiber under side loads, and second, a finite element analysis with the same aim. These analyses serve to provide a qualitative and quantitative model of the change in material properties within the optical fiber core, to the resulting predicted change in the FBG signal.

The purpose of this analysis is to relate the external conditions present to the optical fiber sensor, and the changes within the sensor structure and the FBG signal. This is an important study in considering the application of such devices [17, 76, 77]. In some cases, the embedment of FBGs present a stress concentration into the material being measured. This means that the strain measurement is within this stress concentration and may not match the far-field conditions. A number of factors play into the relationship of the measured strain within a FBG and the strain condition away from the fiber including interface characteristics, fiber placement, loading condition, material properties of the host vs. the FBG, coatings used on the FBG, and in the case of fibrous composites, the size of the FBG vs. the composite fiber diameters, etc..

The first step in analyzing such a complex condition, which comprises the scope of this thesis, is to study the foundational case of loading the optical fiber from a transverse direction, and examining the effect on the FBG signal. These studies can then be applied to an individual application, where a more complex stress and strain analysis
can provide an “input” or loading condition to the fiber itself, and a prediction of the FBG signal can be made. The method of analysis that follows postulates loading an optical fiber, determining the longitudinal and transverse stress that is induced, and then calculating the change in index of refraction that results through photoelastic effects. With knowledge of the change in index of refraction and stresses, a prediction can be made as to the effect on a FBG signal.

4.2 Diametric Load Case, Analytical

The simplest case to begin the analysis of the FBG is that of a diametrically loaded optical fiber. The fiber in this analysis has no coating or jacket and is comprised only of silica. In this first analysis a circular cross section of optical fiber is subjected to loading on two sides (Figure 4.1). This contact can be defined by the Hertz solutions for stress states in disks and spheres under diametrical compression under point loading [78, 79]. The Hertz approximation assumes very small strains and shape change, point loading, and frictionless contact. These assumptions are appropriate for a high Young’s modulus material like silica optical fiber (E=69 GPa). Hertz’s approximations were first formulated for brittle high modulus materials. Plane strain is also assumed.
Hertz formulated the normal stresses within the disk (Figure 4.1) to be:

\[
\sigma_x = \frac{-2P}{\pi t} \left\{ \frac{x^2(R-y) + x^2(R+y)}{\beta_1^4} + \frac{1}{2R} \right\} \tag{4.1}
\]

\[
\sigma_y = \frac{-2P}{\pi t} \left\{ \frac{(R-y)^3}{\beta_1^4} + \frac{(R+y)^3}{\beta_2^4} - \frac{1}{2R} \right\} \tag{4.2}
\]

where:

P is the load (diametric point load per unit thickness), R is the radius, t is the thickness, and \( \beta_1^2 = (R-y)^2 + x^2 \), \( \beta_2^2 = (R+y)^2 + x^2 \). The Hertz solution predicts that maximum stresses will occur in the center of the disk, and that stress is tensile along the x axis. The solution of these equations give the normal stresses in the x and y direction for any location \((x,y)\) within the optical fiber cross section.
Using the stress at these locations, a map of stress for both $\sigma_x$ and $\sigma_y$ can be created for the fiber core. If these values are combined with relation for photoelasticity, which directly relates the state of stress to the change in index of refraction, then a map of index of refraction can be created for the optical fiber core. Photoelasticity relates the change in index of refraction to the stresses in the fiber by:

$$
(\Delta n_{\text{eff}})_x (x, y, z) = -\frac{(n_{\text{eff} 0})^3}{2E} \times \left\{ \frac{(p_{11} - 2\nu p_{12})\sigma_x(x, y, z) + [(1-\nu)p_{12} - \nu p_{11}]}{\sigma_x(x, y, z) + \sigma_y(x, y, z)} \right\} \tag{4.3}
$$

$$
(\Delta n_{\text{eff}})_y (x, y, z) = -\frac{(n_{\text{eff} 0})^3}{2E} \times \left\{ \frac{(p_{11} - 2\nu p_{12})\sigma_y(x, y, z) + [(1-\nu)p_{12} - \nu p_{11}]}{\sigma_x(x, y, z) + \sigma_y(x, y, z)} \right\} \tag{4.4}
$$

where $n_{\text{eff}}$ is the effective index of refraction, $E$ is the Young’s modulus, $\nu$ is the Poisson’s ratio, $p$ is the photoelastic constant, and $\sigma_{x,y,z}$ are normal stresses. The above analysis was completed for the simplest case, a cylindrical uncoated silica fiber with a circular cross section. Equations 4.1 and 4.2 were solved using computer software to solve for stresses of a fiber under diametrical load for all locations within the fiber cross section. Equations 4.3 and 4.4 were then solved to calculate the change in index of refraction resulting from the stresses within the fiber cross section.
Figure 4.2 Normal stress in optical fiber under diametrical loading, x-direction ($\sigma_{xx}$)
Figure 4.3 Normal stress in optical fiber under diametrical loading, y-direction ($\sigma_{yy}$)
Figure 4.4 Index of refraction change under diametric loading, x-direction ($\Delta n_x$)
Equations 4.1-4.4 were solved using a very small nominal load (6E-4g, small due to very small force application area) and the following values [40]:

- $E_{\text{fiber}}$: 69 GPa
- $n_0 = 1.45$
- $\nu = 0.29$
- $p_{11} = 0.121$
- $p_{12} = 0.270$
Fiber diameter = 125 microns

Figures 4.2 to 4.5 show graphically the effect of loading on the side of an optical fiber. At the points of load application there are concentrated areas of stress seen. With the Hertz approximation, this is expected since the area of load application is so small. With the loading seen in Figure 4.1, we expect and see compressive force concentrations at the point of application. Stress contours are seen, creating a compressive stress along the y-axis, and a tensile stress along the x-axis. Stresses in the fiber (with the 6E-4g applied force) ranged from -60 to +60 kPa for the various component level stresses. By graphing equations 4.3 and 4.4 the effect on index of refraction is seen for both x and y components. Index of refraction changes up to 0.002 were seen in this case, with a \( \Delta n_x - \Delta n_y \) in the center region of the fiber (which would correspond to the area in which a FBG would operate) of approximately 0.0016. Changes at the points of application were far greater, but the small change in the center core-like region is a rather high one from an optical standpoint with such a small force, due to the very small force application area. These data show the essence of the bifurcation of the FBG signal under transverse loading. Figure 4.4 and Figure 4.5 show the directional dependence of index of refraction possible, and therefore, the possible splitting of the FBG signal. In other words, orthogonal polarizations of light would experience different indices of refraction, and therefore, produce a different Bragg reflection upon loading. In addition, as the load increases, the difference between the Bragg wavelengths would increase as well.

The Bragg wavelength for two such polarizations can be described in the general case by [40]:
\[ (\lambda_y)_{x} = -\frac{(n_{\text{eff}})^3 \Lambda_{B,0}}{E} \times \{(p_{11} - 2\nu p_{12})\sigma_{x} + [(1 - \nu)p_{12} - \nu p_{11}] \times [\sigma_{y} + \sigma_{z}]\} \]

\[ + 2 \frac{n_{\text{eff},0} \Lambda_{B,0}}{E} \times \{\sigma_{z} - \nu[\sigma_{x} + \sigma_{y}]\} \]

(4.5)

\[ (\lambda_{B})_{y} = -\frac{(n_{\text{eff}})^3 \Lambda_{B,0}}{E} \times \{(p_{11} - 2\nu p_{12})\sigma_{y} + [(1 - \nu)p_{12} - \nu p_{11}] \times [\sigma_{x} + \sigma_{z}]\} \]

\[ + 2 \frac{n_{\text{eff},0} \Lambda_{B,0}}{E} \times \{\sigma_{z} - \nu[\sigma_{x} + \sigma_{y}]\} \]

(4.6)

So for two orthogonal polarizations transmitted along the x and y axis of the fiber, different indices of refraction would be experienced and changes in individual FBG signals would be seen. The last term in each equation is identical, representing effects of longitudinal strain. The key feature is that under loading, given the stress condition, photoelastic constants, and original Bragg wavelength and indices of refraction, changes in the FBG signal can be calculated. Lastly, it is important to note that for any theoretical fiber analyzed, the actual light path is in a small core approximately 10 microns in diameter, as described in Chapter 2.

4.3 Distributed Load Case, Analytical

The next step in the analysis was to consider a more realistic case of simple transverse loading of an optical fiber. A simulation was created for the case of a distributed diametric transverse load on the fiber (Figure 4.6). Again, this is an analysis
with no jacketing or coating on the fiber. It is the case to best represent loading the fiber between two rigid plates (Figure 4.7).

![Figure 4.6 Distributed diametric transverse loading of optical fiber](image)

This analysis is essentially an extension of the Hertz analysis, assuming small strains, but the load is distributed rather than a point load. This extension of Hertz was pioneered by Hondros [80], and has been repeated and verified extensively. Hondros formulated the normal stresses within the fiber cross section to be (in \( r, \theta \) coordinates):
\[ \sigma_r = -\frac{2P}{\pi} \{ \alpha + \sum_{n=1}^{\infty} \left( 1 - \frac{1}{n} \left( \frac{r}{R} \right)^2 \right) \left( \frac{r}{R} \right)^{2n-2} \sin(2n\alpha)\cos(2n\theta) \} \] (4.7)

\[ \sigma_\theta = -\frac{2P}{\pi} \{ \alpha - \sum_{n=1}^{\infty} \left( 1 - \frac{1}{n} \left( \frac{r}{R} \right)^2 \right) \left( \frac{r}{R} \right)^{2n-2} \sin(2n\alpha)\cos(2n\theta) \} \] (4.8)

Where \( P \) is the pressure. In this way the normal stresses can be calculated for various angles of load application. For very small values of \( \alpha \), the Hondros solution collapses to the Hertz formulation presented above.

![Figure 4.7 Transverse loading of FBG by parallel plates](image)
The Hondros equations were solved in the same manner as the Hertz equations above. Equations 4.7 and 4.8 were solved with a $\alpha$ of 30 degrees to provide the largest realistic deviation from the Hertz condition. A nominal load of 0.5 kg was used as an input load, and the other initial conditions were identical to the Hertz solution above. In addition, the radial components had to be translated to Cartesian coordinates so that the data could be used in the relations for index of refraction change and change in FBG signal.

Figure 4.8 Normal stress in optical fiber under distributed loading, x-direction ($\sigma_{xx}$)
Figure 4.9 Normal stress in optical fiber under distributed loading, y-direction (σ_{yy})

As with the Hertz solutions presented above, the stress values calculated above (ranging from -60 to +60 kPa) were transformed into the change in index of refraction through
equations 4.3 and 4.4. A visualization of this data is shown in Figure 4.10 and Figure 4.11.

Figure 4.10  Index of refraction change under distributed loading, x-direction ($\Delta n_x$)
In this data, some differences are seen compared to the data in the Hertz point loading examples. The stress concentrations are more dispersed, to reflect the distributed nature of the loading. This has a direct result on the index of refraction calculations. Index of refraction changes in the highly stressed areas are not as high as that of the Hertz formulation. As in the case with point loading, changes in Bragg wavelength can also be
calculated from equations 4.5 and 4.6. Again, in this case, applied load is related directly to the change in Bragg wavelength.

In the Hondros case, an applied force of as little as 0.5kg was enough to increase or decrease (at the areas of maximum stress) the index of refraction 0.002, and a $\Delta n_x - \Delta n_y$ in the center region of the fiber of approximately 0.0016. This value may appear small, but a change of that magnitude can cause a resultant change (based on equations 4.5 and 4.6) in Bragg wavelength of up to 6nm, and a separation of split peaks of up to 3 nm – a value considered rather high from previous experimental data and the literature. These values are based on the use of the simplified fundamental equations (assuming frictionless loading, small strains, etc.), and only applicable to a one part system (silica fiber only), ignoring the polymer coating that is used on all optical fiber sensors. This would contribute to an elevated value of index of refraction and Bragg wavelength shift.

4.4 Embedded Fiber Case, Analytical

In the final analytical case, FBGs were analyzed while embedded in a simple epoxy host. As stated earlier, the conditions involved in embedding or affixing a FBG to any structure can vary widely. The use of a myriad of host materials, adhesives or coatings as well as using the exact boundaries and interface conditions between fiber and host makes a generic solution virtually impossible. However – an examination of simplified conditions surrounding a hypothetical embedded sensor does yield useful results in showing characteristics of such an application.

Analytically, the problem described above is that of describing the stress and strain within and outside an elliptical inclusion in a solid. Much work has been done on attempting to represent this problem analytically. A good deal of work has been done by
numerous researchers working with the so-called Eshelby relations [41, 81-93]. This work approaches the problem by calculating a “stress-free strain” of the inclusion, and then back-calculates the stresses from the strain. The basis of all modern work was performed by Eshelby [41, 81] in the late 1950’s. Eshelby constructed a set of tensor relation to try and relate the stresses in and around a spherical inclusion embedded in a host material. His work was groundbreaking in its attempt, but was difficult to use as it only worked with an infinite matrix. Later work by Mura and Hasegawa and others refined Eshelby’s approach, but continued to work with infinite matrices and concentrated more on the stresses in the matrix rather than the inclusion [83, 84, 86, 88, 91, 92].

The challenge of these formulations is that they do not provide a solution where the inclusion is embedded in a finite solid, but an infinite space. In addition, closed form solutions are very difficult to process from the Eshelby relations. Researchers at University of California at Berkeley (Dr. Shofan Li, Dr. Roger Sauer) are continuing a year long effort on a finite solution for an elliptical inclusion within a circular solid. More applicable to the case presented here are the relations used by Matthews and Sirkis [94], based on the stress equations developed by Savin [95]. Savin developed equations to describe a large number of situations, including the stress inside and outside of a circular inclusion in a solid. In a sense, a portion of this treatment resembles Eshelby in that the stress is calculated around a hole and then used as an input of stress in the inclusion. Savin’s solutions [94, 95] (in polar coordinates) for the normal stresses in a rectangular host material and an embedded circular inclusion (Figure 4.12) are:
for embedded fiber,

\[
\sigma_{rr}^f = \frac{\sigma_0}{2} \left( a + \frac{1}{2} b \cos(2\theta) \right) \\
\sigma_{\theta\theta}^f = \frac{\sigma_0}{2} \left( a - \frac{1}{2} b \cos(2\theta) \right)
\]  \hspace{1cm} (4.9)

for host,

\[
\sigma_{rr}^h = \frac{\sigma_0}{2} \left( \left(1 - \frac{1}{2} \beta \zeta^{-2}\right) + \left(1 - 2\alpha \zeta^{-2}\right) \cos(2\theta) \right) \\
\sigma_{\theta\theta}^h = \frac{\sigma_0}{2} \left( \left(1 + \frac{1}{2} \beta \zeta^{-2}\right) - \cos(2\theta) \right)
\]  \hspace{1cm} (4.10)

where:

\(\sigma_0\) = Input Stress

\(a = -(1+\chi_h)/(G_h/G_f-1)-(1+\chi_f G_h/G_f)\)

\(b = 2(1+\chi_h)/(\chi_h G_h/G_f)\)

\(\chi = (3-\nu)/(1+\nu)\)

\(G_{h,f}\) = Shear Modulus (host/fiber)

\(\beta = 2+\{2(1+\chi_h)\}/\{(G_h/G_f-1)-(1+\chi_f G_h/G_f)\}\)

\(\zeta = r/R\)

\(r = \text{radius within fiber}\)

\(R = \text{radius of the fiber inclusion}\)

\(\theta = \text{angle within fiber}\)

\(\alpha = 2\{-2(1+\chi_h)/(\chi_h G_h/G_f)\}\)
These relations were used to calculate normal stresses with a FBG embedded in a block with square cross section (Figure 4.12), and an input loading commensurate with data from sections 4.2 and 4.3. The solutions to equations 4.9 and 4.10, converted to Cartesian coordinates are shown in Figure 4.13 and Figure 4.14. The red circle indicates the outer diameter of the fiber. The host material is shown in the immediate area of the fiber.

![Figure 4.12 Loading of FBG embedded in epoxy host](image)

The block is assumed to be epoxy with a 10GPa Young’s modulus, and a Poisson ratio of 0.3, and the fiber properties are the same as listed before. Stresses ranging from
13 MPa to 0.6 MPa were seen in the fiber in this analysis with an input loading of 0.5 kgf/mm along the top of the block.

In this analysis one can see stress concentrations along the x and y axis corresponding to the principle loading axes, both compression and tension. The plots also show the stress in the host material outside of the fiber inclusion. Examining the shape of the responses in the x and y direction, it appears the relations for the inter-fiber stresses may be too symmetrical, but do show stress concentrations along the main compressive and tensile force axes. The Eshelby relations held the same, assuming an infinite host. The relationship of the size of the fiber inclusion and the host would effect the stress distribution if the fiber size were increased to be significant (if diameter were greater than 1/10 the size of a square host cross section for example) compared to the host.
Figure 4.13 Normal stress in x-direction ($\sigma_x$), embedded fiber
Figure 4.14 Normal stress in y-direction ($\sigma_y$), embedded fiber
Chapter 5  Finite Element Analysis of Bragg Gratings Under Transverse Load

5.1 Introduction

After completing the analytical analysis of the effect of transverse loading on the material properties, structural geometry, and projected FBG signal, a finite element analysis was performed. The finite element analyses were created in order to confirm the results of the analytical work, and to allow an easy-to-use tool to vary conditions of the FBG system and analyze more complicated geometries. In addition, this analysis will provide the information needed for better understanding of the FBG signal behavior and changes under multi-axis loading conditions. All finite element analyses were performed using ABAQUS finite element software.

Similar to the procedure in the above analytical sections, the finite element analysis software was used to create the appropriate model, and examine theoretically the effect on FBG signals due to externally applied transverse loading. As with the analytical work, three types of analyses were done, diametric point loading, distributed diametric loading (loading between two rigid plates) and an analysis of an embedded FBG in an epoxy host material. The analysis is a 2D analysis applied to cross sections of silica fiber. The goal is the same as it was in the analytical study – to observe the stresses, change in Bragg periodicity and commensurate change in index of refraction when loading FBGs transversely. In the finite element analysis, stresses and strains were calculated for the fiber under diametric point loading, and for the distributed loading case and the embedded case, changes in index of refraction and FBG signal were calculated. In addition, with the flexibility of the FEA program, the more realistic condition of a
FBG sensor of silica core and cladding and polymer coating was considered for the above data. In order to consider the change in FBG periodicity, plane stress had to be calculated as well as plane strain. Plane stress would approximate the conditions in the future lab test where the ends of the fiber were not fixed. Plane strain approximates more closely the condition of an embedded fiber, when the fiber is more restricted.

5.2 Diametric Load Case, Finite Element Analysis

In the first case, diametric point loading was assumed as described in Figure 4.1. The material properties of the fiber remained the same as the analytical case, E=69GPa and n=0.29, and the loading conditions remained similar. The fiber in the first case was assumed monolithic and silica only, but matched the outer diameter of a typical core/cladding system of 125 microns. The conditions were changed slightly to allow for a frictionless bottom plate to provide loading from the bottom, and to compare in one analysis (and one more practically applicable) the difference between point and distributed loading. The analysis was completed with tetragonal elements. The results of this loading case are shown in Figure 5.1, Figure 5.2, and Figure 5.3. Plots are shown for $\sigma_{xx}$ (Figure 5.1), $\sigma_{yy}$ (Figure 5.2), and also the Mises stress (Figure 5.3) to show the general condition of stress in the fiber. The use of very small tetragonal elements causes aliasing in some of the plots showing what appear to be large elements on the order of 10% of the fiber diameter – this is an artifact of the graphical display.

What is shown in Figure 5.1 - Figure 5.3 confirms information from the analytical work. A stress concentration is seen at the points of loading. At the points of loading a
compressive force is encountered, and along the centerline in the x-direction a tensile stress is seen. The data are largely symmetric, but a difference can be seen between the point loading on top of the fiber and the distributed nature of the force on the bottom of the fiber against the frictionless plate. This is seen most clearly in Figure 5.1, where the

![Figure 5.1 Normal stress in optical fiber under diametric loading, x-direction ($\sigma_{xx}$), via FEA](image)
Figure 5.2 Normal stress in optical fiber under diametric loading, y-direction ($\sigma_{yy}$), via FEA

Figure 5.3 Mises stress in optical fiber under diametric loading, via FEA
stress distribution on the bottom of the fiber is clearly more distributed. As discussed later, this difference, when confined to the plane of light transmission along the y-axis may not have a strong effect on the FBG signal. As can be seen from the data, stresses in this theoretical case can reach MPa levels with a loading of 100gf.

After analyzing the case of loading on a silica fiber, the above analysis was repeated for a case closer to the actual FBG application, which is a silica fiber with a polymer (polyimide type) coating. In this case a polymer coating is modeled with a thickness equal to the radius of the fiber, as found in most commercial fiber. As stated earlier, FBGs are usually manufactured in single mode optical fiber with a core/cladding outer diameter of 125 microns. The coating applied usually increases the outer diameter of the fiber to 250 microns. The analyses were repeated with the same conditions as above, and the results, $\sigma_{xx}$ and $\sigma_{yy}$ are shown below in Figure 5.4 and Figure 5.5. What can be readily seen is that the more compliant polymer coating (E=5GPa vs. silica fiber E=69GPa) experiences the most notable stress. In other words, the polymer layer appears to “cushion” the silica fiber from the transverse loading. A closer look at the fiber core reveals stress contours, but representing much smaller stress levels than the uncoated fiber. Figure 5.6 and Figure 5.7 show a detailed view of the fiber core and cladding inside the polymer coating, and Figure 5.8 shows a Mises stress distribution highlighting the core, but showing the whole fiber. In these plots one can determine the predictable result that the fiber is in compression in the y-axis, and tension in the x-axis. The coating in addition to reducing the level of stress in the fiber, also slightly distributes the forces, it appears, to a greater surface of the fiber.
Figure 5.4 Normal stress in coated optical fiber under diametric loading, x-direction ($\sigma_{xx}$), via FEA
Figure 5.5 Normal stress in coated optical fiber under diametric loading, y-direction ($\sigma_{yy}$), via FEA

Figure 5.6 Normal stress in coated optical fiber under diametric loading ($\sigma_{xx}$), center detail
Figure 5.7 Normal stress in coated optical fiber under diametric loading ($\sigma_{yy}$), center detail
5.3 Distributed Load Case, Finite Element Analysis

After performing the analysis of point loading of fiber on a plate, the analysis was extended to a fiber under compression from two rigid, frictionless plates. As with the analysis in section 5.2, modeling was done for both uncoated and coated fibers to observe the differences between them. This loading case is particularly important as the results of plate loading are addressed in the experimental verification work (see Chapter 6). It also provides information on an easily constructed, and therefore generic, configuration with very little variation.

The first set of data, seen in Figure 5.9, Figure 5.10, and Figure 5.11 shows an uncoated fiber transversely loaded by frictionless plates. The lower plate was a rigid fixed support, and the load was applied through the top plate in a unidirectional
downward force. Figure 5.9 and Figure 5.10 show normal stresses in the x-direction and the y-direction. Figure 5.11 shows the Mises state of stress for the fiber. The stress plots match well to those seen in sections 4.2 and 4.3. In these cases the mesh used for the analyses is shown. The profile of the x-directional and y-directional stress shows compression in the y-direction, and tension in the x-direction. The deformation is exaggerated in these plots to show the direction of compression and tension. Once again, loading of 100 g provided stress levels in the MPa range. As was seen in the previous section, stress concentrations are seen at the points of loading. It is seen (as with the lower plate of the previous section) that the stress concentration at the loading plate is more distributed than the point loading location. The next set of data, seen in Figure 5.12 and Figure 5.13 is a similar setup, but shows the normal stresses in the x and y-direction of a coated fiber. Figure 5.14, Figure 5.15, and Figure 5.16 show a silica fiber detail of the x and y directional normal stresses and the Mises stress respectively. The results from a stress perspective showed a couple of key features. First, that the level of stress in the core of the fiber was diminished rather significantly by the polymer coating. Also, with a completed quantitative analysis relating transverse loading to the change in index of refraction, change in index of refraction, and projected change in FBG signal could be performed. This data is presented in section 5.5.
Figure 5.9 Normal stress in optical fiber under distributed loading, x-direction ($\sigma_{xx}$), via FEA

Figure 5.10 Normal stress in optical fiber under distributed loading, y-direction ($\sigma_{yy}$), via FEA
Figure 5.11 Mises stress in optical fiber under distributed loading, via FEA

Figure 5.12 Normal stress in coated optical fiber under distributed loading, x-direction ($\sigma_{xx}$), via FEA
Figure 5.13 Normal stress in coated optical fiber under distributed loading, y-direction ($\sigma_{yy}$), via FEA

Figure 5.14 Normal stress in coated optical fiber under distributed loading ($\sigma_{xx}$), center detail
Figure 5.15 Normal stress in coated optical fiber under distributed loading ($\sigma_{yy}$), center detail

Figure 5.16 Mises stress in coated optical fiber under distributed loading, center detail
5.4 Embedded Fiber Case, Finite Element Analysis

The final analysis case was a FBG fiber (polymer coated) embedded in a simple block with square cross section, under compression, as described in section 4.4 and in Figure 4.12. In this case an analysis was done using symmetry and ¼ of the total block geometry. As seen in Figure 5.17 the fiber is embedded in the center of the block. Symmetry of the problem allowed an analysis on the upper right hand quadrant of the block to reduce computer computation time. The forces on the top and the bottom of the block are frictionless. In order to avoid any interaction between the shape of the inclusion-induced stress and the edge of the block, the actual dimensions of the block are 20 fiber diameters in the x and y direction. The results displayed in this chapter are represented roughly by the green area in Figure 5.17, a subset of the block, to show the fiber and coating in detail. The embedded fiber case was done assuming standard coated communications type fiber with a total diameter of 250 microns. The Young’s modulus was set at 69GPa for the silica fiber (outer diameter 125 microns) and 5GPa for the polymer coating. The epoxy block host has a Young’s modulus of 10GPa. The input force was a distributed force of 10kgf/cm.

In Figure 5.18 and Figure 5.19 the normal stresses are shown, x-directional and y-directional. In Figure 5.20 the Mises stress is shown. A closer, detailed view of the silica fiber is seen for the x and y-directional normal stresses in Figure 5.21 and Figure 5.22.
Figure 5.17 FEA of embedded FBG showing 1/4 analysis and display zone

Figure 5.18 Normal stress in embedded FBG fiber under compressive loading, \( (\sigma_{xx}) \), via FEA
Figure 5.19 Normal stress in embedded FBG fiber under compressive loading, \((\sigma_{yy})\), via FEA

Figure 5.20 Mises stress in embedded FBG fiber under compressive loading, via FEA
Figure 5.21 Normal stress in embedded FBG fiber under compressive loading ($\sigma_{xx}$), center detail

Figure 5.22 Normal stress in embedded FBG fiber under compressive loading ($\sigma_{yy}$), center detail
Figure 5.20 shows a good overview of the coated fiber embedded in the matrix. Stress concentrations can be seen where the block meeting the coating, and where the coating meets the silica fiber cladding. A closer look at the stress in the coating is seen in Figure 5.18 and Figure 5.19. As seen before under plate loading, the overall stress picture emphasizes large levels of normal stress around the fiber coating, but much smaller changes in the silica fiber itself. When taking a closer look at the silica fiber in Figure 5.21 and Figure 5.22 stress contours are seen within the fiber that are different than the simple plate loading. In this case, the stress contours resemble a linear gradient of stress. As seen in the case of the coated plate loaded case, the gradient of stress across the silica fiber is very low, when compared to the maximum stresses seen elsewhere in the system.

This data show the level of complexity in embedding an optical fiber sensor and predicting its behavior, particularly in assumptions made regarding the interface between the fiber coating (or any additional jacketing that may be present) and the host material. Analyses of this type need to be tailored to the particular application of interest, as it appears the difference in material properties, adhesion characteristics, wetting properties, presence of voids etc. may pose significant effects on the model.

For embedded fiber uncoated fiber will provide a more sensitive measurement. The strain in the host material will be transferred to the fiber directly without a more compliant intermediary such as the polymer jacket. This also presents an opportunity to tune the sensitivity of the FBG sensor. The polymer jacket could be designed to respond
with a certain dynamic range based on its properties. Its thickness, Young’s modulus, and other properties could be designed with consideration of the host to provide a given amount of sensitivity and durability. This design would also provide for an optimum interface with the host material. In most applications – removing the jacket completely for maximum sensitivity is a significant problem as the silica cladding will be exposed to the host, and could be damaged in manufacturing, or could absorb moisture and render the fiber more brittle and susceptible to cracking.

5.5 Calculation of Bragg Wavelength & Index of Refraction Changes

Previous sections have outlined the observation of stress levels within a 2D section of FBG under transverse loading in a number of different configurations. The governing equations to convert this stress data into change in index of refraction and FBG signal were also described. Following the FEA cases listed above, the data were collected and processed to calculate the change in index of refraction, and the change in Bragg wavelength for a number of loading cases. This worked towards a final goal of observing the sensitivity of FBGs to transverse loading, and creating a modeling tool to provide such predictions.

The data processed was that of the plate loaded coated fiber case. This case was the case planned for experimental verification due to the simplicity of the system, and the ability to re-create the setup physically with the greatest degree of accuracy. To process the data, an output database of the FEA run was accessed similarly to that seen in Figure 5.14 and Figure 5.15. As stated earlier, the stress contours suggest a maximum and minimum index of refraction along the major axes of the fiber based on maximum compressive and tensile forces. As a result stress data was collected from the output
database in the region that FBG signal light would pass. The stress data was taken for each element along the axis within the silica fiber. Data was taken from FEA elements that corresponded to the fiber core which carries the FBG signal, representing the center 10 microns (core dimension) with the center four elements in each direction (x and y). In order to have the data comparable to the experimental verification, FBG periodicity and Poisson effects were taken into account in the z-direction, and plane stress was assumed. While plane strain may be a more appropriate approximation for an embedded FBG, plane stress data were more comparable to the laboratory experiments, as the ends of the fiber are free.

The data were averaged for each axis, and then the change in index of refraction and FBG signal were calculated as specified in Equations 4.3-4.6. A sample of stress data is seen in Table 1. Since calculations are being made along each axis (for n and $\Delta\lambda_{b,x,y}$), both x-directional and y-directional normal stresses are taken for each, producing four sets of data.

<table>
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<tr>
<th>Axis in Fiber</th>
<th>x axis</th>
<th>y axis</th>
<th>x axis</th>
<th>y axis</th>
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</thead>
<tbody>
<tr>
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<td>S: S22</td>
<td>S: S11</td>
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</tr>
<tr>
<td>Stress values</td>
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<td>5.3273E-03</td>
<td>5.3383E-03</td>
</tr>
<tr>
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<tr>
<td></td>
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<tr>
<td></td>
<td>-7.6234E-03</td>
<td>-7.6245E-03</td>
<td>5.3426E-03</td>
<td>5.3383E-03</td>
</tr>
</tbody>
</table>

Table 1 Example stress data for plate loaded coated fiber

After collecting and averaging data for four loading cases, the stress data was used as an input to Equations 4.3-4.6 to calculate the change in index of refraction and Bragg
wavelength. The first calculation was that of index of refraction. As index of refraction is a tensor with directional dependencies (related to the nature of the electromagnetic permittivity, etc.), both $n_x$ and $n_y$ are calculated at each point along each axis.

An example is seen in, where $n_x$ and $n_y$ are shown calculated from each element form the FEA analysis along the x axis of the fiber. The data show the values of $n_x$ and $n_y$ for loading values up to 4 kgf (per cm of FBG length) on the upper plate. In Figure 5.24 the corresponding change in Bragg wavelength is seen.

The proposed manifestation of this data in the FBG data on an optical spectrum analyzer is seen notionally in Figure 5.25. The signal is shifted to the right with a changing in the FBG periodicity. The Bragg signal has bifurcated into two components due to the change in index of refraction along the two major axes. Also note that where
the compression and tension in the 2D analysis would cause one of the double peaks to decrease in wavelength, the added component of periodicity shift (increase) is significant enough to shift both bifurcated signals to the right. Also note that a very small change in index of refraction along the axes of light propagation (0 to 0.00014) can provide a significant change in the FBG signal (0-3.5nm). This is principally due to the nature of the FBG signal, and significant observances of changes in the fractions of nanometer possible with the wavelength of light used in the sensor.

![Loading vs. Bragg Wavelength Change](image)

**Figure 5.24 Plate loading force vs. Bragg Wavelength change, coated fiber**

To this end, it was decided that the results achieved with the theoretical analysis & modeling were acceptable and complete at this point. A model was constructed to impart a transverse load on the FBG and to propose the change in the sensor’s signal. The model as constructed easily allows for variations in material properties of both the
optical fiber and the polymer coating, and for pure transverse loading, allows for a useful predictive tool.

An important consideration for the manifestation of the dual peak waveform and the peaks’ values as proposed in Figure 5.24 is necessary. The graphic in Figure 5.25 represents a common manifestation of the split peak signal as seen empirically – but does not represent the “ideal case”. In purely theoretical terms, the FBG waveform should be an extremely narrow reflection, not nearly as wide as seen in practice. The reality is that two major factors are responsible for the waveforms appearance in an unloaded FBG – the lack in uniformity of the lines of gratings, and the resolution of the spectrum analyzer used to measure the FBG signal. Transverse loading may cause broadening due to non-uniformity in both $\Delta n$ changes and $\Delta \Lambda$. The key feature is that while signal splits are very small, they may appear simply as signal broadening. As seen in the literature and in the experimental chapter, the peaks will separate as do the imperfections in the spacing between all the lines in the grating. Once the two peaks are sufficiently separated, they become apparent to the optical spectrum analyzer. The exact manifestation of the split peaks may differ with FBG manufacturer and analysis equipment. In addition, the peaks may not be the same amplitude. While the wavelength split is due to a change in index of refraction, the amplitude difference of the peaks (if any) relates to the energy present in each orthogonal polarization axis. The overall energy reflected by the FBG may change as some of the energy is directed into the cladding. The energy of each peak is related to the boundary conditions along each of the polarized modes propagating through the core. If the polarization modes boundary conditions change relative to each other the split peaks will not be of equal amplitude.
Figure 5.25  Generic effect of transverse loading on FBG signal showing dual peak formation and overall shift
Chapter 6 Experimental Analysis of Bragg Gratings under Transverse Load

6.1 Introduction

After analyzing the response of the FBG to transverse loads though analytical means an experimental procedure was done to verify the results. The goal of the experimental work was to develop a test setup with the most suitable conditions to predict the FBG signal bifurcation, and to show the relationship between loading and the FBG response.

To accomplish these goals a test of loading a FBG between two parallel plates was devised. Loading the FBG transversely between two plates provides a test with the least number of assumptions, and provides an appropriate baseline for future work. Testing under the pure Hertz assumptions is difficult due to the very small area of load application. Based on the assumptions related to the interface characteristics and variation with processing and manufacturing, it was decided that testing an embedded FBG would not be accurate for the analysis. Testing the FBG between two parallel plates provided a more generic confirmation of the analytical work above.

6.2 Experimental Setup

The purpose of the experimental work was to confirm analytical data of a distributed diametrical loading, as done with the Hondros solution, and the FEA solution of FBG loaded between two rigid plates (Figure 4.7). The setup equipment was designed in a way to represent this loading case with the greatest degree of accuracy and applicability.
The experimental setup was comprised of a broadband light source, a loading rig, a FBG sensor, a micrometer, a load cell, and a spectrum analyzer (Figure 6.1, Figure 6.2). The goal of the experiments was to operate the FBG and observe changes in the signal with transverse loading, as done with the FEA experiments above. A broadband light source was used in conjunction with an Anritsu Optical Spectrum Analyzer (OSA) to characterize the FBG signal output. The light source was centered at 1300 nm (Figure 6.3). The OSA had a maximum accuracy of 0.1 nm, and shows optical amplitude (y-axis) vs. wavelength (x-axis). The micrometer and load cell were integrated into the loading rig to obtain load vs. displacement data. The experiments were conducted on an air-damped Newport optical table. The loading rig, in order to ensure proper loading, had to be designed and fabricated from scratch.
Figure 6.1 Experimental Setup Diagram

Figure 6.2 Experimental Setup
A loading rig was fabricated from scratch to apply the transverse loading to the FBG properly (Figure 6.4, Figure 6.5). The loading rig was specifically designed to have the loading plates parallel while loading the fiber. The plates in such an experiment must be parallel to match the FEA, and also to ensure that no chirping or alteration of the FBG signal would be produced. The plates were made of a machined aluminum, and therefore would have some frictional contact with the fiber. The loading fixture is comprised of an upper metal plate fixed to a bracket with thin flexures. These thin flexures allow for translation of the plate downwards, and keeping the plate parallel to the bottom loading plate. The upper plate is actuated with a simple threaded rod, threaded through the top of the bracket, and resting on the plate with a rounded cap screw to minimize friction and lateral forces. An independently supported micrometer with accuracy up to 1 micron
read the displacement of the top plate through a hole in the bracket. Below the upper loading plate is the lower loading plate, on which the test fiber sat. The lower plate sits in a slot in an anvil block.

Figure 6.4 Schematic of FBG transverse loading rig
The lower block has a threaded hole in the bottom to accommodate a threaded rod which then is supported by a precision load cell accurate to 1 g. The load cell’s analog output went to a tabletop readout provided by the manufacturer, Fairbanks scales. The rig was designed so the load cell readout would accurately reflect the actual load applied to the fiber. The loading plates were 4.93cm in length. Figure 6.6 shows the equivalent load diagram of the experimental setup.

The fibers used in the experiments were standard communications grade single mode optical fiber FBGs. The FBGs were manufactured by Blue Road Research, Inc. of Gresham, OR, and Avensys, Inc. of Point-Claire, Quebec, Canada. Two samples of each were tested. The fibers had standard nominal dimensions of 10 micron core, 125 micron
cladding, and a 62.5 micron coating, for a total outer diameter of 250 microns. The Blue Road Research fibers were coated with a polyimide coating, the Avensys fibers were coated with an acrylate coating. All of the samples were 1m in length, and all gratings were 1 cm in length. All of the gratings were manufactured by stripping off the polymer coating, writing the grating, then re-coating the fiber.

Figure 6.6 Load diagram of experimental setup
6.3 Experimental Procedure

Each of the tested four fibers (2 from Blue Road Research and two from Avensys) were connected to the light source and to the OSA. Each fiber was carefully placed on the optical table and run through the loading plates of the loading rig so that no forces were acting on the fiber outside of the loading rig. The FBGs were centered on the loading plates. The loading screw was used to lower the upper loading plate just enough to hold the fiber in place. Data were collected by applying incrementally increasing amount of load on the fiber in the transverse direction. The effects of increasing load on the FBG were noted with the light intensity vs. wavelength data from the OSA. The position of the Bragg peak or peaks was noted and plotted against load. Data was repeated for the fibers, and two fibers of each manufacturer were tested.

6.4 Experimental Data

Experiments were conducted resulting in load vs. FBG response for each FBG. The two Avensys FBGs were indexed as Avensys FBG 2 and 3, and the two Blue Road Research FBGs were indexed as Blue Road FBG 4 and 5 to avoid confusion. The baseline trace of the optical light source was obtained to verify its stability and power level (Figure 6.3). Data were then collected on the four FBG samples up to a total load of normalized load of 4 kg per cm length to produce significant FBG signal splitting data.
without permanently damaging the FBGs. The load data were normalized to reflect the amount of load per length (cm) of FBG. The maximum test load of 20 kgf is equivalent to a normalized load of approximately 4 kgf per cm of fiber length. The Bragg gratings tested were 1 cm in length. As indicated in Figure 5.25, data were taken as the peak value for the FBG reflected wavelength, or wavelengths as load was applied. Data for two Avensys fibers are shown in Figure 6.7 and Figure 6.8. Data for two Blue Road research fibers are shown in Figure 6.9 and Figure 6.10. The data show the load applied along the x-axis and the shift in the Bragg signal along the y-axis. Where one trace is seen, the FBG signal is shifting, but has one peak. After the signal bifurcates, the graph shows the shift of each peak.

Figure 6.7 Load vs. FBG data, Avensys fiber (2)
Figure 6.8 Load vs. FBG data, Avensys fiber (3)
Figure 6.9 Load vs. FBG data, Blue Road fiber (4)
An example of OSA data traces used to obtain the data is shown in Figure 6.11, Figure 6.12, and Figure 6.13. These traces show wavelength on the x-axis, and light amplitude on the y-axis. The traces shown are transmission spectra, with light filtered out by the FBG. In these three figures data is shown with the FBG under no load up to a normalized load of 4 kgf. As seen in Figure 6.11, the FBG signal is not perfectly narrow, but has some width. This can be explained by the resolution of the OSA (0.1nm), and also by possible anisotropy in the grating structure.

After testing the fibers, microscope photographs were taken to examine any damage done to the fiber. Figure 6.14 shows a Blue Road Research fiber after testing to 20 kgf. The fiber remains intact, but some flattening of the polymer coating is exhibited.
Figure 6.15 shows the same fiber after a loading of 40 kg, just after the FBG signal showed failure of the sensor.

Figure 6.11  Avensys FBG Signal (3) No Load
Figure 6.12 Avensys FBG Signal (3) 3 kg Load

Figure 6.13 Avensys FBG Signal (3) 4 kg Load
Figure 6.14 Microscope photo of Blue Road fiber

Figure 6.15 Microscope photo of Blue Road fiber (failure)
6.5 Discussion and Comparison to FEA

All four FBGs (Figure 6.7-Figure 6.10) showed a split peak with increased transverse loading. The peak splitting did not appear in the data immediately. As load was added to the fiber, the FBG peak broadened and shifted to the right with Poisson effects in the major fiber axis direction. Between a normalized force of 2kgf and 3kgf the FBG peak split enough to discriminate, and continued to split through the test. This can be seen in the data in Figure 6.7 through Figure 6.10. The Avensys FBGs showed greater signal shift than the Blue Road Research FBGs. The Avensys showed a minimum shift of 0.6nm, and a maximum of 1.0nm shift, with split peaks approximately 0.3nm apart. The Blue Road Research FBGs showed a minimum shift of 0.05nm and a maximum shift of 0.52nm, with split peaks approximately 0.2nm apart. The two Avensys samples showed similar responses. The Blue Road FBGs displayed a lower response than the Avensys fibers, but also showed responses different from each other. The Blue Road FBG 5 (Figure 6.10) showed a response approximately 50% less than Blue Road FBG 4 (Figure 6.9). As the FBGs were loaded to the maximum load of 4,000gf, some nonlinearity seemed to appear. In the case of the Avensys FBGs the response seemed to decrease with increasing load, and the opposite was seen in the Blue Road Research FBGs. In addition, while up to a load of approximately 2,500 gf, the data were repeatable when the FBG were loaded and unloaded, loads in excess of 2,500 gf seemed to cause some hysteresis.

The fiber response in the experiments compared well to that predicted in the FEA. In Figure 6.16 the experimental data is plotted with the prediction from the FEA model. FEA data in Figure 6.16 includes both an assumption of plane stress and plane strain.
Plane stress data were calculated to determine what effect constraints along the z-axis would impart on the data. The FEA model data shows a split peak from the very beginning of side loading, not seen in the experimental data for both plane stress and plane strain. Because of the small inhomogeneous nature of the FBG lines, and the limitations of the OSA, the split peak was not seen until the magnitude of the difference of the x-axis and y-axis components were large enough to overcome the “noise floor”. The data show the experimental data scattered around the FEA solution. It should be noted that the FEA model made some specific assumptions which, if changed, would change the response as seen in Figure 6.16.

As described in Chapter 4, the numerical model assumed perfect adhesion of polymer coating to glass core/cladding, and also some specific material property values, most importantly that of Young’s modulus and Poisson ratio. These values for actual test specimens can only be ascertained with a moderate degree of accuracy, given manufacturers provision of a range of values for their products (both silica and polymer coatings). In addition, the polymer coating, critical in this analysis, was applied to protect the fibers which were stripped in order to produce the FBG, then re-applied to the fiber. While the manufacturing process has improved over the last decades, this re-applied polymer coating may vary from the ideal in a more significant way than coatings applied during fiber manufacturing. The length of fiber where polymer was re-applied is generally greater than the length of the FBG (1 cm) but less than the length of the loading
Figure 6.16 Comparison of FEA and experimental FBG data

plates (4.93cm). As with most models if the material properties of the components can be accurately ascertained, the model will predict the performance of the FBG with improved accuracy.
Figure 6.17  Loading vs. FBG signal for Avensys fiber (tests 2 and 6)

Figure 6.18 Loading vs. FBG signal for Blue Road fiber (tests 5 and 7)
Figure 6.17 and Figure 6.18 show FBGs tested to 4,000g/cm, unloaded, and then tested to failure. It appears that testing to 4,000 gf resulted in some permanent change in the FBG that altered the fiber’s performance. This could have been plastic deformation of the polymer coating, a disturbance of the interface between the polymer coating and the silica cladding, or a disturbance of the core/cladding system or interface. In addition, it appears that the sensor’s response dissipated for the Avensys fiber above 4,000 gf, possibly indicating that additional loading was damaging the polymer coating and not increasing load in the optical fiber core. The Blue Road FBGs performance did not dissipate as greatly, but did fail at a lower load than the Avensys fiber (6,300gf/cm vs. 8,100gf/cm).

It is thought that part of the difference in the performance of the two manufacturer’s fibers were due to the impact of the choice of polymer coating. Blue Road Research uses a polyimide coating, and Avensys an acrylate coating. In theory the response of the grating (the amount of birefringence) is dependent on the material properties of the coating, most importantly Young’s modulus. In Figure 6.19 the response of coatings with different Young’s modulii can be seen. According to the numerical model, a decrease in E makes the FBG more sensitive to transverse load, and vice versa. By this measure, the Avensys FBG should have been more sensitive than the Blue Road Research (if we assume common mean values for E for acrylate and polyimide) FBG, and the experimental work verifies this. More importantly, this concept shows the ability to tune the FBG – to design the polymer coating in size and stiffness to match the user’s requirements for performance (sensitivity and dynamic range). Polymer
coatings can limit the use of FBGs by limiting the upper temperature limit, and by limiting the total strain limit by failing mechanically at a lower strain than the silica.

Figure 6.19 Comparison of FBG response with different coatings
7.1 Conclusions

It has been established analytically, numerically, and experimentally that transverse loading of FBGs will cause a signal broadening, and signal bifurcation. For the user of such technology, the salient results are that a local compressive side load of approximately 200gf per cm (1.12 lbf per inch) of sensor will be enough (largely through Poisson effects) to shift a FBG signal about 0.5nm, and to begin signal broadening. Also indicated is that after approximately 1,000gf per linear cm (5.59 lbf per linear inch) of FBG sensor, it is likely that a bifurcation will occur. These conclusions are indicated from the evidence outlined in Chapters 2 through 6.

In this thesis work a simple analytical analysis of a monolithic fiber structure indicated quantitative data to show a difference in index of refraction (through stress, and the stress-optic law) along the major cross sectional axes of an optical fiber under transverse compressive load. This coupled with the general theory of FBG operation laid a foundation of proposing bifurcation of the FBG signal, and a shifting of the signal to higher wavelengths with Poisson effects.

In addition a finite element model was used to generate data of the change in index of refraction in the FBG cross section under compressive transverse load with a more complicated geometry, mimicking that found in the actual device. The model data of change in index of refraction with increasing load was processed to provide FBG response data. The model used the basic assumptions that the polymer coating was perfectly adhered to the cladding, the loading from two opposed plates was frictionless, and also certain material properties obtained from eventual experimental test specimens.
The model, differing from models in the literature, did not assume a uniform stress (and therefore index of refraction) in the x and y axes under loading. While variations in the fiber core were indeed small, they were significant with respect to the eventual calculation of FBG signal. FEA were conducted for a number of loading cases, including compressive loading by two plates, and fiber embedded in a matrix host.

Finally, experiments were conducted by loading commercial FBGs from two manufacturers to verify the model. At higher loads, some permanent changes to the FBGs performance were noted. From a practical standpoint, it appears loading the fiber higher than 2,500g per linear cm of FBG sensor length (13.97lb per linear inch) will initiate these permanent changes in these particular fibers. These limits will be different for various manufacturers and various fiber & coating types.

This work has taken a unique comprehensive approach to the characterization of FBGs under embedded conditions focusing on transverse loading. Specifically, relating analytical and numerical analyses to examine the change in material properties within the fiber core, and the resulting change in the FBG signal. Of specific contribution to the state of the art were the small but notable changes in index of refraction from the center to the edge of the optical fiber core, and the visualization of property changes throughout the optical fiber system.

7.2 Additional Considerations & Recommendation for Future Work

The work described above provides a detailed analysis of the foundational case of transversely loaded FBG. A logical follow-on consideration is to propose what more complicated loading cases, and other factors, would have on the FBG signal. These
propositions would provide the ideal case for future endeavors in this subject. First, the simple case of changing the compressive transverse loading to tensile loading could be considered. In theory, a tensile load in the same form of the compressive loads described above would create a similar biaxial index of refraction change as seen with the compressive load. A tensile stress along the loading axis, and a compressive load along the orthogonal axis would create a similar bifurcation. Experimental verification of such a scenario would be difficult – embedding in a solid host may make such a test possible. In addition, its unclear if the fiber/polymer coating would behave similarly in tension as it did in compression.

Also of interest are more complicated loading profiles (Figure 7.1). These profiles vary the application of force on the grating, and will theoretically effect the operation in different ways. In the work above, the compressive load case assumed plane strain, and loading constant in the z-direction. In real world applications, such simple loading profiles can not always be ensured, and in most cases fibers are used to measure strain along the fiber’s main axis. In addition, short gage lengths (~1cm) are typically used to minimize effects of complicated transverse loading profiles.

If the force were varied linearly along the fiber length, (Figure 7.1 a.), this would change the FBG response. In this case a “chirped” grating would be created. The lines of the FBG would vary in distance from each other in a linear fashion. Applying this to the governing equation, \( \lambda_b = 2n\Lambda \), the result would be a broad reflection \( \lambda_b \) to correspond with the range of \( \Lambda \) values in other words, rather than create a narrow reflection, \( \Lambda(z) \) would create a broad reflective range. This mode of operation is actually used in communications systems for multiplexing and demultiplexing broadband light signals.
The other complication is that the amount of index of refraction change, and therefore the amount of FBG signal broadening and bifurcation, would change as a function of distance along the z-direction. This would make use or detection of the transverse loading, qualitatively or quantitatively, difficult.

If the load along the FBG is more complicated, as seen in Figure 7.1 b., this would cause numerous changes of $\Delta \Lambda(z)$, as well as various changes in index of refraction throughout the fiber’s cross section. This could allow for multiple FBG peaks and bifurcations and/or broadening in various parts of the spectrum. This situation would be further complicated if the complex loading were present in 3d, or over the whole surface of the FBG (Figure 7.1 c.). If a constant force were applied in a non-linear pattern (for example, a helix or curved path as seen in Figure 7.1 d.), the FBG may still behave in a predictable manner. In theory, if a clearly bifurcated signal is created by a transverse force, if the force is moved (in the cross sectional view, this would be akin to rotating the force to a different point of application) the light will oscillate between the two peaks with that rotation.

The last point of interest is that of interface characteristics. This topic is relevant for both the interface between the optical fiber cladding and the polymer coating, and also for the interface between the polymer coating (or bare fiber) and a host, whether a matrix material or a composite (particularly fibrous composites). The quality of the interface can have a great effect on the translation of loading from an external source to the FBG. In addition, this interface may not behave the same in compression as in tension, or in various directions. The presence of voids, fiber wetting characteristics,
varied component material properties (Poisson, Young’s Modulus, frictional constants, geometry, size etc.) of the fiber coating can all factor into the interface characteristics.

Future work in the area could focus on a full three dimensional analysis of the FBG. Of value to the body of knowledge in this area would be a study in more complicated loading and unloading conditions, as well as an in-depth study of the interface characteristics of the polymer coating both with the silica fiber and with any host material it is embedded in. This could lead to more refined and accurate models of FBG response in field applications. In addition, experimental work could be expanded into testing FBGs embedded in a composite matrix, and observing the FBG response to far field loading in any direction.
Figure 7.1 Loading profiles of FBG sensor
References


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Vita

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