Temperature-Dependent Transmission Line Models for Electric Power Systems

and Their Impacts on System Studies

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Electric power transmission line models for large scale system studies are characterized by uniformly distributed parameters or a lumped parameter configuration. These models have been historically developed for studies under nominal operating conditions. The following assumptions are often made: uniform current density, constant material characteristics, and constant external conditions, including temperature. However, in recent years, the increased use of renewable energy resources and corresponding enabling technologies has had an impact on the electric power system, and several previous operating assumptions are no longer valid.

This thesis focused on electric power lines as a critical network component of the power system, and aimed at developing line models to be used in steady-state system analysis applications. By relaxing historical modeling assumptions and by taking into account parameters not previously considered, more accurate results are expected by the decision-making tools. Specifically, two parameters were considered:

1. Environmental conditions – ambient temperature gradients along a line,
2. Electrical frequencies – the expected increase in non-fundamental frequencies due to augmented use of power electronic devices.

Contributions include formulating an optimization problem to incorporate these parameters into static line models. Then, a focus was put on developing a line modeling approach to take into account ambient temperature variations along the line. The
proposed line models are temperature-dependent in terms of both structure and line parameters; therefore, they enable capturing of temperature effects, including more accurate determination of power handling capabilities. The line modeling approach was automated and incorporated into multi-bus system analysis tools. The proposed models’ impacts on large-scale, steady-state system studies, such as power flow analysis, were investigated.
CHAPTER 1: INTRODUCTION

1.1 OVERVIEW

This thesis focused on developing electric power transmission line models for large-scale system studies that take into account parameters not previously considered, by relaxing select historical modeling assumptions. In [1] and [2], line modeling for studies at non-fundamental frequencies was investigated, and a multiple segmentation of the line model was proposed. Here, the problem of incorporating ambient temperature information into line models was addressed and a multiple, non-uniform segment model structure was used. A line modeling tool was developed accordingly and integrated within state-of-the-art power flow analysis tools.

In this chapter, the following topics are presented:

- A background and motivation for the work;
- A list of objectives and a summary of contributions;
- An overview of the thesis organization.

1.2 BACKGROUND AND MOTIVATION

Electric power transmission lines are the critical network component of the electric power system. For large scale power system studies, transmission lines are traditionally modeled through a uniformly distributed parameter or a lumped parameter configuration. These line models have been historically developed for studies at the fundamental frequency and under nominal operating conditions. Several assumptions are
often made when modeling transmission lines, including: uniform current density, constant material characteristics, and constant external conditions, including temperature.

However, especially in recent years, the increase in electric power demand and in renewable energy resources, with corresponding enabling technologies (e.g. power electronic devices), has had an impact on the electric power system. For example, with the increase in power generation, it is critical to accurately determine the power handling capabilities of the lines, especially when close to their thermal limits. Moreover, with the augmented use of power electronic devices, an increase in non-fundamental frequency components in the power system can be expected. Also, recent advances in synchronized phasor measurement techniques could aid in the development of more detailed line models as well as dynamic parameter estimation.

Several previous operating assumptions are no longer valid, and parameters not previously fully considered, such as environmental conditions and non-fundamental frequencies, gain importance. Thus, this work has re-evaluated historical line modeling assumptions, as well as the development of new component and integrated system models. As such, this thesis has focused on developing transmission line models for steady-state studies that take into account these parameters. In order to separately identify effects of non-fundamental frequencies and environmental conditions, the line modeling problem was decoupled into two sub-problems:

Sub-Problem 1: Line modeling for studies at non-fundamental frequencies, addressed in [1] and [2];

Sub-Problem 2: Incorporating ambient temperature variations within the line model.
In this thesis, a mathematical formulation for both sub-problems was developed. It should be noted, an approach addressing Sub-Problem 1 was presented in [1] and [2]. Therefore, this thesis focuses on Sub-Problem 2. A brief background and motivation for each sub-problem is presented in the next sub-sections.

1.2.1 STUDIES AT NON-FUNDAMENTAL FREQUENCIES

An emerging characteristic of modern power systems is the increased level of non-fundamental frequency components present in the network and attributed to the augmented use of power electronic switches. In addition, approximately up to the 15th harmonic component is introduced in the network in a capacitor switching scenario, while common power electronic devices introduce up to the 39th harmonic [3][4]. Under the increasing presence of non-fundamental frequency components, transmission line models were investigated in this work.

Thus far research work on frequency-dependent transmission line models has been performed with a focus on transient analysis [5]-[7]. These studies consider models with frequency–dependent parameters, but not frequency-dependent structures. Limited research has been performed on the relationship between line model segmentation and model accuracy [8]-[11]. In general, previous work on the subject has revolved around switching studies.

In [1] and [2], it was shown that the simple lumped equivalent circuit, while sufficient for studies at fundamental frequencies, may not be suitable in studies concerning higher frequencies. A hardware validated transmission line modeling tool was then developed to determine the appropriate model structure for system studies under
non-fundamental frequencies. A uniform, multi-segment line model structure, as shown in Figure 1.1, was adopted, and an automated tool was developed to determine:

i) number of segments, and

ii) segment parameters,

based on a desired level of accuracy and given frequencies of interest.

Figure 1.1 $K$-Segment $\Gamma$ Line Model of a Lossless Line, where:

$L_1, \ldots, L_K$ are series inductances and $C_1, \ldots, C_K$ are shunt capacitances

In this thesis, a formal problem statement for line modeling at non-fundamental frequencies was developed. Then, a similar framework and model structure was proposed to incorporate non-uniformity of line parameters along the line due to temperature gradients.

1.2.2 INCORPORATING AMBIENT TEMPERATURE INFORMATION

Transmission line models for system studies are characterized by uniformly distributed parameters or a single lumped parameter configuration. The following assumptions are often made when deriving these line models:

A1. uniform current density along the line,

A2. constant material characteristics, and
A3. constant external conditions, including temperature.

It is also noted that the geometrical configuration of three-phase lines is often assumed such that any unbalance between phases can be neglected (i.e. transposition) [15].

The line models that are currently utilized in power flow tools and state estimators are static and do not take into consideration changes in ambient temperature along the line [12]-[14]. Line parameters are usually calculated assuming a predefined seasonal temperature, \( T_{\text{Seas}} \). The conductor temperature and consequently the line parameters are assumed constant along the line, i.e. not dependent on position, \( x \) [15].

Ambient temperature gradients along transmission lines are experienced normally and particularly in extreme weather conditions [16]. These cause differentials in conductor temperature and therefore non-uniformity in line parameters along the line. See Figure 1.2 for a graphical representation of parameter non-uniformity due to a temperature gradient.

![Figure 1.2 Graphical Representation of Parameter Non-Uniformity with Temperature](image)

In the past, temperature information has been utilized to calculate line ratings. Significant work has been performed in dynamic / thermal line ratings [17]-[23], and it indicates that the variability in weather conditions along a line is to be taken into account when estimating worst case line ratings. Previous work also illustrates the importance of multi-station weather monitoring.
In the literature, it has been shown that changes in transmission line resistance due to temperature have a non-negligible impact on state estimation accuracy [12]. In [12], the authors propose an updated state estimation algorithm with a resistance correction module that utilizes weather data and the Heat-Balance equation [24]. By incorporating available ambient temperature information within the line models, more accurate results from energy management systems and state estimation tools can then be expected [12].

This thesis presents a line modeling approach for steady-state studies that incorporates ambient temperature information into the model, with respect to both line parameters and line model structure. The approach investigates uniform and non-uniform modeling as well as distributed and lumped parameter models of the line. As a result, a non-uniform multi-segment model structure is proposed.

Previously, a uniform, multi-segment modeling technique for frequency variation was introduced in [2], where this model structure was developed to increase model accuracy at non-fundamental frequencies. Studies on non-uniform line modeling have been presented in several works, such as [25]-[28], and were mainly focused on non-uniform line parameters due to longitudinal variations in line geometry. Also, they were used for simulation of electromagnetic transients, while in this thesis the main application focus was steady-state system analysis tools. Non-uniform transmission lines have also been modeled using finite element analysis; these models are utilized to describe high-frequency signal propagation along power lines with non-uniform geometry, such as in [29], or non-uniform microwave transmission lines [30].
1.3 OBJECTIVES

The objectives of this thesis include:

- Re-evaluating select line modeling assumptions:
  - Constant conductor temperature along the line, and
  - Uniform distribution of line parameters along the line.

- Investigating:
  - Various ways of incorporating temperature data into traditional line models, including:
    - distributed, and
    - lumped parameter models (single and multi-segment);
  - The sensitivity of line models to ambient temperature variations, through a circuit-based approach.

- Developing and testing:
  - Equivalent circuit representations of a line that take into account temperature changes along the line;
  - A tool to model the line given a set of ambient temperature measurements.

- Integrating line models’ impacts on large-scale system studies.

Figure 1.3 graphically shows the thesis’ framework: the intended application of this work was line modeling for system studies, rather than component studies. Therefore, transmission line models were developed and then integrated as part of the network. Although material properties were not a focus of this work, line parameters are included in Figure 1.3 because their relationship to temperature played a role in line
model determination. Transmission line parameters, as well as commonly used line models and network models are briefly reviewed in the next chapter.

1.4 SUMMARY OF CONTRIBUTIONS

The thesis’ contributions are summarized here:

- Formulation of an optimization problem:
  - Given: non-fundamental frequencies of interest and desired accuracy,
  - Determine: optimum line model structure, i.e. uniform segmentation;

- Formulation of an optimization problem:
  - Given: ambient temperature information,
  - Determine: 1. Optimum line model structure, i.e. non-uniform segmentation, and
  - 2. Each segment start and end points, i.e. segment length;

- Development of transmission line model structures that incorporate ambient temperature variations along the line:
  - Representation of parameter non-uniformity with respect to position along the line, through
    - A non-uniformly distributed line model,
• A model composed of multiple non-uniform lumped segments to be used in power flow and state estimation algorithms;

• Development and implementation of an automated line modeling tool to obtain:
  - Line model segmentation, and
  - Parameter values of each segment;

• Integration of the line modeling tool into network models to be used in state-of-the-art power flow software;

• Simulation results with respect to the developed models’ impacts on power flow analysis.

1.5 ORGANIZATION OF THESIS

This thesis is organized as follows:

• In Chapter 2, fundamentals of transmission line modeling for system studies are reviewed, with a focus on relationship between line parameters and temperature.

• In Chapter 3, the problem statement is presented, including modeling assumptions and hypotheses, and problem formulations.

• In Chapter 4, the transmission line modeling approach is introduced, including ambient temperature models, line models, and specifics of the line modeling tool.

• In Chapter 5, test results and evaluation of the proposed line models are presented.
- In Chapter 6, the integration of the line modeling tool for system studies is discussed. Experimental test results for a multi-bus system are presented.
- Finally, in Chapter 7 the research accomplishments and contributions of this thesis are summarized, and a discussion of the future work and vision is presented.
2.1 OVERVIEW

Transmission lines are an important interconnection component of the electric power system. They enable transmission of electricity from the power plants to the consumers by delivering electric power from one end of the line (sending-end) to the other (receiving-end) [15][31].

This chapter presents a review of transmission line modeling, including:

- determination of line parameters, with a focus on their dependency on temperature;
- distributed and lumped parameter line models; and
- network models.

2.2 TRANSMISSION LINE PARAMETERS

A transmission line has four distributed electrical parameters affecting the way it transmits electric power from sending to receiving end:

\[ r : \quad \text{series resistance (Ω) per unit length} \]
\[ l : \quad \text{series inductance (H) per unit length} \]
\[ g : \quad \text{shunt conductance (S) per unit length} \]
\[ c : \quad \text{shunt capacitance (F) per unit length} \]

The distributed resistance and inductive reactance form the line series impedance, \( z \):
\[
z = r + j\omega l = r + jx_L : \text{series impedance (}\Omega\text{) per unit length}
\]

where

\[
\omega : \text{operating angular frequency (rad/s)}
\]

\[
x_L : x_L = \omega l, \text{ inductive reactance (}\Omega\text{) per unit length}
\]

while the conductance and the capacitive susceptance between conductors or conductor to neutral form the shunt admittance of the line, \(y\):

\[
y = g + j\omega c = g + jb_c : \text{shunt admittance (S) per unit length}
\]

where

\[
b_c : b_c = \omega c, \text{ capacitive susceptance (S) per unit length}
\]

Please note that inductive susceptance in the shunt admittance of the line is usually ignored.

The values of these parameters depend mainly on the cable material characteristics and on the electric and magnetic fields along and around the conductors [15]. The parameters’ dependency on temperature is addresses in sub-section 2.2.1.

The series resistance of a line is affected by three factors: frequency, spiraling and temperature. For direct current, the current distribution throughout a conductor is uniform, and its series resistance, at a specific temperature, can be defined by the dc resistance:

\[
r_{dc} = \frac{\rho}{A} \quad (2-1)
\]

where

\[
r_{dc} : \text{dc resistance of a conductor (}\Omega\text{) per unit length}
\]
\[ \rho : \text{conductor resistivity} \]
\[ A : \text{cross-sectional area} \]

With alternating current as the frequency increases, the current density tends to be greater toward the surface of the conductor, effectively decreasing its cross-sectional area, i.e. increasing the resistance \[32\]. The effective (ac) resistance can be calculated by:

\[
\quad r_{ac} = \frac{P_{Loss}}{|I|^2 \ell} \tag{2-2}
\]

where

\[ r_{ac} : \text{effective (ac) resistance of a conductor (}\Omega\text{) per unit length} \]
\[ P_{Loss} : \text{real power loss in the conductor (W)} \]
\[ |I| : \text{rms current in the conductor (A)} \]
\[ \ell : \text{total length of the line} \]

Spiraling, e.g. for stranded conductors, also slightly increases the resistance of a conductor, though this effect can be considered negligible \[32\]. At temperatures far from the melting point, the conductor resistance increases with temperature in a linear manner. Details on the relationship between conductor resistance and temperature are presented in sub-section 2.2.1. Resistance values of given conductors, for both dc and 60 Hz, are provided by the manufacturers through empirical tables (e.g. in [33]).

The series inductance represents the relationship between the voltage induced by flux changes and the rate of change of current, and it depends on line geometry, including cable size and configuration. In general, the inductance per phase of a three-phase line is given by:
\[ l = \frac{\mu}{2\pi} \ln \frac{D_{eq}}{D_s} \]  \hspace{1cm} (2-3)

where

- \( l \): inductance (H) per unit length
- \( \mu \): permeability of the conductor
- \( D_{eq} \): equivalent distance between conductors
- \( D_s \): conductor geometric mean radius

Please note that in this thesis per phase analysis is conducted. Single-phase or three-phase systems with transposed conductors and balanced phases are considered.

The shunt conductance exists between conductors and conductors to ground, and it accounts for leakage currents along the insulators. Especially for overhead lines, this parameter is very small and is often ignored in the determination of the line shunt admittance element, \( g = 0 \).

The shunt capacitance of a transmission line results from the difference in potential between the conductors. In general, based on of the geometrical characteristics of the line, it is given by:

\[ c = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{d}} \]  \hspace{1cm} (2-4)

where

- \( c \): capacitance (F) per unit length
- \( \varepsilon \): permittivity of the material surrounding the conductor surface
- \( D_{eq} \): equivalent distance between conductors
- \( d \): conductor radius
For a three-phase line, including the effects of the earth:

\[ c = \frac{2\pi \epsilon}{\ln \frac{D_{eq}}{d} - \ln \sqrt[3]{\sqrt[3]{H_{12}H_{23}H_{31}}}} \]  

(2-5)

where

- \( H_1, H_2, H_3 \): distances between each conductor and its image below the surface of the earth
- \( H_{12}, H_{23}, H_{31} \): distances between each conductor and the other conductors’ images below the surface of the earth

### 2.2.1 DEPENDENCY OF LINE PARAMETERS ON TEMPERATURE

The line effective resistance changes with the temperature of the conductor, \( T_C \), mainly due to the dependence of the resistivity, \( \rho \), on temperature, and to the thermal expansion of the conductor. The following linear relationship of resistance versus temperature holds:

\[ r(T_C) = r(T_0) \cdot \left[ 1 + \alpha (T_C - T_0) \right] \]  

(2-6)

where

- \( T_C \): conductor temperature
- \( T_0 \): reference temperature – usually 20°C
- \( \alpha \): temperature coefficient of resistivity (1/°C)

and the temperature coefficient of resistivity, \( \alpha \), depends on the physical properties of the cable [34].

The inductive reactance of the line varies similarly with temperature, mainly due
to a change in cross-sectional area, affecting cable size [35]:

\[ x_L(\omega, T_C) = x_L(\omega, T_0) \cdot \left[ 1 + \beta(T_C - T_0) \right] \]  

(2-7)

where

\[ \beta \quad : \quad \text{temperature coefficient of reactance (1/oC)} \]

Conductor temperature, \( T_C \), is dependent upon the temperature resulting from external conditions, i.e. ambient temperature, \( T_A \), wind, etc., and a uniform temperature rise due to the current flowing through the conductor, \( I \). Figure 2.1 shows an example of graphical representation of conductor temperature versus distance along the line, \( x \).

![Figure 2.1 Example Graphical Representation of Conductor Temperature, \( T_C \), vs. Distance along the Line, \( x \)](image)

The reference line parameters, \( r(T_0) \) and \( x_L(T_0) \), are determined empirically at a reference conductor temperature, \( T_0 \), which is considered constant along the line. Reference line parameters can usually be found in look-up tables for negligible loading conditions and \( T_0 = 20^\circ \text{C} \), or at 75% loading and \( T_0 = 50^\circ \text{C} \) [33].

Under steady-state conditions, the current-temperature relationship of an overhead line is expressed by the heat-balance equation [24]:

\[ I^2 R(T_C) + q_s = q_c(T_C) + q_r(T_C) \]  

(2-8)
where

\[ R \quad : \quad R = r \ell, \text{ line series resistance (Ω)} \]
\[ q_s \quad : \quad \text{solar heat gain (W)} \]
\[ q_c \quad : \quad \text{convection heat loss (W)} \]
\[ q_r \quad : \quad \text{radiated heat loss (W)} \]

Determining conductor temperature, \( T_C \), then requires either:

- the installation of additional hardware to measure temperature within the cable (e.g. in [36] and [37]), or
- an iterative calculation, since \( q_c \) and \( q_r \) are not linearly dependent upon \( T_C \) [17].

In general, if loading conditions are kept constant, i.e. \( I \) is constant, and if the only varying external condition is the ambient temperature, \( T_A \), an increase/decrease in \( T_A \) results in an equal increase/decrease in \( T_C \) [24]. Please note that traditionally in power system studies, \( T_C \) is assumed constant along the line because mainly driven by the temperature rise due to current, which does not depend on \( x \).

When looking at the effects of temperature on shunt capacitance, \( c \), two considerations must be made:

- temperature effects on the permittivity, \( \varepsilon \), of the dielectric, and
- temperature effects on line geometry.

The permittivity of the dielectric is defined as:

\[ \varepsilon = \varepsilon, \varepsilon_0 \quad \text{(2-9)} \]
where

\[ \varepsilon_0 : \text{permittivity of free space (constant and not dependent on temperature)} \]

\[ \varepsilon_r : \text{permittivity of the material surrounding the surface relative to the permittivity of free space} \]

In open-wire lines, the principal dielectric, i.e. insulation between conductors, is air. The relative permittivity, or dielectric constant, \( \varepsilon_r \), of air does not change appreciably with temperature [38].

The relative distance between conductors, \( D_{eq} \) in (2-4) and (2-5), is considered insensitive to temperature variations, since line sagging is equal for all conductors. Moreover, although the distances between conductors and ground, \( H_1, H_2, H_3, H_{12}, H_{23}, H_{31} \) in (2-5), change with temperature (due to sagging), for overhead lines, it is often assumed that the conductors are high above ground compared to distances between them, i.e. \( \sqrt[3]{H_{12}H_{23}H_{31}}/\sqrt[3]{H_1H_2H_3} = 1 \). Therefore, temperature effects on line shunt capacitance are usually neglected.
2.3 TRANSMISSION LINE MODELS

Traditional power transmission line models assume uniformly distributed parameters or a lumped parameter configuration, such as gamma ($\Gamma$) or pi ($\pi$) forms. These line models imply constant resistance, inductance, and capacitance values, calculated at a pre-set seasonal temperature and independent of frequency. In addition, when modeling a line, the following main historical assumptions are made:

A1. uniform current density along the line,
A2. constant material characteristics, and
A3. constant external conditions, including temperature.

Please note that A1 implies an assumption on the operating electrical frequency, considered constant at the fundamental (i.e. 50-60 Hz), as well as on the line geometry (e.g. negligible line sagging, transposed conductors, etc.); while A3 implies a zero-gradient ambient temperature along the line.

In order to study the line as part of the power system, it is important to be able to determine how line parameters influence the flow of power through the system. The line behavior in terms of wave propagation is of interest. Specifically, voltage and current relationships along the lines are needed. Given A1-A3, the transmission line can be modeled through a uniformly distributed parameter configuration, which is reviewed in sub-section 2.3.1.

A simplification of this model is more commonly utilized for studies on power system behavior, such as power flow and state estimation tools. These tools determine the nodal behavior of the system, and are therefore mostly interested on line terminal behavior. Also, they are time-sensitive tools, in which calculation using the distributed
parameter model would be time-consuming and relatively difficult. Therefore, a simplified line model is often used. The simplification of the distributed parameter model consists of adopting a lumped parameter configuration of the line that maintains the same terminal behavior as the distributed line model. Examples of these lumped parameter models are the $\pi$ and $\Gamma$ forms, briefly reviewed in sub-section 2.3.2.

2.3.1 THE UNIFORMLY DISTRIBUTED PARAMETER MODEL

In the traditional distributed parameter model, the line parameters are uniformly distributed throughout the length of the line. The four line parameters previously identified (series resistance, series inductance, shunt capacitance, and shunt conductance) are in fact quantified in per unit length. The model is then obtained as a summation of the differential sections [39]. A differential section of the distributed line model (per-phase analysis) is shown in Figure 2.2 for a section of length $dx$.

![Figure 2.2 Representation of the Uniformly-Distributed Parameter Model](image)
Please note that:

- The \( S \) subscript stands for sending-end, which corresponds to the point \( x = \ell \), where \( \ell \) is the line length, and
- The \( R \) subscript stands for receiving-end of the line, where \( x = 0 \).

The following notation holds:

- \( V_S \): sending-end voltage (V)
- \( I_S \): sending-end current (A)
- \( V_R \): receiving-end voltage (V)
- \( I_R \): receiving-end current (A)
- \( x \): position along the line (miles, ft, m, …)
- \( dx \): length of the differential element (miles, ft, m, …)
- \( \ell \): total line length (miles, ft, m, …)
- \( z \, dx = (r + j\omega l) \, dx \): series impedance of differential element \( dx \) (Ω)
- \( y \, dx = (g + j\omega c) \, dx \): shunt admittance of differential element \( dx \) (S)

The relationships between the terminal voltages and currents can be defined with the following 1\(^{st}\)-order ordinary differential equations:

\[
\frac{dV}{dx} = zI \\
\frac{dI}{dx} = yV
\]  

resulting in the following 2\(^{nd}\)-order equations:
\[ \frac{d^2V}{dx^2} = yzV = \gamma^2 V \]
\[ \frac{d^2I}{dx^2} = yzI = \gamma^2 I \]  

(2-11)

where
\[ \gamma \equiv \sqrt{yz} : \text{ propagation constant} \]

Solving the differential equations gives the following voltage and current equations in terms of \( x \), point along the line:
\[ V(x) = V_R \cosh(\gamma x) + Z_c I_R \sinh(\gamma x) \]
\[ I(x) = I_R \cosh(\gamma x) + \frac{V_R}{Z_c} \sinh(\gamma x) \]  

(2-12)

where
\[ Z_c \equiv \sqrt{\frac{z}{y}} : \text{ line characteristic impedance} \]

The steady-state voltages and currents can then be determined at any point along the line, \( x \). However, the relationship between the terminal voltages and currents, i.e. at the sending and receiving ends, are sufficient in power system studies since nodal analysis is conducted. In matrix form:

\[
\begin{bmatrix}
V_R \\
I_R
\end{bmatrix} =
\begin{bmatrix}
\cosh \gamma \ell & -Z_c \sinh \gamma \ell \\
-\frac{1}{Z_c} \sinh \gamma \ell & \cosh \gamma \ell
\end{bmatrix}
\begin{bmatrix}
V_S \\
I_S
\end{bmatrix}
\equiv
\begin{bmatrix}
V_{R,\text{Dist}} \\
I_{R,\text{Dist}}
\end{bmatrix}
\]  

(2-13)

Please note that the traditional distributed parameter model of (2-13) does not represent non-uniformity along the line, i.e. \( z \) and \( y \) are assumed constant with \( x \).
2.3.2 LUMPED PARAMETER MODELS

Commonly adopted power system steady-state analysis tools are concerned with the steady-state nodal behavior of the system. Therefore, to determine the non-measured states, they utilize a single lumped parameter model of the transmission line [40][41], which greatly reduces computational burden by avoiding integration.

A lumped parameter model of the line is developed to maintain the same terminal voltage and current relationships, but information on voltage and current propagation along the line is lost. Common examples of lumped parameter models are the pi (π) and the gamma (Γ) models shown in Figure 2.3(a) and (b), respectively. The lumped-equivalent circuits are obtained by selecting the model components $Z'$ and $Y'$ so that the terminal behavior of the distributed line model (2-13) is preserved with the use of passive elements.

![Lumped Parameter Models](image)

*Figure 2.3 Examples of Lumped Parameter Models: a) π-Equivalent and b) Γ-Equivalent Circuits*

where:

$$Z' = Z_c \sinh (\gamma \ell) = Z \frac{\sinh (\gamma \ell)}{\gamma \ell}$$

$$\frac{Y'}{2} = \frac{1}{Z_c} \tanh \left( \frac{\gamma \ell}{2} \right) = \frac{Y \tanh (\gamma \ell/2)}{\gamma \ell/2}$$  \hspace{1cm} (2-14)
Note that for small values of $\gamma \ell \ (\gamma \ell \ll 1)$:

$$Z' = Z = z \ell$$

$$Y' = Y = y \ell$$

(2-15)

The lumped parameter model is considered valid at the fundamental frequencies because these frequencies are well below the cutoff frequency, $f_c$, of the lumped parameter model as a low-pass filter, which in general is defined by:

$$f_c = \frac{1}{\pi \ell \sqrt{l c}}$$

(2-16)

The accuracy with which the lumped parameter model represents the distributed parameter model decreases with increasing frequency [42][43][2].

The next section presents a brief review of a network model for power system studies and of the formulation of the power flow problem.

2.4 NETWORK MODELS AND POWER FLOW ANALYSIS

In order to study the steady-state behavior of the power system, network equations can be formulated in a variety of forms [15][31]. The system is often assumed to be balanced across phases, and is therefore represented by a single-phase network. A single-phase diagram of an example 7-bus power system is shown in Figure 2.4.
A common method to represent the power system is the node-voltage method, based on nodal analysis of the network equations:

\[ I_{Bus} = Y_{Bus} V_{Bus} \]  \hspace{1cm} (2-17)

Where, assuming per-phase analysis:

- \( I_{Bus} \in \mathbb{C}^\mathcal{N} \): \((\mathcal{N} \times 1)\) vector of currents injected at each bus,
- \( V_{Bus} \in \mathbb{C}^\mathcal{N} \): \((\mathcal{N} \times 1)\) vector of bus voltages,
- \( Y_{Bus} \in \mathbb{C}^{\mathcal{N} \times \mathcal{N}} \): \((\mathcal{N} \times \mathcal{N})\) bus admittance matrix,
- \( \mathcal{N} \in \mathbb{Z}^+ \): number of buses in the system, \( n = 1, \ldots, \mathcal{N} \)

Transmission lines are represented by their lumped equivalent model, and values of each branch’s series impedance, \( Z \), and shunt admittance, \( Y \), are needed to determine the bus admittance matrix, \( Y_{Bus} \), which is a \((\mathcal{N} \times \mathcal{N})\) symmetric matrix defined by:
A typical element, $Y_{nm}$, of the bus admittance matrix is given as:

$$Y_{nm} = |Y_{nm}| \angle \delta_{nm}$$  \hspace{1cm} (2-19)

where $n$ and $m$ are any two buses. Please note that the bus admittance matrix of a typical power system is large and sparse.

The power flow problem is the foundation of most power system studies, including planning and operation analyses. It consists of determining magnitudes, $|V_n|$, and phase angles, $\theta_n$, of the voltage at each bus $n$, and consequently real and reactive power flow in each branch. The bus admittance matrix is often used to solve the power flow problem. A summary of the power flow problem is shown in Table 2.1 below, and the polar form of the power flow equations is shown in (2-20).

**Table 2.1  Summary of the Power Flow Problem [15]**

<table>
<thead>
<tr>
<th>Bus Type</th>
<th># of Buses</th>
<th>Known Quantities</th>
<th># of Equations</th>
<th># of State Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slack</td>
<td>$n = 1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Generator</td>
<td>$n = 2, \ldots, \mathcal{N}_g+1$</td>
<td>$\mathcal{N}_g$</td>
<td>$\mathcal{N}_g$</td>
<td>$\mathcal{N}_g$</td>
</tr>
<tr>
<td>Load</td>
<td>$n = \mathcal{N}_g+2, \ldots, \mathcal{N}$</td>
<td>$\mathcal{N}_g-1$</td>
<td>$2(\mathcal{N}_g-1)$</td>
<td>$2(\mathcal{N}_g-1)$</td>
</tr>
</tbody>
</table>

**Totals:** $\mathcal{N}$  \hspace{1cm} $2\mathcal{N}$  \hspace{1cm} $2\mathcal{N}_g-2$  \hspace{1cm} $2\mathcal{N}_g-2$
\[ P_n = \sum_{m=1}^{N} Y_{nm} V_n V_m \cos(\delta_{nm} + \theta_m - \theta_n) \]  
\[ Q_n = -\sum_{m=1}^{N} Y_{nm} V_n V_m \sin(\delta_{nm} + \theta_m - \theta_n) \]  

(2-20)

where \( P_n \) and \( Q_n \) define real and reactive power, respectively, entering the network at bus \( n \).

Solutions to the power flow problem can be obtained through iterative techniques, such as the Newton-Raphson or the Gauss-Seidel methods [15][31][44][45]. The computational complexity of the power flow problem is related to matrix inversion required at each iteration, which is of the order \( O(N^3) \). This can be improved substantially by utilizing the sparsity properties of the \( Y_{Bus} \) matrix [46][47].

The work presented in [1][2] focused on re-evaluating and developing transmission line models for steady-state studies at non-fundamental frequencies. In this thesis, a formal problem formulation was developed. Then, a similar framework was utilized to design transmission line models that take into consideration ambient temperature gradients along a line. In the next chapter, the problem statement is presented in terms of modeling assumptions and mathematical formulation.
CHAPTER 3: PROBLEM STATEMENT

3.1 OVERVIEW

The problem addressed in this work consisted of developing transmission line models for steady-state system analysis by relaxing select historical modeling assumptions and by taking into account parameters not previously considered. Specifically, two parameters were investigated:

- Electrical frequencies – the expected increase of non-fundamental frequency components due to augmented use of power electronic devices, formulated in this thesis and addressed in [1][2];
- Environmental conditions – ambient temperature gradients along a line, main focus of this thesis;

The main objective of the work was to develop equivalent circuit representations of a line, for use in power flow and state estimation studies, that would take into account these parameters. A multiple lumped segment structure is proposed; appropriate line model segmentation and lumped parameter values of each segment were identified. Computational efficiency and system size were also considered in the development of these models.

In this chapter, the problem statement is articulated in terms of the hypotheses the work is based upon as well as the modeling assumptions, section 3.2. The proposed line model structures are then introduced, section 3.3, followed by a mathematical
formulation of the overall line modeling problem, section 3.4, and of the two decoupled sub-problems, defined based on the selected parameters of interest:

Sub-Problem 1: line modeling for studies at non-fundamental frequencies (3.4.1);
Sub-Problem 2: line modeling to incorporate ambient temperature variations (3.4.2).

3.2 HYPOTHESES AND MODELING ASSUMPTIONS

Given the following assumptions for the overall line modeling problem:

A1. Electrical frequency is constant with respect to $x$, position along the line;
A2. Current density is uniform along the line;

the work presented in this thesis is based upon two main hypotheses:

H1. Steady-state terminal voltages and line currents are affected by the operating electrical frequency, $f$, and by ambient temperature variations along the line, $T_A(x)$;
H2. Line model, i.e. structure and parameters, can change to account for frequency and temperature effects.

Therefore, given:

- Line length, $\ell$;
- Cable type, i.e. $r(T_0)$, $x_L(T_0)$, $b$, $\alpha$ and $\beta$;
- Frequencies of interest, $f$;
- Ambient temperature along the line, $T_A(x)$;

the overall line modeling problem consists of obtaining an appropriate equivalent circuit representation of the transmission line, as shown in Figure 3.1, that would depend on $f$ and $T_A(x)$. 
In order to isolate the effects on steady-state line terminal behavior due to frequency and the effects due to temperature variations, two decoupled sub-problems were identified, as shown in Figure 3.2:

**Sub-Problem 1**: To model the line for studies at non-fundamental frequencies – assuming constant ambient temperature;

**Sub-Problem 2**: To develop line models that incorporate ambient temperature variations along the line – keeping frequency fixed.

For studies at non-fundamental frequencies – **Sub-Problem 1**, the following modeling assumptions (A3-A4) were then added:

A3. The ambient conditions are fixed, including ambient temperature;

A4. The line per unit length impedance and admittance, $z$ and $y$, are constant along the line;
Based on the modeling assumptions A1-A4, for Sub-Problem 1 the following main consideration was made:

- For all frequencies, the benchmark line model is the uniformly distributed parameter model (2.3.1) [1][2].

When incorporating ambient temperature variations along the line – Sub-Problem 2, the following modeling assumptions (A5-A8) were added to A1 and A2:

A5. Frequency is fixed at the fundamental, i.e. 50 or 60 Hz;
A6. Loading conditions are fixed, i.e. conductor current, $I$, is constant;
A7. Ambient conditions such as humidity, insolation, wind speed etc. are taken into account through ambient temperature measurements;
A8. Effects of changes in temperature on line charging are negligible.

Based on assumptions A1, A2, A5, A6, and A7 for Sub-Problem 2, the following considerations were made:

- A change in conductor temperature, $T_C$, would depend solely on ambient temperature, $T_A(x)$, which varies with distance along the line, $x$. This implies a dependency of conductor temperature on $x$, i.e. $T_C(x)$; while $T_C$ is historically considered constant along the line because mainly driven by conductor current, $I$, which is a uniform parameter.
- Given (2-6) and (2-7), the line series impedance varies with ambient temperature, $T_A(x)$, as follows:

$$\begin{align*}
r(T_A(x)) &= r(T_0) \cdot [1 + \alpha(T_A(x) - T_0)] \\
x_L(T_A(x)) &= x_L(T_0) \cdot [1 + \beta(T_A(x) - T_0)]
\end{align*}$$

(3-1) (3-2)
Note that the reference impedance values, \( r(T_0) \) and \( x_L(T_0) \), are given by empirical look-up tables (e.g. [33]), in which the temperature of the conductor is assumed to equal ambient temperature, i.e. negligible temperature rise due to current.

Effects of these line models on line terminal behavior were then investigated and compared to commonly used line models. The line model structures used in this work are presented in the following section; then, the mathematical problem formulation is presented in section 3.4.

### 3.3 LINE MODEL STRUCTURES

Given the described assumptions, equivalent circuit representations of a line were developed in this work to evaluate the hypotheses. Following the framework presented in [1], this work utilized finitely segmented line models, created by dividing the single lumped parameter circuit into \( K \) number of segments (Figure 1.1).

To address Sub-Problem 1, a uniform line model segmentation was applied, i.e. all segments have equal length (3-3) and parameter values (3-4).

\[
d_k = d_k \quad \forall k = 1,\ldots,K
\]

\[
Z_k(d_k) = \frac{Z}{K}, \quad Y_k(d_k) = \frac{Y}{K} \quad \forall k = 1,\ldots,K
\]

where:

\( K \in \mathbb{Z}^+ \): number of segments in the model

\( d_k \in \mathbb{R}^+ \): length of segment \( k \) (e.g. miles, ft)

\( Z \in \mathbb{C} \): total series impedance of the line (Ω)
$Y \in \mathbb{C}$: total shunt admittance of the line (S)

$Z_k(d_k) \in \mathbb{C}$: total series impedance of segment $k$ (Ω)

$Y_k(d_k) \in \mathbb{C}$: total shunt admittance of segment $k$ (S)

It is noted that a uniform segmentation was selected given assumption A1 by which all frequency components are assumed uniformly distributed along the line. The relationship of terminal voltages and currents for $K$ uniform Γ-type segments are then given in (3-5):

$$
\begin{bmatrix}
V_R \\
I_R
\end{bmatrix}
= 
\begin{bmatrix}
1 & -Z_k(d_k) \\
-Y_k(d_k) & 1+Y_k(d_k)Z_k(d_k)
\end{bmatrix}
^K
\begin{bmatrix}
V_S \\
I_S
\end{bmatrix}
= 
\begin{bmatrix}
1 & -\frac{Z}{K} \\
-\frac{Y}{K} & 1+\frac{YZ}{K}
\end{bmatrix}
^K
\begin{bmatrix}
V_S \\
I_S
\end{bmatrix}
$$

(3-5)

In this thesis, to address Sub-Problem 2, a non-uniform segmentation of the line model was introduced, i.e. segments can vary in length and parameter values. Taking into account variations in ambient temperature along the line, expressed by $T_A(x)$, implies based on (3-1) and (3-2):

1. A change in line parameters with ambient temperature at a particular point, $x$;

2. A non-uniformity in line parameters with position along the line, $x$, based on the ambient temperature gradient along the line.

The uniformly distributed parameter model and a single lumped parameter line model cannot represent non-uniformity of line parameters along the line, because they inherently assume a single value of conductor temperature for the entire length of the line. A line model structure as shown in Figure 3.3 was therefore utilized, and the relationship between its terminal voltages and currents $(V_S, V_R, I_S, I_R)$ is given (for Γ-equivalent circuits) by equation (3-6), which follows.
In Figure 3.3, the following notation holds:

\[ T_{\text{Seg}k} \in \mathbb{R} : \text{ambient temperature for segment } k \text{ (e.g. } ^\circ\text{C, } ^\circ\text{F}) \]

\[ Z_k(d_k, T_{\text{Seg}k}) \in \mathbb{C} : \text{total series impedance of segment } k \text{ (} \Omega \text{)} \]

\[ Y_k(d_k) \in \mathbb{C} : \text{total shunt admittance of segment } k \text{ (} S \text{)} \]

Please note the dependency of the segment series impedance on segment length, \( d_k \), and segment ambient temperature, \( T_{\text{Seg}k} \), i.e. \( Z_k(d_k, T_{\text{Seg}k}) \), while the segment shunt admittance is only dependent on \( d_k \), \( Y_k(d_k) \), (A8).

\[
\begin{bmatrix}
V_R \\
I_R
\end{bmatrix} =
\begin{bmatrix}
1 & -Z_1 & 1 & -Z_2 & \cdots & 1 & -Z_K \\
-Y_1 & 1+Y_1Z_1 & -Y_2 & 1+Y_2Z_2 & \cdots & -Y_K & 1+Y_KZ_K
\end{bmatrix}
\begin{bmatrix}
V_S \\
I_S
\end{bmatrix}
\]

(3-6)

In both cases, as the number of segments, \( K \), increases, the model tends to a distributed parameter representation of the line:

- for Sub-Problem 1, it tends to the uniformly distributed model (2.3.1), in which the line parameters, \( z \) and \( y \), do not change with \( x \), and
• for Sub-Problem 2, to a non-uniformly distributed parameter model, in which the line series impedance is a function of $x, z(x)$. This model was developed in this work and will be introduced in the next chapter.

Therefore, it is reasonable to assume that the model’s accuracy monotonically increases as the number of segments increases. However, one of the main advantages of using lumped parameter models over the distributed parameter model is to avoid the computational challenges posed by solving differential equations. Moreover, when considering using multi-segment line models in power system studies, the system size is of concern. In fact, as an example, the size of the power flow problem, $N$, which is related to the number of buses in the system, would increase linearly with the number of segments composing each branch, $K_b$:

$$N^1 = N^0 + \sum_{b=1}^{B} K_b - B$$

where:

- $N^0$: size of the original power flow problem, i.e. number of buses
- $N^1$: size of the updated power flow problem
- $B$: number of branches (i.e. lines) in the system
- $K_b$: number of segments in branch $b$

While the proposed line model structure increases system sparsity, in order to maintain the calculation qualities given by lumped parameter models, and to minimize the problem size for system studies, the model segmentation should be minimized.
3.4 MATHEMATICAL FORMULATION

Based on the proposed multi-segment line model structure, the overall line modeling problem can be formulated as a mixed-integer optimization problem. Specifically, the number of segments in the model, $K$, can be minimized given a set of constraints, some of which are common to the overall problem; others are specific to each of the sub-problems.

The following parameters are assumed to be known:

- $\ell$: total line length
- $x_S$: position of the sending-end of the line, usually $x_S = \ell$
- $x_R$: position of the receiving-end of the line, usually $x_R = 0$

The vector of decision variables, $x$, is then identified as representative of the points of intersection between each segment in the model:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{K-1} \end{bmatrix} \in \mathbb{R}^{+K-1}$$ (3-8)

with elements:

$$x_k \in \left\{ x_k : x_R < x_k < x_{k+1} < x_S \right\}, \ \forall k = 1, \ldots, K-1$$ (3-9)

representing the points of intersection between segments, with respect to a common reference point, i.e. usually the receiving-end of the line (e.g. miles, ft, m), and

$$K \in \mathbb{Z}^+: \ \text{positive integer number of segments representing the line (unitless)}$$
Figure 3.4 below shows a generalized graphical representation of the decision variables.

\[ x_0 \equiv x_R \]
\[ x_1 \]
\[ \cdots \]
\[ x_{K-2} \]
\[ x_{K-1} \]
\[ x_K \equiv x_S \]
\[ 0 \]
\[ x \]
\[ \ell \]

**Figure 3.4** Graphical Representation of x With Respect To Sending \((x_S)\) and Receiving \((x_R)\) -Ends of the Line

For ease of notation, as shown in Figure 3.4:

\[
x_K \equiv x_S \\
x_0 \equiv x_R
\]  \hspace{1cm} (3-10)

The number of segments, \(K\), which is the objective function to be minimized, is then related to the dimension of \(x\):

\[ K = \dim(x) + 1 \] \hspace{1cm} (3-11)

The overall minimization problem is formulated as follows:

\[
\min_{x \in \mathbb{R}^{K+1}} \left\{ K \right\}
\] \hspace{1cm} (3-12)

\[
\text{s.t. } x_k < x_{k+1} \forall k = 0, ..., K - 1
\] \hspace{1cm} (3-13)

\[
g(x) \leq 0
\] \hspace{1cm} (3-14)

where (3-14) represents generic inequality constraints that will be defined individually for each sub-problem in the following two sub-sections.

### 3.4.1 LINE MODELING FOR STUDIES AT NON-FUNDAMENTAL FREQUENCIES

Given assumptions A1-A4, the benchmark for all frequencies is considered to be the uniformly distributed parameter model. The objective function to be minimized for
this sub-problem is then formulated as in the overall problem (3-12). (3-13) holds, and two additional inequality constraints are defined:

\[
\left| V_{R_{\text{Dist}}}^{\text{Dist}}(f) - V_{R}(f, x) \right| \leq \Delta V_{\text{Threshold}} \quad (3-15)
\]

\[
\left| \theta_{R_{\text{Dist}}}^{\text{Dist}}(f) - \theta_{R}(f, x) \right| \leq \Delta \theta_{\text{Threshold}} \quad (3-16)
\]

In (3-15) and (3-16):

\[
V_{R}(f, x) = |V_{R}| \angle \theta_{R} : \text{receiving-end voltage of the segmented line model}
\]

\[
V_{R_{\text{Dist}}}^{\text{Dist}}(f) = |V_{R_{\text{Dist}}}| \angle \theta_{R_{\text{Dist}}} : \text{receiving-end voltage of the uniformly distributed model} - \text{benchmark (2-13)}
\]

\[
\Delta V_{\text{Threshold}} : \text{threshold value on difference in receiving-end voltage magnitude}
\]

\[
\Delta \theta_{\text{Threshold}} : \text{threshold value on difference in receiving-end voltage phase angle}
\]

Please note that the notation \( V_{R_{\text{Dist}}}^{\text{Dist}}(f) \) is used to underline the dependency of the distributed parameter model receiving-end voltage (both magnitude and phase) on operating frequency, \( f \); while \( V_{R}(x, f) \) is used to underline the dependency of the receiving-end voltage of the segmented line model not only on the operating frequency, but also on the decision variables, \( x \) (and therefore on the number of segments in the model, \( K \)).

The dependency of \( V_{R} \) and \( I_{R} \) on \( x \) and \( K \) is expressed in (3-5). Also note that the segment length, \( d_{k} \), in (3-5) is dependent on \( x \) as follows:

\[
d_{k} = x_{k} - x_{k-1} \quad \forall k = 1, \ldots, K \quad (3-17)
\]
The threshold values, $\Delta V_{\text{Threshold}}$ and $\Delta \theta_{\text{Threshold}}$, are given scalar values. $V^\text{Dist}_R$, as well as $I^\text{Dist}_R$ in (2-13), represent the benchmark behavior, and are obtained by solving the voltage and current relationship of a uniformly-distributed line model (2-13).

In [1] and [2], a uniform line model segmentation was proposed to solve this problem. While no formal problem statement was included, the developed modeling approach found a solution through an exhaustive algorithm which incrementally increased the number of segments. This thesis therefore focused on developing and testing an approach to provide a solution for Sub-Problem 2: incorporating ambient temperature variations within the line model. A formulation for this sub-problem is defined in the following sub-section.

### 3.4.2 INCORPORATING AMBIENT TEMPERATURE VARIATIONS

The objective function to be minimized for this sub-problem is formulated in (3-12). (3-13) holds, and an additional inequality constraint is defined as follows:

$$
|T_A(x_k) - T_A(x_{k-1})| \leq \Delta T_{\text{Threshold}}, \quad \forall k = 1, \ldots, K
$$

(3-18)

where:

$T_A(x_k)$: ambient temperature at point $x_k$

$\Delta T_{\text{Threshold}}$: allowable temperature difference across any segment of the line model

$\Delta T_{\text{Threshold}}$ is given/set. $T_A(x_k)$ and $T_A(x_{k-1})$ are either given or can be defined based on available temperature measurements and their locations, e.g. with a piecewise linear model.

If a benchmark model is defined, such as a non-uniformly distributed parameter model (introduced in the next chapter, section 4.2), or a finitely segmented model
obtained by setting $\Delta T_{\text{Threshold}}$ in (3-18) very small (i.e. as $\Delta T_{\text{Threshold}} \to 0$, $K \to \infty$), the constraints for this sub-problem can be reformulated based on electrical parameters of interest. For example, thresholds can be put on the difference between maximum power transfer or load voltage magnitude, between the benchmark and the segmented model. Examples of additional constraints for Sub-Problem 2 in the presence of an electrical benchmark are shown below:

$$
\left| V_R^{\text{Bench}}(T_A(x)) - V_R(T_A(x), x) \right| \leq \Delta V_{\text{Threshold}} \\
\left| P_{\text{max}}^{\text{Bench}}(T_A(x)) - P_{\text{max}}(T_A(x), x) \right| \leq \Delta P_{\text{Threshold}} \\
\left| pf^{\text{Bench}}(T_A(x)) - pf(T_A(x), x) \right| \leq \Delta pf_{\text{Threshold}} \quad (3-19)
$$

where:

- $V_R(T_A(x), x)$: receiving-end voltage of the segmented line model
- $V_R^{\text{Bench}}(T_A(x))$: receiving-end voltage of the benchmark model
- $P_{\text{max}}(T_A(x), x)$: maximum real power transfer point of the segmented line model
- $P_{\text{max}}^{\text{Bench}}(T_A(x))$: maximum real power transfer point of the benchmark model
- $pf(T_A(x), x)$: power factor of the segmented line model
- $pf^{\text{Bench}}(T_A(x))$: power factor of the benchmark model
- $\Delta V_{\text{Threshold}}$: threshold value on difference in receiving-end voltage magnitude
- $\Delta P_{\text{Threshold}}$: threshold value on difference in maximum power transfer
- $\Delta pf_{\text{Threshold}}$: threshold value on difference in power factor

Please note that the dependency of $V_R$ and $I_R$ on $x$ and $K$ for the non-uniformly segmented line model is expressed in (3-6).
In order to address the line modeling problem, various techniques were investigated and included the use of both distributed and lumped parameter models. The proposed line model structures are presented in the next chapter and include:

- a non-uniformly distributed parameter model, and
- a multiple non-uniform-segment model.

The line modeling procedure developed to determine an appropriate line model segmentation given a set of ambient temperature measurements is also discussed in the next chapter.
CHAPTER 4: LINE MODELING APPROACH

4.1 OVERVIEW

This chapter discusses the line modeling approach used to incorporate ambient temperature information. Specifically, it presents:

- In Section 4.2, a non-uniformly distributed parameter line model, developed to represent a continuous non-uniformity of line parameters along the line, caused by a temperature gradient;
- In Section 4.3, a lumped parameter model composed of multiple non-uniform segments;
- In Section 4.4, a line modeling procedure developed to determine appropriate line model segmentation and segment parameter values based on temperature information. The procedure provides a solution to the problem formulated in Section 3.4.2. Two line model segmentation algorithms were developed and are presented in 4.4.1 and 4.4.2. The first algorithm (4.4.1) is utilized in the subsequent chapters;
- In Section 4.5, alternative line models that account for ambient temperature information by utilizing traditional line model structures.

4.2 THE NON-UNIFORMLY DISTRIBUTED PARAMETER MODEL

In order to address the non-uniformity of line parameters along the line, caused by an ambient temperature gradient, a non-uniformly distributed parameter model was
developed. It is noted that the traditional uniformly distributed parameter model (presented in 2.3.1) is not able to represent this non-uniformity of line parameters along the line, i.e. per unit length parameters $z$ and $y$ are not a function of distance along the line, $x$.

The non-uniformly distributed parameter model is characterized by a dependency of the line distributed impedance on distance along the line $x$:

$$z(x) = z + \Delta z(x)$$

(4-1)

Please note that the line distributed shunt admittance, $y$, is not considered to be a function of $x$ based on assumption A8. If this assumption was relaxed, the dependency of $y$ on $x$ could be treated in a similar manner.

A representation of a differential section, of length $dx$, for this model is shown in Figure 4.1, and a derivation of the model follows.

![Figure 4.1 Representation of a Non-Uniformly Distributed Parameter Model](image)

The following notation holds:

- $V_S$ and $V_R$: sending-end and receiving-end voltages (V)
- $I_S$ and $I_R$: sending-end and receiving-end current (A)
- $x$: position along the line (miles, ft, m, ...)

...
The distributed series impedance of the line, \( z \), is a function of ambient temperature \( T_A(x) \), which changes with distance along the line, \( x \):

\[
z(T_A(x)) = r(T_A(x)) + jx_L(T_A(x))
\]

(4-2)

where, given (3-1) and (3-2):

\[
r(T_A(x)) = r(T_0) \cdot \left[ 1 + \alpha(T_A(x) - T_0) \right] = \underbrace{r_0 - \alpha r_0 T_0}_{\text{not dependent on } x} + \underbrace{\alpha r_0 T_A(x)}_{\text{dependent on } x}
\]

\[
x_L(T_A(x)) = x_L(T_0) \cdot \left[ 1 + \beta(T_A(x) - T_0) \right] = \underbrace{x_{L,0} - \beta x_{L,0} T_0}_{\text{not dependent on } x} + \underbrace{\beta x_{L,0} T_A(x)}_{\text{dependent on } x}
\]

(4-3)

and

\[
T_0: \quad \text{reference temperature}
\]

\[
r_0: \quad \text{reference series resistance (} \Omega \text{) per unit length, } r_0 = r(T_0)
\]

\[
x_{L,0}: \quad \text{reference series inductive reactance (} \Omega \text{) per unit length, } x_{L,0} = x_L(T_0)
\]

Therefore, a portion, \( z \), of the series impedance is constant along the line, and a portion, \( \Delta z(x) \), depends on \( x \):

\[
z = (r_0 - \alpha r_0 T_0) + j(x_{L,0} - \beta x_{L,0} T_0)
\]

(4-4)

\[
\Delta z(x) = \alpha r_0 T_A(x) + j \beta x_{L,0} T_A(x)
\]

(4-5)

The relationships between the terminal voltages and currents can then be defined with the following 1st-order differential equations:
\[
\frac{dV}{dx} = I(x)(z + \Delta z(x)) \\
\frac{dI}{dx} = V(x)y
\]  
resulting in the following 2\textsuperscript{nd}-order equations:

\[
\frac{d^2V}{dx^2} = (z + \Delta z(x))\frac{dI(x)}{dx} + I(x) \frac{d\Delta z}{dx} = y(z + \Delta z(x)) V(x) + \frac{1}{(z + \Delta z(x))} \frac{d\Delta z}{dx} \frac{dV(x)}{dx}
\]

(4-7)

where:

\[
\frac{d\Delta z}{dx} = (\alpha r_0 + j \beta x_{L_0}) \frac{dT_A(x)}{dx}
\]

(4-8)

If a continuously differentiable function of ambient temperature \(T_A(x)\) is given, solutions to the differential equations in (4-7) can be found. \(V(x)\) and \(I(x)\) would then provide voltage and current at any specified point along the line. Additional details on the derivation of the non-uniformly distributed line model can be found in Appendix B.

A lumped parameter model composed of multiple non-uniform segments (3-6) can be used to approximate the non-uniformly distributed parameter model. A methodology developed to segment the line model based on ambient temperature information is presented in the next section and provides a solution to the line modeling problem as formulated in Section 3.4.2.

4.3 MULTIPLE NON-UNIFORM LUMPED SEGMENTS

In order to address the non-uniformity of line parameters along the line caused by an ambient temperature gradient, this work proposed a multiple segmentation of the line model. In this model structure composed of \(K\)-segments, each segment, \(k\), is characterized
by its own per unit length resistance and inductance values, \( r_k \) and \( l_k \), as well as its own length, \( d_k \). Line parameters \( r_k \) and \( l_k \) are a function of temperature and therefore can vary based on the segment’s position along the line, while the per unit length capacitance, \( c \), is assumed to be constant with temperature (A8), i.e. constant along the line. Therefore, for segment \( k = 1, \ldots, K \):

\[
R_k(d_k, T_{Segk}) = r_k(T_{Segk}) \ d_k : \text{series resistance of segment } k \ (\Omega)
\]

\[
L_k(d_k, T_{Segk}) = l_k(T_{Segk}) \ d_k : \text{series inductance of segment } k \ (H)
\]

\[
C_k(d_k) = c \ d_k : \text{shunt capacitance of segment } k \ (F)
\]

Each segment length, \( d_k \), is selected to model a section of the line with an ambient temperature variation below a given temperature threshold. The segment ambient temperature, \( T_{Segk} \), is the temperature value used to calculate the per unit length parameters for segment \( k \), \( r_k(T_{Segk}) \) and \( l_k(T_{Segk}) \).

Figure 4.2 in the next page shows a graphical representation of an example ambient temperature profile, \( T_A(x) \), and corresponding line model segmentation of 3 non-uniform \( \Gamma \)-type segments. Figure 4.2 also shows the example temperature value used to calculate parameters \( R_k \) and \( L_k \) for each of the three segments \( T_{Seg1}, T_{Seg2}, T_{Seg3} \) as well as the length of each segment \( (d_1, d_2, d_3) \).

A line modeling approach was developed to determine model segmentation and segment parameter values to account for ambient temperature variations along the line. Detailed step-by-step procedure is presented in the following sub-section. Please note that in the limit, the segmentation procedure would result in a non-uniformly distributed parameter line model as described in Section 4.2.
Figure 4.2 Graphical Representation of an Example Temperature Profile and Corresponding Non-Uniformly Segmented Line Model, $K = 3$

4.4 LINE MODELING PROCEDURE

In order to capture the non-uniformity of line parameters caused by ambient temperature gradients, a line modeling procedure was developed. In the limit, as the allowable temperature difference across any segment of the line model, $\Delta T_{\text{Threshold}}$ in (3-18), tends to zero, the modeling approach tends to a distributed parameter line model, in which non-uniformity could result due to the rate of change of ambient temperature along the line, $dT_d(x)/dx$ in (4-8). As $\Delta T_{\text{Threshold}}$ increases, the modeling procedure then gives a finite number of multiple lumped segment: the number of segments, $K$, their lengths, $d_k$, and corresponding parameter values, $Z_k \left(d_k, T_{\text{Segk}} \right)$ and $Y_k \left(d_k \right)$.

The line model segmentation technique is presented here. It utilizes the framework in [1] and [2], but differs from it in two major aspects:
the segmentation is based on temperature information,

- the segments can be non-uniform in length and in per unit length parameter values (due to temperature).

Given a set of temperature measurement points, the inputs and outputs of the modeling approach are shown in Figure 4.3 below.

![Figure 4.3 Line Modeling Approach – Inputs and Outputs](image)

The modeling approach requires the following inputs:

1. Transmission line length, $\ell$;
2. Cable type, i.e., $r_0, x_{L0}, b$, as well as $\alpha$ and $\beta$ coefficients;
3. Desired circuit model type, such as $\pi$ or $\Gamma$ forms;
4. Temperature measurement locations, and
5. Temperature measurements.

The user-defined parameter $\Delta T_{\text{Threshold}}$ is selected and it identifies an acceptable temperature variation across a single segment of the line model (3-18). Then, the tool outputs consist of:
O1. Number of segments, \( K \), and their location,
\[ i.e. \quad x_k, \forall k = 1, ..., K - 1 \quad \text{and} \quad d_k, \forall k = 1, ..., K; \]

O2. Lumped parameter values for each segment,
\[ i.e. \quad R_k, L_k, C_k, \quad \forall k = 1, ..., K. \]

A relatively high value of \( \Delta T_{\text{Threshold}} \) would result in a single or few lumped segments. On the contrary, as \( \Delta T_{\text{Threshold}} \) tends to zero, the number of segments would tend to infinity. Studies on the sensitivity of the resulting model to \( \Delta T_{\text{Threshold}} \) are included in Chapter 5.

Two variations of the line modeling approach were developed. The two algorithms differ in the way the line model segmentation is determined, i.e. number of segments and length of each segment. Once the segmentation has been defined, the calculation of segment parameter values is identical.

Algorithm 1: Temperature checkpoints are created at uniform intervals along the line, based on a temperature profile derived from temperature measurements. The model segmentation is then determined based on the difference in temperature between checkpoints. The resulting segmentation provides a solution to the problem in Section 3.18. Details of this algorithm are presented in sub-section 4.4.1.

Algorithm 2: Moving from the sending-end of the line towards the receiving-end, segmentation occurs at points along the line between which the temperature difference reaches the threshold, \( \Delta T_{\text{Threshold}} \). Details of this algorithm are presented in sub-section 4.4.2.
Please note that, for both algorithms, a monotonic function of temperature is assumed between any two successive temperature measurement points. Moreover, Algorithm 2 provides a solution to a stricter constraint than (3-18). Specifically, the temperature difference between any two points on a segment is within the temperature threshold $\Delta T_{\text{Threshold}}$. The resulting model satisfies:

$$
\left| T_A(x_i) - T_A(x_j) \right| \leq \Delta T_{\text{Threshold}}, \\
\forall x_i, x_j \in \left\{ x_i, x_j : x_k \leq x_i, x_j \leq x_{k-1} \quad \forall k = 1, ..., K \right\}
$$

where $x_i$ and $x_j$ are any two points along the segment between $x_k$ and $x_{k-1}$.

### 4.4.1 ALGORITHM 1

This line modeling algorithm consists of four main steps. A procedural overview of the methodology is shown in Figure 4.4, while detailed sub-steps are described below.

- **Step 1.** Identification of the temperature profile
- **Step 2.** Designation of temperature checkpoints
- **Step 3.** Determination of line model segmentation
- **Step 4.** Calculation of segment parameter values

Figure 4.4 Algorithm 1. Procedural Overview
Step 1. Algorithm 1: Identification of the temperature profile

**Step 1.1** Define $T_A(x)$, temperature as a function of distance along the line, $x$, based on the temperature measurement points (I4 and I5).

**Step 1.2** Based on $T_A(x)$, determine, if unknown, $T_R$ and $T_S$, the temperature at the receiving ($x_R$) and sending ($x_S$) – ends of the line, respectively.

Please note that a monotonic function of temperature $T_A(x)$ is assumed between any two successive temperature measurement points. For example, a piecewise linear model of temperature can be used, as shown in Figure 4.5(a).

Step 2. Algorithm 1: Designation of temperature checkpoints

**Step 2.1** For the evaluation of temperature difference, checkpoints are designated every $\Delta x$ distance along the line, creating a uniform discretization of the $x$-axis, i.e. of the line length. Determine the distance between temperature checkpoints, $\Delta x$:

\[
\Delta x \leq \min\left(m_{ab}, m_{Sa}, m_{Ra}\right) \forall a, b \tag{4-11}
\]

and

\[
\Delta x = \ell/M \tag{4-12}
\]

where:

$\Delta x$ : distance between temperature checkpoints

$m_{ab}$ : distance between measurement points $a$ and $b$

$m_{Sa}$ : distance between the line sending-end, $x_S$, and measurement point $a$

$m_{Ra}$ : distance between the line receiving-end, $x_R$, and measurement point $a$
\[
M = \left\lceil \frac{\ell}{\min(m_{ab}, m_{ba}, m_{na})} \right\rceil \quad \forall \ a, b
\] (4-13)

Note: \( \lceil \cdot \rceil \) denotes the ceiling function.

**Step 2.2** Create \( M + 1 \) temperature checkpoints: starting at \( x_S \) and moving towards \( x_R \), place a checkpoint every \( \Delta x \) distance along the line \((u \cdot \Delta x, \forall u = 0,1,\ldots,M)\). Please note that the first and the last checkpoints, i.e. \( 0\Delta x \) and \( M\Delta x \), correspond to the sending, \( x_S \), and receiving, \( x_R \), – ends of the line, respectively.

**Step 2.3** Based on \( T_A(x) \), calculate the temperature at each checkpoint.

An example temperature profile with temperature checkpoints and corresponding temperatures is shown in Figure 4.5(b).

![Temperature Profile](image)

**Figure 4.5** Example of Piecewise Linear Temperature Profile Showing \( T_R \) and \( T_S \) as well as:
(a) Measurement Points Mst 1-4, and (b) Temperature Checkpoints 0-5 used in Algorithm 1

**Step 3. Algorithm 1: Determination of line model segmentation**

**Step 3.1** Identify:

\( i) \) number of segments, \( K \), and

\( ii) \) length of each segment, \( d_k, \forall k = 1,\ldots,K \),
by following the segmentation procedure shown in the flow chart of Figure 4.6.

Starting at the sending-end of the line and moving towards the receiving-end, segmentation occurs based on temperature difference between checkpoints. This temperature difference is normalized against the parameter $\Delta T_{\text{Threshold}}$ and is defined as:

$$n_{ij} := \frac{|T_i - T_j|}{\Delta T_{\text{Threshold}}}$$  \hspace{1cm} (4-14)

where $T_i$ and $T_j$ are ambient temperatures calculated at checkpoints $i$ and $j$, respectively. In the flow chart, $i$ and $j$ represent current temperature checkpoints, while $q$ represents position along the line. As segmentation occurs, $q$ tracks the end of the most recently segmented portion of the line.

Please note that the decision variables, $x$ as expressed in (3-8), are determined in Step 3.1, and correspond to the selection of the temperature checkpoints at which segmentation occurs, i.e. $\left\lceil n_{ij} \right\rceil \geq 1$, as well as points in between checkpoints where $\left\lceil n_{ij} \right\rceil \geq 2$. The decision variables $x_k$ are related to the segment length $d_k$ as follows:

$$d_k = x_k - x_{k-1} \quad \forall k = 1,\ldots, K$$  \hspace{1cm} (4-15)

A segmentation sequence example (Step 3) can be found in Appendix C.
Figure 4.6 Algorithm 1. Step 3 - Flow Chart for Determining Line Model Segmentation:

i) # of segments, and ii) beginning and end of each segment, i.e. segment lengths
Step 4. Algorithm 1: Calculation of segment parameter values

Step 4.1 Calculate an average temperature for each segment. For segment \( k \), between points \( x_k \) and \( x_{k-1} \):

\[
T_{\text{Seg}k} = \text{Average}(T_{n_k}^{k})
\]  

(4-16)

Step 4.2 Following equations (3-1) and (3-2), determine the corresponding parameter values of each segment:

\[
R_k(d_k, T_{\text{Seg}k}), L_k(d_k, T_{\text{Seg}k}) \text{ and } C_k(d_k).
\]

In the limit, as \( \Delta T_{\text{Threshold}} \) tends to zero, the algorithm would result in a distributed parameter model of the line. Specifically, if no temperature variation is experienced along the line, the result would be a uniformly distributed model; if a temperature variation occurs, a non-uniformly distributed model would result instead.

Moreover, the modeling tool results in a 1:1 mapping between its inputs (I1-I5) and outputs (O1-O2). Although in general, starting at the sending-end of the line, versus the receiving-end, may not result in the same model segmentation, as \( \Delta T_{\text{Threshold}} \rightarrow 0 \), the model segmentation converges to the same solution.

Given \( \Delta T_{\text{Threshold}} \neq 0 \), the number of segments \( K \) can always be determined and is bounded by:

\[
1 \leq K \leq \sum_{i=1}^{M} \left\lceil \frac{T_i - T_{i+1}}{\Delta T_{\text{Threshold}}} \right\rceil
\]

(4-17)

where

\( i \) : temperature checkpoint, \( i = 1, \ldots, M + 1 \)
Also, through a relaxation of the inequality constraint in (3-18) given by:

$$\frac{2}{3} \left| T_A (x_k) - T_A (x_{k-1}) \right| < \Delta T_{\text{Threshold}}, \quad \forall k = 1, \ldots, K$$

(4-18)

where:

$$T_A (x_k) : \text{ ambient temperature at point along the line, } x_k$$

the line modeling tool provides a feasible solution to the minimization problem described in 3.4.2. Please note that the coefficient 2/3 in (4-18) comes from rounding off to the nearest integer in the segmentation algorithm (Step 3).

The line modeling procedure utilizes temperature measurement data to recreate a temperature profile along the line. The entire length of the line, i.e. the $x$-axis, is then uniformly discretized through temperature checkpoints used for evaluation of temperature differences. It is noted that through this discretization process, the actual temperature measurement points may not exactly correspond to temperature checkpoints.

Algorithm 1 is utilized in the subsequent chapters to determine multiple non-uniform-segment models.

### 4.4.2 ALGORITHM 2

This algorithm consists of three main steps:

Step 1. Identification of the temperature profile – same as Algorithm 1;

Step 2. Determination of line model segmentation, utilizing temperature measurement points as checkpoints;

Step 3. Calculation of segment parameter values – same as Algorithm 1.

Detailed steps are described below.
Step 1. **Algorithm 2: Identification of the temperature profile**

**Step 1.1** Define $T_A(x)$, temperature as a function of distance along the line, $x$, based on the temperature measurement points (I4 and I5).

**Step 1.2** Based on $T_A(x)$, determine, if unknown, $T_R$ and $T_S$, the temperature at the receiving ($x_R$) and sending ($x_S$) – ends of the line, respectively.

Please note that a monotonic function of temperature $T_A(x)$ is assumed between any two consecutive temperature measurement points. For example, a piecewise linear model of temperature can be used, as shown in Figure 4.7 below.

![Figure 4.7 Example of Piecewise Linear Temperature Profile: $T_R$ and $T_S$ and Mst 1-4](image)

Step 2. **Algorithm 2: Determination of line model segmentation**

**Step 2.1** Starting at the sending-end of the line ($x_S$) and moving towards the receiving-end ($x_R$), segmentation occurs at every point along the line at which the threshold on temperature difference across a segment, $\Delta T_{Threshold}$, is reached. The detailed segmentation procedure is shown in the flow chart of Figure 4.8 on page 58. An illustrative example of temperature profile and corresponding segmentation based on this method is shown in Figure 4.9.
Figure 4.8 Algorithm 2. Step 2 - Flow Chart for Determining Line Model Segmentation:

i) # of segments, and ii) beginning and end of each segment, i.e. segment lengths
Please note that in the flow chart the following notation holds:

\( x_i \) and \( x_j \): points along the line for evaluation of temperature difference

\( x_{\text{mst}} \): point along the line tracking a local extremum, i.e. a point at which the temperature function changes sign.

Figure 4.9 Example of Temperature Profile and Corresponding Line Model Segmentation Obtained with Algorithm 2

**Step 3. Algorithm 2: Calculation of segment parameter values**

**Step 3.1** Calculate an average temperature for each segment, as in (4-16).

**Step 3.2** Following equations (3-1) and (3-2), determine the corresponding parameter values of each segment: \( R_k \left( d_k, T_{\text{Seg}_k} \right) \), \( L_k \left( d_k, T_{\text{Seg}_k} \right) \) and \( C_k \left( d_k \right) \).
Another line model segmentation procedure could also be developed to include actual temperature measurements as checkpoints for evaluation of temperature difference, and perform segmentation following the methodology presented in Section 4.4.1 (Figure 4.6). For example, the temperature checkpoints could be created at:

- the temperature measurement points, and
- between any two consecutive temperature measurement points, at intervals based on the rate of change of temperature between the two measurement points, which would result in a non-uniform discretization of the $x$-axis.

Line segmentation algorithm 1 (presented in Section 4.4.1) has been automated and utilized in the subsequent chapters for single-line model evaluation, and for the investigation of line model’s impacts on system studies. Alternative line and temperature models developed and used for model comparison are presented in the next section.

### 4.5 ALTERNATIVE PROPOSED LINE AND TEMPERATURE MODELS

Line models that incorporate ambient temperature information by utilizing traditional line model structures, i.e. uniformly distributed parameter and single-segment lumped parameter models, were also developed and were utilized for comparison to the multiple non-uniform segment line model (4.3).

The uniformly distributed parameter model, as described in 2.3.1, is characterized by a uniform distribution of the line parameters, quantified in per unit length, along the entire length of the line. Therefore, temperature information can be embedded in this line model by adjusting per unit length parameters, through equations 3.1 and 3.2, based on a single “lumped” value of temperature, considered constant along the line.
Similar to the uniformly distributed parameter model, the single segment line models, such as a \( \pi \) or \( \Gamma \)-type (2.3.2), cannot represent non-uniformity in line parameters, i.e. the lumped parameters are calculated at a single temperature value. Therefore, in order to incorporate temperature information, a “lumped” temperature value is used to adjust the line parameters.

Four ambient temperature models were considered:

1. \( T_{Seas} \) – a pre-set seasonal temperature.

   This is the currently used method to include ambient temperature in power system analysis tools.

   If ambient temperature measurements along the line are available, other possible modeling techniques were considered:

2. \( T_{avg} \) – average temperature value calculated based on measured or estimated temperatures at the sending and receiving-ends of the line (\( T_S \) and \( T_R \), respectively), i.e.

   \[
   T_{avg} = \frac{T_S + T_R}{2}
   \]  

(4-19)

3. \( T_{Wavg} \) – weighted average value based on all available measurements and their locations along the line, i.e.

   \[
   T_{Wavg} = \sum_{a=1}^{N+1} \left[ \frac{T_a + T_{a+1}}{2} \times \frac{m_{a,a+1}}{\ell} \right]
   \]  

(4-20)

where

\( N \): total # of temperature measurement points

\( a \): temperature measurement point (\( a = 1, \ldots, N \))
Temperature measured at point $a$ \\

Distance between measurement point, $a$, and the subsequent one, $a+1$ \\

Line length \\

4. $T_{\text{Seg}}$ – average value calculated based on estimated temperatures at the beginning and end of each section (i.e. segment) of the line.

The last temperature model, $T_{\text{Seg}}$, implies a non-uniformly segmented line model structure (as described in 4.3), selected in order to capture ambient temperature variations along the line. Parameters of each segment are then calculated based on the corresponding $T_{\text{Seg}}$.

Several considerations can be made on the described temperature models:

- $T_{\text{Seas}}$ is a predefined value, not based on current temperature measurements, and a single value of $T_{\text{Seas}}$ is used for every branch in the power system;
- $T_{\text{Seas}}$, $T_{\text{avg}}$, and $T_{\text{Wavg}}$ define a single (“lumped”) value of ambient temperature for the entire length of the line;
- $T_{\text{Seg}}$ defines multiple ambient temperature values along the line;
- $T_{\text{avg}}$, $T_{\text{Wavg}}$ and $T_{\text{Seg}}$ take into account available temperature information, and line parameters of each branch in the system can be computed accordingly.

The line modeling algorithm presented in 4.4.1 has been automated and implemented in Matlab. Results of the modeling tool for an example case study composed of a single line are presented in chapter 5, as well as comparisons, through simulation, of the different line models, i.e. “lumped”-temperature and non-uniformly segmented models. In order to investigate proposed models’ impacts on multi-bus systems, the tool was integrated within state-of-the-art load flow software. Tool integration as well as test results for a multi-bus case study are presented in Chapter 5.
CHAPTER 5:  MODEL EVALUATION AND SIMULATION RESULTS

5.1 OVERVIEW

For validation and demonstration purposes, the line modeling approach has been tested on hypothetical lines for given ambient conditions. Its outcome has been compared to traditional line model structures using the different ambient temperature models presented in the previous chapter. The differences between the existing state-of-the-art models and the ones incorporating temperature information are investigated through a single line case study. Metrics for comparison were selected as: line attenuation characteristics and maximum power transfer.

5.2 SINGLE-LINE CASE STUDY 1: ACTUAL AMBIENT TEMPERATURE MEASUREMENTS

In order to illustrate the proposed modeling approach to incorporate ambient temperature variations, a hypothetical medium-length line running from Philadelphia, PA to Orange County, NY was selected. Details on the line characteristics are given in sub-section 5.2.1. The multiple non-uniform segment line models that result from the proposed line modeling approach (Algorithm 1, Section 4.4.1) are presented in sub-section 5.2.2 for three different values of $\Delta T_{Threshold}$. Several models of the given line were then developed and simulated in order to perform model comparisons using Cadence PSpice [48][49]. Their performance was characterized through line propagation and system metrics:
• Voltage attenuation,
• Maximum power transfer.

All developed line models of the given line are presented in 5.2.3; the simulation setup used to compare the models is briefly described in 5.2.4; results of the model comparison are then discussed in 5.2.5.

5.2.1 LINE AND TEMPERATURE MEASUREMENT DATA

The hypothetical line has the following characteristics, presented in terms of inputs to the multiple-segment modeling approach (Figure 4.3):

1. Line length: 138.35 miles
2. Cable type: Falcon [33] in delta configuration
3. Lumped model type: gamma
4. Temperature measurement locations: See Table 5.1
5. Temperature measurements: See Table 5.1

Temperature measurement data of an example date and time was obtained from NOAA [50] at select weather stations based on their geographical location.

The Falcon cable in delta configuration has the following characteristics, defined at a reference temperature of 20 °C and at a frequency of 60 Hz:

\[ r = 0.0612 \, \Omega/\text{mi} \]
\[ x_L = 0.6081 \, \Omega/\text{mi} \]
\[ x_c = 0.1423 \, \text{M} \Omega/\text{mi} \]

Also, the temperature coefficient of resistance for Falcon cable was determined to be:

\[ \alpha = 0.003 \, (1/\text{°C}) \]
interpolated from values in [33], and the temperature coefficient of reactance $\beta$ was assumed to equal $\alpha$.

Table 5.1 Single-Line Case Study 1:
Temperature Measurement Location and Corresponding Distance along the Line

(Sending-End, $x_S = \ell$, to Receiving-End, $x_R = 0$)

<table>
<thead>
<tr>
<th>Mst Point</th>
<th>Location (Lat / Lon)</th>
<th>$x$ (mi)</th>
<th>$T_A$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>39°52'N / 75°14'W</td>
<td>138.35</td>
<td>26.1</td>
</tr>
<tr>
<td>2</td>
<td>40°05'N / 75°01'W</td>
<td>119.4</td>
<td>26.1</td>
</tr>
<tr>
<td>3</td>
<td>40°17'N / 74°49'W</td>
<td>102</td>
<td>23.9</td>
</tr>
<tr>
<td>4</td>
<td>40°37'N / 74°40'W</td>
<td>77.6</td>
<td>22.2</td>
</tr>
<tr>
<td>5</td>
<td>40°48'N / 74°25'W</td>
<td>59.2</td>
<td>22.2</td>
</tr>
<tr>
<td>6</td>
<td>40°53'N / 74°17'W</td>
<td>50.2</td>
<td>22.2</td>
</tr>
<tr>
<td>7</td>
<td>41°31'N / 74°16'W</td>
<td>0</td>
<td>22.8</td>
</tr>
</tbody>
</table>

For the example case study:

- $T_{Seas} = 19.7$ °C (May – October)
- $T_{avg} = 24.4$ °C
- $T_{Wavg} = 23.3$ °C

For the given line and ambient temperature data, the multiple non-uniform segment line models obtained from the line model segmentation algorithm 1 (4.4.1) are presented in the next sub-section for three different $\Delta T_{Threshold}$ values.
5.2.2 MODELING TOOL OUTPUT: MULTIPLE NON-UNIFORM LUMPED SEGMENTS

For the single-line case study 1 with given ambient temperature measurements, the output of the modeling tool (Algorithm 1) presented in 4.4.1, i.e. O1. Number of segments, K, and their location, and O2. Lumped parameter values for each segment, are presented in this section for three values of the user-defined parameter ΔT_{Threshold}:

• ΔT_{Threshold} = 1°C result in a line model composed of 8 non-uniform segments; tool outputs are shown in Table 5.2 and, graphically, in Figure 5.1.

• ΔT_{Threshold} = 1.5°C result in a line model composed of 5 non-uniform segments; tool outputs are shown in Table 5.3 and Figure 5.2.

• For ΔT_{Threshold} = 2°C result in a line model composed of 4 non-uniform segments; tool outputs are shown in Table 5.4 and Figure 5.3.

Tables 5.2-5.4 show the number of segments, K, the location and length of each segment, the average temperature, \( T_{Seg} \), and the corresponding parameter values of each segment \( k \) in the resulting line model.

Figures 5.1-5.3 show plots of ambient temperature versus distance along the line, \( x \), and indicate the temperature measurement points as well as \( T_{Seg} \) for each segment in the line model.
Table 5.2  Single-Line Case Study 1: Modeling Tool Outputs O1 and O2 for $\Delta T_{\text{Threshold}} = 1^\circ\text{C}$

$\Delta T_{\text{Threshold}} = 1^\circ\text{C}$

<table>
<thead>
<tr>
<th>Seg #, $k$</th>
<th>Location, $x$</th>
<th>Length, $d_k$ (mi)</th>
<th>$T_{\text{Seg}}$ ($^\circ\text{C}$)</th>
<th>Segment Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>from to</td>
<td></td>
<td></td>
<td>$R_k$ (Ω)</td>
</tr>
<tr>
<td>1</td>
<td>0 51.88</td>
<td>51.88</td>
<td>22.50</td>
<td>3.1989</td>
</tr>
<tr>
<td>2</td>
<td>51.88 77.82</td>
<td>25.94</td>
<td>22.23</td>
<td>1.5982</td>
</tr>
<tr>
<td>3</td>
<td>77.82 86.47</td>
<td>8.65</td>
<td>22.53</td>
<td>0.5332</td>
</tr>
<tr>
<td>4</td>
<td>86.47 95.12</td>
<td>8.65</td>
<td>23.12</td>
<td>0.5341</td>
</tr>
<tr>
<td>5</td>
<td>95.12 103.77</td>
<td>8.65</td>
<td>23.77</td>
<td>0.5352</td>
</tr>
<tr>
<td>6</td>
<td>103.77 112.42</td>
<td>8.65</td>
<td>24.67</td>
<td>0.5366</td>
</tr>
<tr>
<td>7</td>
<td>112.42 121.07</td>
<td>8.65</td>
<td>25.66</td>
<td>0.5382</td>
</tr>
<tr>
<td>8</td>
<td>121.07 138.36</td>
<td>17.29</td>
<td>26.11</td>
<td>1.0778</td>
</tr>
</tbody>
</table>

Figure 5.1  Single-Line Case Study 1: Temperature vs. Position Along the Line, Showing Temperature Measurements and Corresponding Line Model Segmentation ($\Delta T_{\text{Threshold}} = 1^\circ\text{C}$)
Table 5.3  Single-Line Case Study 1: Modeling Tool Outputs O1 and O2 for $\Delta T_{\text{Threshold}} = 1.5^\circ\text{C}$

<table>
<thead>
<tr>
<th>$\Delta T_{\text{Threshold}} = 1.5^\circ\text{C}$</th>
<th>K = 5</th>
<th>Location, x</th>
<th>Length, $d_k$</th>
<th>$T_{\text{Seg}}$</th>
<th>Segment Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seg #, $k$</td>
<td>from to (mi)</td>
<td>(°C)</td>
<td>$R_k$ (Ω)</td>
<td>$L_k$ (mH)</td>
<td>C (μF)</td>
</tr>
<tr>
<td>1</td>
<td>0 86.47</td>
<td>86.47</td>
<td>22.8029</td>
<td>5.3364</td>
<td>140.6500</td>
</tr>
<tr>
<td>2</td>
<td>86.47 103.76</td>
<td>17.29</td>
<td>23.4710</td>
<td>1.0694</td>
<td>28.1859</td>
</tr>
<tr>
<td>3</td>
<td>103.76 112.41</td>
<td>8.65</td>
<td>24.6662</td>
<td>0.5366</td>
<td>14.1430</td>
</tr>
<tr>
<td>4</td>
<td>112.41 121.06</td>
<td>8.65</td>
<td>25.6647</td>
<td>0.5382</td>
<td>14.1847</td>
</tr>
<tr>
<td>5</td>
<td>121.06 138.35</td>
<td>17.29</td>
<td>26.1111</td>
<td>1.0778</td>
<td>28.4068</td>
</tr>
</tbody>
</table>

Figure 5.2  Single-Line Case Study 1: Temperature vs. Position Along the Line, Showing Temperature Measurements and Corresponding Line Model Segmentation ($\Delta T_{\text{Threshold}} = 1.5^\circ\text{C}$)
Table 5.4  Single-Line Case Study 1: Modeling Tool Outputs O1 and O2 for $\Delta T_{\text{Threshold}} = 2^\circ\text{C}$

<table>
<thead>
<tr>
<th>Seg #, $k$</th>
<th>Location, $x$ from to (mi)</th>
<th>Length, $d_k$ (mi)</th>
<th>$T_{\text{Seg}}$ ($^\circ\text{C}$)</th>
<th>$R_k$ (Ω)</th>
<th>$L_k$ (mH)</th>
<th>$C$ (μF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 86.47</td>
<td>86.47</td>
<td>22.8029</td>
<td>5.3364</td>
<td>140.6500</td>
<td>1.5677</td>
</tr>
<tr>
<td>2</td>
<td>86.47 103.76</td>
<td>17.29</td>
<td>23.4710</td>
<td>1.0694</td>
<td>28.1859</td>
<td>0.3135</td>
</tr>
<tr>
<td>3</td>
<td>103.76 112.41</td>
<td>8.65</td>
<td>24.6662</td>
<td>0.5366</td>
<td>14.1430</td>
<td>0.1568</td>
</tr>
<tr>
<td>4</td>
<td>112.41 138.35</td>
<td>25.94</td>
<td>25.6647</td>
<td>1.6145</td>
<td>42.5542</td>
<td>0.4703</td>
</tr>
</tbody>
</table>

Figure 5.3  Single-Line Case Study 1: Temperature vs. Position Along the Line, Showing Temperature Measurements and Corresponding Line Model Segmentation ($\Delta T_{\text{Threshold}} = 2^\circ\text{C}$)
5.2.3 MODEL COMPARISONS

The models marked by an “x” in Table 5.5 were created for the hypothetical line. Distributed and single-segment line models with parameters at seasonal temperature represent traditionally used line models. These are shaded in Table 5.5. In order to differentiate between the impacts of:

- segmentation due to temperature and
- non-uniformity of the parameters with temperature,

multiple-segment models with parameters at $T_{\text{Seas}}$, $T_{\text{avg}}$ and $T_{\text{Wavg}}$ were also developed. The segmentation of these models, i.e. the length of each segment, was obtained through the step-by-step procedure presented in 4.4.1, then the per unit length parameters of each segment are equal to each other and are calculated for:

- $T_{\text{Seg}_{ij}} = T_{\text{Seas}}, \ \forall \ i, j$
- $T_{\text{Seg}_{ij}} = T_{\text{avg}}, \ \forall \ i, j$, as defined in (4-19)
- $T_{\text{Seg}_{ij}} = T_{\text{Wavg}}, \ \forall \ i, j$, as defined in (4-20).

Table 5.5 Line Models Developed for Comparison

<table>
<thead>
<tr>
<th>Ambient Temperature Model</th>
<th>$T_{\text{Seas}}$</th>
<th>$T_{\text{avg}}$</th>
<th>$T_{\text{Wavg}}$</th>
<th>$T_{\text{Seg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributed</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>n.a.</td>
</tr>
<tr>
<td>1 Segment</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Multiple Segments</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

In Table 5.5, the column corresponding to $T_{\text{Seg}}$ represents models in which the
parameters are calculated at the segment average temperature. Therefore, $T_{\text{Seg}}$ is not applicable to a distributed parameter model structure, and it is equivalent to $T_{\text{avg}}$ when looking at the single segment model structure. The single-segment model at $T_{\text{avg}}$ is in fact a subset of the multiple-segment modeling algorithm described in 4.4. Model structure and parameter values for each of the developed models are shown in tabular form in Appendix D, Tables D.1.

In order to compare the different models, simulations were performed using Cadence PSpice. The simulation setup is described in the following sub-section.

### 5.2.4 SIMULATION SETUP

The source voltage (or sending-end voltage), $V_S$, was set to 115 kVrms, which was chosen because the Falcon cable type is commonly utilized for 115-139 kV lines. Please note that qualitatively similar simulation results would be expected if other source voltage levels were adopted.

The metrics chosen for model comparisons were voltage attenuation and maximum power transfer. The load was kept fixed and the load voltage (or receiving-end voltage), $V_R$, was monitored to determine attenuation. In order to determine the maximum power transfer point for all models, a load sensitivity study was performed. Using a constant resistive impedance load, a variation in load from approximately 1.5 to 76 MW was considered.

Note that the distributed model behavior of the line is obtained through simulation using the Cadence PSpice distributed model [39].
5.2.5 SIMULATION RESULTS

Load voltage magnitude and phase for all simulated models and for a resistive load of 90 Ω, are shown in Tables 5.6-5.7. Tables of results for current flowing through the load can be found in Appendix D, Table D.2. From the load sensitivity studies, PV curves were created for all simulated models and are shown in Figure 5.4 and 5.5, zoomed-in around the maximum power point. The maximum real power values are then presented in Table 5.8.

Table 5.6 Single-Line Case Study 1 – Simulation Results: \(|V_R|\) for Each Line Model

<table>
<thead>
<tr>
<th>Model</th>
<th>(T_{Seas})</th>
<th>(T_{avg})</th>
<th>(T_{Wavg})</th>
<th>(T_{Seg})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributed</td>
<td>81.85</td>
<td>81.33</td>
<td>81.45</td>
<td>-</td>
</tr>
<tr>
<td>1 Segment</td>
<td>82.85</td>
<td>82.30</td>
<td>82.43</td>
<td>-</td>
</tr>
<tr>
<td>Multi Seg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 Seg – (\Delta T_{Threshold} = 1^\circ C)</td>
<td>82.09</td>
<td>81.13</td>
<td>81.25</td>
<td>81.25</td>
</tr>
<tr>
<td>5 Seg – (\Delta T_{Threshold} = 1.5^\circ C)</td>
<td>82.29</td>
<td>81.63</td>
<td>81.76</td>
<td>81.73</td>
</tr>
<tr>
<td>4 Seg – (\Delta T_{Threshold} = 2^\circ C)</td>
<td>82.36</td>
<td>81.77</td>
<td>81.89</td>
<td>81.87</td>
</tr>
</tbody>
</table>

Table 5.7 Single-Line Case Study 1 – Simulation Results: \(\theta_{VR}\) for Each Line Model

<table>
<thead>
<tr>
<th>Model</th>
<th>(T_{Seas})</th>
<th>(T_{avg})</th>
<th>(T_{Wavg})</th>
<th>(T_{Seg})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributed</td>
<td>-41.35</td>
<td>-41.72</td>
<td>-41.64</td>
<td>-</td>
</tr>
<tr>
<td>1 Segment</td>
<td>-42.88</td>
<td>-43.27</td>
<td>-43.183</td>
<td>-</td>
</tr>
<tr>
<td>Multi Seg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 Seg – (\Delta T_{Threshold} = 1^\circ C)</td>
<td>-41.68</td>
<td>-42.06</td>
<td>-41.97</td>
<td>-41.97</td>
</tr>
<tr>
<td>5 Seg – (\Delta T_{Threshold} = 1.5^\circ C)</td>
<td>-42.05</td>
<td>-42.44</td>
<td>-42.35</td>
<td>-42.37</td>
</tr>
<tr>
<td>4 Seg – (\Delta T_{Threshold} = 2^\circ C)</td>
<td>-42.06</td>
<td>-42.45</td>
<td>-42.36</td>
<td>-42.37</td>
</tr>
</tbody>
</table>
Figure 5.4  Single-Line Case Study 1 – Simulation Results: PV Curves for All Simulated Models

(Multi-Segment Models Obtained with $\Delta T_{\text{Threshold}} = 1.5^\circ \text{C}$)
Figure 5.5 Single-Line Case Study 1 – Simulation Results: PV Curves for All Simulated Models

\((\Delta T = 1.5^\circ C \text{ for Multi-Seg. Models})\) Zoomed-In Around Maximum Power Transfer Point

Table 5.8 Single-Line Case Study 1 – Simulation Results: \(P_{\text{max}}\) for Each Line Model

<table>
<thead>
<tr>
<th>Model</th>
<th>(T_{\text{Seas}})</th>
<th>(T_{\text{avg}})</th>
<th>(T_{\text{Wavg}})</th>
<th>(T_{\text{Seg}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributed</td>
<td>74.82</td>
<td>73.86</td>
<td>74.08</td>
<td>-</td>
</tr>
<tr>
<td>1 Segment</td>
<td>76.67</td>
<td>75.65</td>
<td>75.89</td>
<td>-</td>
</tr>
<tr>
<td>Multi-Seg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 Seg – (\Delta T_{\text{Threshold}} = 1^\circ C)</td>
<td>75.15</td>
<td>74.17</td>
<td>74.4</td>
<td>74.41</td>
</tr>
<tr>
<td>5 Seg – (\Delta T_{\text{Threshold}} = 1.5^\circ C)</td>
<td>75.42</td>
<td>74.43</td>
<td>74.66</td>
<td>74.61</td>
</tr>
<tr>
<td>4 Seg – (\Delta T_{\text{Threshold}} = 2^\circ C)</td>
<td>75.49</td>
<td>74.51</td>
<td>74.73</td>
<td>74.7</td>
</tr>
</tbody>
</table>
From the simulation results, it is evident that differences in behavior exist between all tested models and in terms of both metrics, $V_R$ and $P_{\text{max}}$. These differences can be seen across different temperatures, as well as different structures; specifically between:

- models with same structure type and different temperatures (going across a row in Tables 5.6-5.8);
- models of different structures with parameters at the same temperature (going down a column);

Moreover, the following considerations can be made:

- As expected, both load voltages and maximum real power are higher at lower temperatures.
- The model that presents the greatest difference in behavior from the multiple segment model at $T_{\text{Seg}}$ is the single segment at $T_{\text{Seg}}$, with almost 3% difference in $P_{\text{max}}$ and 1.5% difference in $|V_R|$.
- The model that behaves most closely to the multiple segment model at $T_{\text{Seg}}$ is the multiple segment at $T_{\text{Wavg}}$, with less than a 0.1% difference in $P_{\text{max}}$ and 0.05% difference in $|V_R|$.

Please note that the % differences between the multi-segment model at $T_{\text{Seg}}$ and the other models were calculated as follows:

\[
\text{\% diff} = \left(\frac{V_R^{\text{Multi-Seg} \ T_{\text{seg}}} - V_R^{\text{Comparison Model}}}{V_R^{\text{Multi-Seg} \ T_{\text{seg}}}}\right) \times 100 \quad (5-1)
\]

\[
\text{\% diff} = \left(\frac{P_{\text{Max}}^{\text{Multi-Seg} \ T_{\text{seg}}} - P_{\text{Max}}^{\text{Comparison Model}}}{P_{\text{Max}}^{\text{Multi-Seg} \ T_{\text{seg}}}}\right) \times 100 \quad (5-2)
\]
5.3 SINGLE-LINE CASE STUDY 2: SEVERE AMBIENT TEMPERATURES

In order to illustrate the results of the modeling approach for an example with more severe ambient temperature variations, a 150-mile transmission line was selected. Across the line a hypothetical ambient temperature variation of 20°C was assumed. Please note that this level of temperature changes can be experienced, particularly on a storm front [16].

Two scenarios were analyzed:

Scenario 1. Ambient temperature decreases from receiving to sending-end,

Scenario 2. Ambient temperature rises from receiving to sending-end.

The temperature profiles for the two scenarios are shown in Figure 5.6(a) and (b).

Figure 5.6 Single-Line Case Study 2: Temperature Profiles

(a) Scenario 1 - 20°C decrease from $x_R$ to $x_S$ and (b) Scenario 2 - 20°C increase from $x_R$ to $x_S$
The 150-mile line has the following characteristics, presented in terms of inputs to the multiple-segment modeling approach:

1. Line length: 150.00 miles
2. Cable type: Falcon in delta configuration
3. Lumped model type: gamma
4. Temperature measurement locations: See Table 5.9
5. Temperature measurements: See Table 5.9

The inputs I1-I4 to the modeling tool are the same for both severe temperature scenarios. I5 changes based on Table 5.9.

Table 5.9 Single-Line Case Study – Severe Ambient Temperatures:

<table>
<thead>
<tr>
<th>Temperature Measurement Locations and Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
</tr>
<tr>
<td>Mst Point</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

For an ambient temperature threshold set to:

$$\Delta T_{Threshold} = 1.5 \, ^\circ$C$$

outputs O1 and O2 of the modeling tool are shown in Table 5.10 for Scenario 1, and in Table 5.11 for Scenario 2. Please note that 13 uniform-length segments result for both scenarios. Also, segment parameter values are symmetrical across the two scenarios from sending to receiving end, e.g. segment # 1 for Scenario 1 is equivalent to segment # 13 for Scenario 2 and so on, which is expected based on their symmetrical temperature profiles (Figure 5.6).
Table 5.10  Single-Line Case Study 2 – Scenario 1:  Modeling Tool Outputs O1 and O2

<table>
<thead>
<tr>
<th>Seg #, k</th>
<th>Location, x</th>
<th>Length, $d_k$ (mi)</th>
<th>$T_{seg}$ (°C)</th>
<th>Segment Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>11.5385</td>
<td>34.2308</td>
<td>$R_k$ (Ω) 19.4065</td>
</tr>
<tr>
<td>2</td>
<td>11.5385</td>
<td>23.0769</td>
<td>32.6923</td>
<td>$L_k$ (mH) 19.3206</td>
</tr>
<tr>
<td>3</td>
<td>23.0769</td>
<td>34.6154</td>
<td>31.1538</td>
<td>$C_k$ (μF) 19.2347</td>
</tr>
<tr>
<td>4</td>
<td>34.6154</td>
<td>46.1538</td>
<td>29.6154</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>46.1538</td>
<td>57.6923</td>
<td>28.0769</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>57.6923</td>
<td>69.2308</td>
<td>26.5385</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>69.2308</td>
<td>80.7692</td>
<td>25.0000</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>80.7692</td>
<td>92.3077</td>
<td>23.4615</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>92.3077</td>
<td>103.8462</td>
<td>21.9231</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>103.8462</td>
<td>115.3846</td>
<td>20.3846</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>115.3846</td>
<td>126.9231</td>
<td>18.8462</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>126.9231</td>
<td>138.4615</td>
<td>17.3077</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>138.4615</td>
<td>150</td>
<td>15.7692</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.11  Single-Line Case Study 2 – Scenario 2:  Modeling Tool Outputs O1 and O2

<table>
<thead>
<tr>
<th>Seg #, k</th>
<th>Location, x</th>
<th>Length, $d_k$ (mi)</th>
<th>$T_{seg}$ (°C)</th>
<th>Segment Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>11.5385</td>
<td>15.7692</td>
<td>$R_k$ (Ω) 18.3757</td>
</tr>
<tr>
<td>2</td>
<td>11.5385</td>
<td>23.0769</td>
<td>17.3077</td>
<td>$L_k$ (mH) 18.4616</td>
</tr>
<tr>
<td>3</td>
<td>23.0769</td>
<td>34.6154</td>
<td>18.8462</td>
<td>$C_k$ (μF) 18.5475</td>
</tr>
<tr>
<td>4</td>
<td>34.6154</td>
<td>46.1538</td>
<td>20.3846</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>46.1538</td>
<td>57.6923</td>
<td>21.9231</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>57.6923</td>
<td>69.2308</td>
<td>23.4615</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>69.2308</td>
<td>80.7692</td>
<td>25.0000</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>80.7692</td>
<td>92.3077</td>
<td>26.5385</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>92.3077</td>
<td>103.8462</td>
<td>28.0769</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>103.8462</td>
<td>115.3846</td>
<td>29.6154</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>115.3846</td>
<td>126.9231</td>
<td>31.1538</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>126.9231</td>
<td>138.4615</td>
<td>32.6923</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>138.4615</td>
<td>150</td>
<td>34.2308</td>
<td></td>
</tr>
</tbody>
</table>
The models shown in Table 5.5 were constructed for the 150-mile line, where:

- \( T_{\text{Seas}} = 20 \, ^\circ\text{C} \)
- \( T_{\text{avg}} = T_{\text{Wavg}} = 25 \, ^\circ\text{C} \)

Model structure and parameter values for each of the developed models are shown in tabular form in Appendix D, Tables D.3.

All models were simulated using Cadence PSpice and resulting load voltage magnitude and phase, as well as maximum power transfer point for all models are shown in Tables 5.12 - 5.14. Results for receiving-end current can be found in Table D.4 in Appendix D. Note that \( T_{\text{avg}} \) and \( T_{\text{Wavg}} \) equal each other; therefore, the results of models with parameter calculated at these two temperatures have been combined in the tables.

**Table 5.12 Single-Line Case Study 2 – Simulation Results: \( |V_R| \)**

<table>
<thead>
<tr>
<th>Model</th>
<th>( T_{\text{Seas}} )</th>
<th>( T_{\text{avg}} )</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributed</td>
<td>77.62</td>
<td>77.05</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1 Segment</td>
<td>78.52</td>
<td>77.92</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Multi Seg.</td>
<td>( \Delta T_{\text{Threshold}} = 1.5 , ^\circ\text{C} )</td>
<td>77.73</td>
<td>77.16</td>
<td>77.14</td>
</tr>
</tbody>
</table>

**Table 5.13 Single-Line Case Study 2 – Simulation Results: \( \theta_{VR} \)**

<table>
<thead>
<tr>
<th>Model</th>
<th>( T_{\text{Seas}} )</th>
<th>( T_{\text{avg}} )</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributed</td>
<td>-43.61</td>
<td>-44.01</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1 Segment</td>
<td>-45.41</td>
<td>-45.83</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Multi Seg.</td>
<td>( \Delta T_{\text{Threshold}} = 1.5 , ^\circ\text{C} )</td>
<td>-43.72</td>
<td>-44.12</td>
<td>-44.10</td>
</tr>
</tbody>
</table>
Table 5.14 Single-Line Case Study 2 – Simulation Results: $P_{\text{max}}$

<table>
<thead>
<tr>
<th>Model</th>
<th>$T_{\text{Seas}}$</th>
<th>$T_{\text{avg}}$</th>
<th>Scenario 1 $T_{\text{Seg}}$</th>
<th>Scenario 2 $T_{\text{Seg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributed</td>
<td>69.55</td>
<td>68.61</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1 Segment</td>
<td>71.65</td>
<td>70.67</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Multi Seg.</td>
<td>$\Delta T_{\text{Threshold}} = 1.5^\circ$C</td>
<td>69.77</td>
<td>68.85</td>
<td>68.82</td>
</tr>
</tbody>
</table>

Please note that the models in the first and second column of Tables 5.12-5.14, i.e. all model structures at $T_{\text{Seas}}$, $T_{\text{avg}}$, and $T_{\text{Wavg}}$ cannot differentiate between the temperature profiles in Scenario 1 and in Scenario 2. Therefore, they would not be able to capture effects given by a temperature rise at the receiving-end or at the sending-end of the line. Moreover, the following considerations can be made based on the simulation results:

- For both scenarios, the model that presents the greatest difference in behavior from the multiple segment models at $T_{\text{Seg}}$ is the single segment at $T_{\text{Seas}}$, with 1.8% and 1.75% difference in $|V_R|$ for Scenarios 1 and 2, respectively, and more than 4% difference in $P_{\text{max}}$ for both scenarios.

- The differences between Scenario 1 and Scenario 2 are approximately 0.05% in $|V_R|$ and 0.1% in $P_{\text{max}}$. These differences are calculated based on (5-1) and (5-2).

The results of the model comparisons overall show that the line model performance is sensitive to ambient temperature variations along the line. While computational effort may be increased by the use of a multi-segment model for system
studies, the single segment may not accurately portray the line when undergoing high temperature gradients. For example, models that incorporate this non-uniformity of line parameters due to temperature are needed to determine an accurate estimate of the line capacity, i.e. maximum power transfer point.

5.4 SENSITIVITY OF OUTPUT MODEL TO ΔT_{THRESHOLD}

In general, as the temperature threshold, ΔT_{Threshold}, decreases, the output of the line modeling approach would give an increasing number of segments, while as ΔT_{Threshold} increases, the number of segments would tend to 1. An example plot of number of segments, K, vs. the temperature threshold ΔT_{Threshold} for the line and temperature data in Section 5.3 - Scenario 1, is shown in Figure 5.7.

![Figure 5.7 Example Plot of K vs ΔT_{Threshold}](image)


5.5 SUMMARY OF RESULTS AND OBSERVATIONS

Simulations using state-of-the-art software were performed to illustrate differences between models in terms of metrics of voltage attenuation and power handling capabilities. Currently adopted models give the same solution for an electrical operating point (sending end voltage and loading conditions), regardless of ambient temperature. The proposed models are functions of temperature in addition to structure. Therefore, they allow capturing the temperature effects on the line. Their behavior, as shown in the simulation results, changes based on temperature and independently of the electrical operating point. This is believed to be a more accurate representation of the line performance.

The use of line models that incorporate available temperature information is then believed to allow for a more accurate determination of the power flows and the voltage profile in the network. Therefore, the line modeling procedure was incorporated into a power flow tool, providing a platform to study the impacts of using the proposed line models for system studies. The line models’ integration and resulting system impacts are described in the following chapter.
CHAPTER 6: SYSTEM IMPACTS

6.1 OVERVIEW

In order to investigate the proposed line models’ impacts on large-scale, steady-state system studies, such as power flow analysis and state estimation, the models were incorporated into multi-bus systems. As an example, the line modeling tool was integrated into network models for power flow analysis. This chapter presents the integration of the modeling tool into a widely available power flow solver [51], as well as some considerations on the impacts the models have on system size and computational effort. For demonstration purposes, power flow results for a multi-bus system with hypothetical ambient temperature profiles are presented and compared to the base case in which ambient temperature variations are not considered. It is noted that the term “base case” indicates the system in which each branch is modeled by a single lumped segment with parameters at a seasonal temperature, constant throughout the system; the term “updated case” indicates the system in which each branch is modeled by multiple non-uniform segments, obtained using the algorithm presented in 4.4.1.

6.2 LINE MODELING TOOL FOR POWER FLOW ANALYSIS

A multiple lumped segment model structure was developed in this work to capture ambient temperature variations along a line. Each segment parameter values and lengths were determined based on the ambient temperature profile along the line. In the next sub-section, the impacts of utilizing this multi-segment model structure for multi-bus
system studies are discussed. Then, in sub-section 6.2.2 the integration of the line modeling tool into a power flow solver is described.

### 6.2.1 IMPACTS ON SYSTEM SIZE AND SPARSITY

As reviewed in Chapter 2, the steady-state behavior of an interconnected power system is often given by performing nodal analysis on the network, where the network is represented through a bus admittance matrix, $Y_{Bus}$ (2-14). Given $N$ buses, or nodes, in the system, the $Y_{Bus}$ is a $(N \times N)$ symmetric matrix (2-15), where:

- The diagonal elements: $Y_{nn} = \text{sum of all admittances incident to the } n\text{-th bus},$
- The off-diagonal elements: $Y_{nm} = \text{negative admittance of all components connecting buses } n \text{ and } m.$

Please note that the shunt element in the lumped parameter line models, $Y$, affect only the diagonal entries of the $Y_{Bus}$ matrix. Moreover, the $Y_{Bus}$ is usually sparse, since each bus is connected only to a few of the other buses.

When utilizing a multi-segment line model, a single line, or branch, is divided into $K$ number of segments, which result in the creation of additional nodes, or buses, to the network. Figure 6.1(a) shows the base case for an example 7-bus power system, in which each branch is modeled by a single lumped segment. Figure 6.1(b) shows the same system updated to incorporate a hypothetical ambient temperature profile, in which each branch is composed of multiple non-uniform segments, resulting in 18 additional buses. Specifically, the number of buses, and therefore the size of the $Y_{Bus}$ matrix, increases with the number of segments in each branch, $K_b$, as follows:
\[ M^1 = M^0 + \sum_{b=1}^{B} K_b - B \]  \hspace{1cm} (6-1)

where:

- \( M^0 \in \mathbb{Z}^+ \): number of buses in the base case,
- \( M^1 \in \mathbb{Z}^+ \): number of buses in the updated case
- \( B \in \mathbb{Z}^+ \): number of branches (i.e. lines) in the system
- \( K_b \in \mathbb{Z}^+ \): number of segments in branch \( b \), \( K_b \geq 1 \)

Figure 6.1  One-Line Diagram of an Example Power System:

(a) Base Case - 7 Buses and (b) Updated Case - 24 Buses
As the number of segments in a branch increases, so does the sparsity of the $Y_{Bus}$ matrix. The number of nonzero entries is in fact related to the number of buses in the updated case, $N^1$, and the number of segments in each branch, $K_b$, as follows:

$$\# \text{ NonZero Entries} = N^1 + 2 \sum_{b=1}^{B} K_b$$

(6-2)

When analyzing the percentage of the elements of the $Y_{Bus}$ matrix that are nonzero,

$$\% \text{ NonZero Entries} = \frac{\# \text{ of NonZero Entries}}{\text{Total # of Entries}} \times 100\%$$

(6-3)

given $K_b \geq 1$, it can be seen that this percentage for the updated system is always less than or equal to the one for the original system:

$$\frac{N^1 + 2 \sum_{b=1}^{B} K_b}{N^1 \times N^1} \times 100\% \leq \frac{N^0 + 2B}{N^0 \times N^0} \times 100\%$$

(6-4)

The use of the proposed multi-segment line models would then increase the number of buses and therefore the size of the system, but it would also at least preserve, if not increase, the sparsity of the network matrix. Several computational techniques can be used to exploit this sparsity. In terms of decreasing storage requirements, techniques are often used in power system studies that enable storing only the nonzero entries. In terms of computational requirements, ordering schemes of the problem equations and variables can be used to preserve the matrix sparsity [52][53].

In the next sub-section, the integration of the line modeling tool into a power flow solver is introduced as an example of the use of the proposed line models in power system studies.
6.2.2 INTEGRATION INTO POWER FLOW SOLVER

The line modeling tool presented in Chapter 4 (Algorithm 1) was embedded within MATPOWER [51], which is a MATLAB-based power flow solver. The solver takes in relevant information to perform power flow through a case file, specific to the system under analysis, which contains bus, branch and generator data in a format similar to PTI [54]. In order to incorporate ambient temperature information, an added input file to the power flow solver was applied. This added input file contains:

1. Temperature coefficients $\alpha$ and $\beta$ for each branch in the system,
2. Temperature threshold parameter, $\Delta T_{Threshold}$,
3. Temperature measurement locations for each branch,
4. Beginning and end of each branch with respect to measurement locations,
5. Temperature measurements at each measurement point.

Utilizing the temperature-related inputs in addition to the case file as used by MATPOWER, the developed tool segments each branch in the system according to temperature, and determines the parameters for each segment in each branch. The newly created nodes are modeled as load buses, at which the real and reactive power demands are specified to be zero:

$$
\begin{align*}
    P_{\text{new}} &= 0 \\
    Q_{\text{new}} &= 0
\end{align*}
$$

(6-5)

where the subscript new refers to all newly created nodes.

Given the temperature input data, updated line model structures are developed for each branch in the system, followed by the building of the $Y_{Bus}$ matrix. Inputs and
outputs of the power flow solver that incorporates temperature information within the line models are shown in Figure 6.2.

For demonstration purposes, power flow analysis was performed on an example 9-bus power system for assumed temperature profiles.

6.3 MULTI-BUS CASE STUDY

A 9-bus, 3-generator, power system was used to illustrate the power flow solver. The generator, branch and bus data for this system was taken from the test case described in [55]. A one-line diagram of the test case with specified quantities is shown in Figure 6.3.
It is noted that in Figure 6.3 the subscripts “d” and “g” indicate “demanded” and “generated”, respectively.

Two ambient temperature scenarios have been analyzed:

Scenario 1. Mild temperature gradients (max. variation of 3.5°C across a branch and 4°C throughout the whole system),

Scenario 2. Severe temperature gradients (max. variation of 20°C across a branch) [16].

Given the temperature scenarios, temperature information and power flow results are presented in the next sub-sections for the base case, in which each branch is modeled by a single segment with parameters at a seasonal temperature (1 Seg. at $T_{\text{Seas}}$, where $T_{\text{Seas}}$ equals 20 °C) ad for the updated case, in which each branch is modeled using Algorithm
1 in Chapter 4 (multi-seg. at $T_{Seg}$). Maximum power transfer point is also presented for base and updated cases, as well as for cases in which each branch is modeled using the other models for comparison described in Section 4.5.

6.3.1 EXPERIMENTAL RESULTS: MILD TEMPERATURE GRADIENTS

A selection of bus, generator and branch data provided in the case file is shown in Tables E.1 – E.3 of Appendix E and represents input $I_1$ of the power flow solver (Figure 6.2). Input $I_2$ includes temperature-related information, as follows:

- $\alpha$ and $\beta$ for all branches in the system:
  \[\alpha = \beta = 0.03 \, (^oC)\]

- $\Delta T_{Threshold}$ for all branches in the system:
  \[\Delta T_{Threshold} = 1.5^oC\]

- Temperature measurement locations and measurement values for all branches in the system – as shown in Table 6.1.

Table 6.1  Multi-Bus Case Study – Mild Temperature Gradients:
Ambient Temperature Measurements for All Branches in the 9-Bus System

<table>
<thead>
<tr>
<th>Branch (From Bus-To Bus)</th>
<th>1-4</th>
<th>1-5</th>
<th>5-6</th>
<th>3-6</th>
<th>6-7</th>
<th>7-8</th>
<th>8-2</th>
<th>8-9</th>
<th>9-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x , mi$</td>
<td>$T_A , ^oC$</td>
<td>$x , mi$</td>
<td>$T_A , ^oC$</td>
<td>$x , mi$</td>
<td>$T_A , ^oC$</td>
<td>$x , mi$</td>
<td>$T_A , ^oC$</td>
<td>$x , mi$</td>
<td>$T_A , ^oC$</td>
</tr>
<tr>
<td>0</td>
<td>20</td>
<td>0</td>
<td>19</td>
<td>0</td>
<td>23</td>
<td>20</td>
<td>19</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>15</td>
<td>20.5</td>
<td>25</td>
<td>21.5</td>
<td>22</td>
<td>19</td>
<td>22.5</td>
<td>19</td>
<td>25</td>
<td>21</td>
</tr>
<tr>
<td>35</td>
<td>22</td>
<td>95</td>
<td>20.5</td>
<td>80</td>
<td>20</td>
<td>22</td>
<td>20</td>
<td>15</td>
<td>21</td>
</tr>
<tr>
<td>50</td>
<td>21.5</td>
<td>130</td>
<td>19</td>
<td>110</td>
<td>21</td>
<td>20</td>
<td>20</td>
<td>85</td>
<td>19</td>
</tr>
<tr>
<td>75</td>
<td>180</td>
<td>180</td>
<td>20.5</td>
<td>150</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>165</td>
<td>22.5</td>
</tr>
</tbody>
</table>
Results from the power flow analysis for the base case (i.e. single segment branches at $T_{\text{Seas}}$) and for the updated system (i.e. multi-segment branches at $T_{\text{Seg}}$, obtained using line model segmentation Algorithm 1 – 4.4.1) are shown in Table 6.2.

Table 6.2 Multi-Bus Case Study – Mild Temperature Gradients:

<table>
<thead>
<tr>
<th>Bus, $n$</th>
<th>Voltage</th>
<th>Generation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
<td>V_n</td>
</tr>
<tr>
<td>Base Case (1Seg at $T_{\text{Seas}}$)</td>
<td>Updated (MultiSeg at $T_{\text{Seg}}$)</td>
<td>Base Case (1Seg at $T_{\text{Seas}}$)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.987</td>
<td>0.987</td>
</tr>
<tr>
<td>5</td>
<td>0.975</td>
<td>0.976</td>
</tr>
<tr>
<td>6</td>
<td>1.003</td>
<td>1.003</td>
</tr>
<tr>
<td>7</td>
<td>0.986</td>
<td>0.986</td>
</tr>
<tr>
<td>8</td>
<td>0.996</td>
<td>0.996</td>
</tr>
<tr>
<td>9</td>
<td>0.958</td>
<td>0.958</td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that the line model segmentation based on temperature resulted in an additional 19 buses, for a total of 28 buses. Select branch data for the updated case can be found in Table E.4 of Appendix E. As expected, the differences in power flow results between the two cases are small. The maximum difference in voltage magnitude is found to be approximately 0.1% at bus 5, $|V_5|$, while in voltage phase angle is 8.5% at bus 7, $\theta_7$. The difference in generated power at the slack bus, Bus 1, is approximately 0.03% for
real power, $P_1$, and 1.7% for reactive power, $Q_1$. Please note that the % difference in power flow results is calculated as follows:

$$\% \ \text{Difference \ in \ Result} = \left| \frac{\text{Result}_{\text{Base \ Case}} - \text{Result}_{\text{Updated \ Case}}}{\text{Result}_{\text{Base \ Case}}} \right| \times 100\% \quad (6-6)$$

Also note that for both the base case and the updated case, the power flow solver, which is based on a standard Newton’s method [44], converges in 4 iterations. The real power flows on the branches are shown in Table 6.3 below for both the base case and the updated case. It is noted that the difference in real power flow on each branch between the updated case and the base case stays within approximately 0.2% (calculated by 6-6).

Table 6.3 Multi-Bus Case Study – Mild Temperature Gradients:

Real Power Flows on Each Branch in the 9-Bus Test System

<table>
<thead>
<tr>
<th>Branch</th>
<th>$P$ (MW)</th>
<th>Base Case (1 Seg at $T_{\text{Seg}}$)</th>
<th>Updated Case (Multi-Seg at $T_{\text{Seg}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td></td>
<td>71.95</td>
<td>71.97</td>
</tr>
<tr>
<td>4-5</td>
<td></td>
<td>30.73</td>
<td>30.7</td>
</tr>
<tr>
<td>5-6</td>
<td></td>
<td>60.89</td>
<td>60.94</td>
</tr>
<tr>
<td>3-6</td>
<td></td>
<td>85</td>
<td>85</td>
</tr>
<tr>
<td>6-7</td>
<td></td>
<td>24.11</td>
<td>24.06</td>
</tr>
<tr>
<td>7-8</td>
<td></td>
<td>76.5</td>
<td>76.55</td>
</tr>
<tr>
<td>8-2</td>
<td></td>
<td>163</td>
<td>163</td>
</tr>
<tr>
<td>8-9</td>
<td></td>
<td>86.5</td>
<td>86.45</td>
</tr>
<tr>
<td>9-4</td>
<td></td>
<td>41.23</td>
<td>41.27</td>
</tr>
</tbody>
</table>
By changing the real power demanded by the loads, the point of maximum power transfer was determined and corresponds to the point of voltage collapse at a particular bus. The maximum power, $P_{\text{max}}$, obtained for the base case (1 Segment at $T_{\text{Sea}}$), for the updated case (Multi-Segment at $T_{\text{Seg}}$), and for other line models developed for comparison, as in Table 5.5, are shown in Table 6.4.

<table>
<thead>
<tr>
<th>Model</th>
<th>$T_{\text{Sea}}$</th>
<th>$T_{\text{avg}}$</th>
<th>$T_{\text{Wavg}}$</th>
<th>$T_{\text{Seg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Segment</td>
<td>264.9</td>
<td>264.44</td>
<td>264.23</td>
<td>-</td>
</tr>
<tr>
<td>Multi-Seg. ($\Delta T_{\text{Threshold}} = 1.5^\circ C$)</td>
<td>265.31</td>
<td>264.85</td>
<td>264.7</td>
<td>264.7</td>
</tr>
</tbody>
</table>

In the next sub-section severe ambient temperature variations of $20^\circ C$ are introduced. Solutions of the power flow solver for the updated case are presented.

6.3.2 EXPERIMENTAL RESULTS: SEVERE TEMPERATURE GRADIENTS

In this scenario, a heat wave is assumed to pass through parts of the 9-bus 3-generator power system test case, affecting mostly buses 2, 8, and 9. A $20^\circ C$ increase in temperature is hypothesized along the branches connecting buses 4 and 9 and buses 7 and 8. A graphical representation of the 9-bus system, subject to severe ambient temperature variations is shown in Figure 6.4.

Input $I_1$ of the power flow solver (Figure 6.2) is the same as for the previous scenario; the temperature-related information, input $I_2$, is as follows:

- $\alpha$ and $\beta$ for all branches in the system:
\( \alpha = \beta = 0.03 \text{ (1/°C)} \quad – \text{Same as for Scenario 1} \)

- \( \Delta T_{\text{Threshold}} \) for all branches in the system:

\[
\Delta T_{\text{Threshold}} = 1.5^\circ \text{C} \quad – \text{Same as for Scenario 1}
\]

- Temperature measurement locations and measurement values for all branches in the system – as shown in Table 6.5.

![One-Line Diagram of the 9-Bus Test Case Subject to Severe Temperature Gradients](image.png)

**Table 6.5 Multi-Bus Case Study – Severe Temperature Gradients:**

*Ambient Temperature Measurements for All Branches in the 9-Bus System*

<table>
<thead>
<tr>
<th>Branch (From Bus-To Bus)</th>
<th>1-4</th>
<th>1-5</th>
<th>5-6</th>
<th>3-6</th>
<th>6-7</th>
<th>7-8</th>
<th>8-2</th>
<th>8-9</th>
<th>9-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x ) mi</td>
<td>( T_A ) °C</td>
<td>( x ) mi</td>
<td>( T_A ) °C</td>
<td>( x ) mi</td>
<td>( T_A ) °C</td>
<td>( x ) mi</td>
<td>( T_A ) °C</td>
<td>( x ) mi</td>
<td>( T_A ) °C</td>
</tr>
<tr>
<td>0</td>
<td>20</td>
<td>0</td>
<td>20</td>
<td>0</td>
<td>22</td>
<td>0</td>
<td>22</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>50</td>
<td>20</td>
<td>130</td>
<td>22</td>
<td>180</td>
<td>20</td>
<td>50</td>
<td>20</td>
<td>110</td>
<td>20</td>
</tr>
<tr>
<td>50</td>
<td>20</td>
<td>165</td>
<td>40</td>
<td>95</td>
<td>20</td>
<td>50</td>
<td>40</td>
<td>165</td>
<td>40</td>
</tr>
</tbody>
</table>
Power flow results for the base case and for the updated system are shown in Table 6.6. The line model segmentation based on this temperature profile resulted in an additional 24 buses, for a total of 33 buses. Select branch data for the updated case can be found in Table E.5 of Appendix E. The maximum difference in voltage magnitude is found to be 0.3% at bus 9, $|V_9|$, while in voltage phase angle is approximately 30% at bus 7, $\theta_7$. The difference in generated power at the slack bus, Bus 1, is approximately 0.3% for real power, $P_1$, and 4% for reactive power, $Q_1$. These % differences are calculated based on (6-6). Please note that the differences in power flow results between the base case and the updated case are consistently higher for Scenario 2 (severe temperature variations) over Scenario 1 (mild variations).

**Table 6.6 Multi-Bus Case Study – Severe Temperature Gradients:**

Power Flow Results for the 9-Bus Test System

<table>
<thead>
<tr>
<th>Bus, $n$</th>
<th>Voltage</th>
<th>Generation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
<td>V_n</td>
</tr>
<tr>
<td></td>
<td>Base Case</td>
<td>Updated</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.987</td>
<td>0.986</td>
</tr>
<tr>
<td>5</td>
<td>0.975</td>
<td>0.975</td>
</tr>
<tr>
<td>6</td>
<td>1.003</td>
<td>1.003</td>
</tr>
<tr>
<td>7</td>
<td>0.986</td>
<td>0.985</td>
</tr>
<tr>
<td>8</td>
<td>0.996</td>
<td>0.996</td>
</tr>
<tr>
<td>9</td>
<td>0.958</td>
<td>0.955</td>
</tr>
<tr>
<td></td>
<td>Total: 319.95</td>
<td>320.14</td>
</tr>
</tbody>
</table>
The real power flows on the branches are shown in Table 6.7 below. It is noted that, for this case of more severe temperature gradients, the difference in real power flow on each branch between the updated case and the base case reaches approximately 3.5% (calculated by 6-6) on branch 6-7.

<table>
<thead>
<tr>
<th>Branch</th>
<th>P (MW)</th>
<th>Base Case</th>
<th>Updated Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>71.95</td>
<td>72.14</td>
<td></td>
</tr>
<tr>
<td>4-5</td>
<td>30.73</td>
<td>29.95</td>
<td></td>
</tr>
<tr>
<td>5-6</td>
<td>60.89</td>
<td>61.71</td>
<td></td>
</tr>
<tr>
<td>3-6</td>
<td>85</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>6-7</td>
<td>24.11</td>
<td>23.29</td>
<td></td>
</tr>
<tr>
<td>7-8</td>
<td>76.5</td>
<td>77.33</td>
<td></td>
</tr>
<tr>
<td>8-2</td>
<td>163</td>
<td>163</td>
<td></td>
</tr>
<tr>
<td>8-9</td>
<td>86.5</td>
<td>85.67</td>
<td></td>
</tr>
<tr>
<td>9-4</td>
<td>41.23</td>
<td>42.19</td>
<td></td>
</tr>
</tbody>
</table>

Please note that the power flow results for the base case are the same in Scenario 1 and Scenario 2, since the base case does not consider temperature variations along the line and $T_{Seas} = 20^\circ C$ is the same for both scenarios.

The point of maximum power transfer, $P_{max}$, was determined for the base case (1-Seg. at $T_{Seas}$), for the updated case (Multi-Seg. at $T_{Seg}$), and for other line models developed for comparison, as in Table 5.5. $P_{max}$ values are shown in Table 6.8.
Table 6.8  Multi-Bus Case Study – Severe Temperature Gradients: $P_{\text{max}}$ for Each Line Model

<table>
<thead>
<tr>
<th>Model</th>
<th>$T_{\text{Seas}}$</th>
<th>$T_{\text{avg}} - T_{\text{Wavg}}$</th>
<th>$T_{Seg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Segment</td>
<td>264.9</td>
<td>262.54</td>
<td>-</td>
</tr>
<tr>
<td>Multi-Seg. ($\Delta T_{\text{Threshold}} = 1.5^\circ\text{C}$)</td>
<td>265.05</td>
<td>262.7</td>
<td>262.7</td>
</tr>
</tbody>
</table>

The PV curves at load bus 9 for the base case and the updated cases for Scenario 1 and Scenario 2 are shown in Figure 6.5. Note that the maximum power transfer point in scenario 2, with more severe temperature gradients, is lower than for scenario 1. Also note that the updated cases (for both temperature scenarios) have a lower maximum power transfer point than the base case. In scenario 2, the voltage collapses at approximately 262.7 MW for the updated case and at 264.9 MW for the base case. This corresponds to a difference of approximately 0.85%.

![Figure 6.5 PV Curve at Bus 9 – for Base Case and Updated Case (Scenarios 1 and 2)](image-url)
Real and reactive power flows and losses in each branch can be found in Appendix E, Tables E.6-E.8, for the base case (1-Seg at $T_{Seas}$) and the updated cases (Multi-Seg at $T_{Seg}$) for mild and severe temperature gradients.

An example application for this tool includes solving power flow on the base case to determine $|V_n|$, $\theta_n$, $P_n$, $Q_n$ at each bus in the system. Then, by checking operating constraints, e.g. overloads, voltage minima and maxima, the need to re-run power flow analysis by taking into consideration temperature information can be determined. For example:

- If at a bus $n$:

$$
|V_n| \leq V_{\text{min}} + \varepsilon_1, \quad |V_n| \geq V_{\text{max}} - \varepsilon_2
$$
$$
P_n \leq P_{\text{min}} + \varepsilon_3, \quad P_n \geq P_{\text{max}} - \varepsilon_4
$$
$$
Q_n \leq Q_{\text{min}} + \varepsilon_5, \quad Q_n \geq Q_{\text{max}} - \varepsilon_6
$$

(6-7)

where

$\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$ : set threshold values

$V_{\text{min}}, V_{\text{max}}$ : minimum and maximum bus voltage magnitude

$P_{\text{min}}, P_{\text{max}}$ : minimum and maximum real power

$Q_{\text{min}}, Q_{\text{max}}$ : minimum and maximum reactive power

- then, the temperature profile along all branches connected to bus $n$ can be taken into account and those branches can be modeled accordingly;

- power flow analysis can then be performed on the updated system and more accurate results can be expected.
6.4 SUMMARY OF RESULTS AND OBSERVATIONS

The work presented in this thesis focused on electric power lines as a critical network component of the power system, and aimed at re-evaluating their models to be used in steady-state system analysis applications. By taking into account available ambient temperature information, more accurate results are expected by the decision-making tools. In this chapter, the line modeling tool to incorporate ambient temperature information was embedded within network models in order to perform system analysis. Considerations on the impact the models would have on the system were made in terms of system size and computation, and differences in power flow results due to temperature.

Although the proposed model structure increases the size of the system by adding nodes in the network and therefore adding state variables, the sparsity of the network matrix is maintained or increased. A brief discussion on techniques that could exploit this sparsity to relieve computational intensity was presented.

The modeling tool was embedded within a power flow solver. Power flow solutions for a 9-bus system under mild ambient temperature variations were presented and compared to solutions for the same system where temperature information was not taken into account. Differences in the solutions were noted. When severe ambient temperature variations were assumed along the branches in the system, the differences in power flow solutions between the updated and the base case were more noticeable. In terms of real power flows on the branches, a difference of approximately 3.5% was noted between the updated and the base case for the severe temperature gradient scenario. This is an important consideration, especially when the lines are close to their maximum power handling capabilities.
CHAPTER 7: CONCLUSIONS

7.1 CONCLUDING REMARKS

The work presented in this thesis addressed the problem of modeling electric power transmission lines for steady-state system analysis. It took into account parameters not previously fully considered by relaxing select historical modeling assumptions. In this thesis, a formal problem formulation was developed for the work presented in [1][2], where transmission line models were developed to maintain a certain accuracy level at non-fundamental frequencies. Then, the focus was put on incorporating another parameter into static line models: ambient temperature, by relaxing two modeling assumptions:

- constant conductor temperature along the cable, and
- uniform distribution of line parameters along the cable.

Various ways of incorporating available temperature information were studied, including:

- distributed parameter, and
- lumped parameter models (single and multi-segment).

A multi-non-uniform-segment line modeling approach was developed and tested. The newly developed line models were then integrated within network models in order to provide a platform to evaluate line models’ impacts on steady-state system studies.

Through testing and evaluation of the developed line models, ambient temperature variations along the line were shown to affect steady-state terminal voltages and line currents. These effects, though mild for mild temperature changes, increased for
severe temperature gradients, especially in terms of the line power handling capabilities. Moreover, currently used line models cannot account for temperature variations along the line, and their steady-state terminal behavior remains constant, regardless of ambient temperature. The developed line models are temperature-dependent, in terms of both their structure and line parameters; therefore, they enable capturing of temperature effects.

Specific research contributions and accomplishments of the work are summarized in Section 7.2; a future vision for the work is then discussed in Section 7.3.

7.2 SUMMARY OF RESEARCH CONTRIBUTIONS

The thesis’ main contributions include:

- Identification and formulation of transmission line modeling problems that take into account electrical frequencies and ambient temperature. Specifically,
  
  Sub-Problem 1. An optimization problem was formulated:
  
  - given: non-fundamental frequencies of interest and desired accuracy,
  
  - determine: optimum line model structure, i.e. segmentation;
  
  Sub-Problem 2. An optimization problem was formulated:
  
  - given: ambient temperature information,
  
  - determine: 1. optimum line model structure, i.e. segmentation, and
  
  2. segment start and end points, i.e. segment length.

- Investigation and development of various methods of incorporating ambient temperature data into traditional line model structures ($T_{Seas}$, $T_{avg}$, $T_{Wavg}$, $T_{Seg}$).
• Development of temperature-dependent transmission line models, in terms of both structure and parameter values. These models capture line parameter non-uniformity with respect to position along the line, caused by a temperature gradient, through:
  – non-uniformly distributed parameters,
  – a non-uniformly segmented structure (multiple lumped segments).

• Determination of a line modeling algorithm to determine model segmentation and segment parameter values. In the limit, the algorithm would result in a non-uniformly distributed parameter model.

• Development of an automated MATLAB-based line modeling tool.

• Evaluation and testing of the line modeling approach through simulation. Model comparison was made in terms of system and line propagation metrics, and sensitivity studies of line models to ambient temperature variations was performed through a circuit-based approach. Several observations were made based on those results:
  – regardless of ambient temperature, currently adopted models give the same solution for an electrical operating point, while
  – the proposed line models are functions of temperature in addition to structure. Their behavior changes based on temperature and independently of the electrical operating point, capturing the temperature effects on the line. This is believed to be a more accurate representation of the line performance.
Incorporation of the developed line models within network models to be used in power flow and state estimation algorithms. Several observations were made:

- in general, the multi-segment line models would cause an increase in system size,
- the sparsity of the network matrix would be maintained or increased, and techniques are available to exploit this sparsity.

As an example, the line modeling approach was embedded within a widely available power flow solver, providing a platform to study the impacts of ambient temperature on power flows and voltage profile in the network.

7.3 FUTURE WORK

Several considerations and suggestions can be made in terms of future work:

- Thus far, line modeling for multi-frequency systems and for varying ambient conditions has been approached as two decoupled sub-problems. Future work could include formulating the line modeling problem within an overall framework, resulting in frequency and temperature dependent models.

- In a similar manner as ambient temperature, other ambient conditions could be included in the analysis and incorporated into static line models. These external parameters include wind speed and direction, insolation, humidity, which would all affect the temperature of the conductor. Please note that if varying humidity is considered, line charging may also vary along the length of the line, i.e. assumption A8 would have to be relaxed.
• A different reference conductor temperature due to loading conditions can be included in the analysis by adjusting segment parameter values based on the new reference values. Please note that the line model segmentation would still be based upon non-uniformity of line parameters along the line given by external temperature changes.

• In terms of reducing the system size, the impacts of Kron reducing the newly created buses could be investigated. In fact, these buses do not have external loads, nor generators, connected to them, therefore their current injection due to the shunt element would be relatively small as opposed to the other buses. Kron reducing the system would then decrease the size of the $Y_{Bus}$ matrix by returning to the size of the original system, and updating the remaining entries.

• Additional analytical study of the non-uniformly distributed parameter model could lead to the determination of a closed-form solution for voltages and currents at any point along the line, given an ambient temperature function $T_A(x)$. The development of an equivalent transmission line matrix that incorporates the effects of the non-uniformly distributed line model can then be investigated.

• Alternative solutions to the optimization problems presented in 3.4.1 for studies at non-fundamental frequencies, and in 3.4.2 for incorporating temperature variations can be investigated. Other approaches can include the use of non-uniformly segmented line models in line modeling for studies at non-fundamental frequencies. The problem of incorporating temperature
information can be viewed as a matrix rank minimization or a large mixed-integer problem.

- Stochastic analysis can be performed on the power system as having uncertain network parameters, where the uncertainty comes from external conditions.
- Moreover, this work can be extended to include real time measurements of parameters or states of the power system by utilizing emerging sensing equipment and technologies, such as phasor measurement units.
LIST OF REFERENCES


APPENDICES
APPENDIX A  LIST OF NOMENCLATURE

In order of appearance:

\( K \)  number of segments in the line model
\( L \)  series inductance
\( C \)  shunt capacitance
\( T_{\text{Seas}} \)  predefined seasonal temperature
\( x \)  position along the line
\( r \)  series resistance per unit length
\( l \)  series inductance per unit length
\( g \)  shunt conductance per unit length
\( c \)  shunt capacitance per unit length
\( z \)  series impedance per unit length
\( \omega \)  operating angular frequency
\( x_L \)  inductive reactance per unit length
\( y \)  shunt admittance per unit length
\( b_c \)  capacitive susceptance per unit length
\( r_{dc} \)  dc resistance of a conductor per unit length
\( \rho \)  conductor resistivity
\( A \)  cross-sectional area
\( r_{ac} \)  effective (ac) resistance of a conductor per unit length
\( P_{\text{Loss}} \)  real power loss
\( I \)  current in the conductor
\( \ell \)  
 total length of the line

\( l \)  
 inductance per unit length

\( \mu \)  
 permeability of the conductor

\( D_{eq} \)  
 equivalent distance between conductors

\( D_s \)  
 conductor geometric mean radius

\( c \)  
 capacitance per unit length

\( \varepsilon \)  
 permittivity of the material surrounding the conductor surface

\( d \)  
 conductor radius

\( H_{1, 2, 3} \)  
 distances between each conductor and its image below the surface of the earth

\( H_{12, 23, 31} \)  
 distances between each conductor and the other conductors’ images below the surface of the earth

\( T_C \)  
 conductor temperature

\( T_0 \)  
 reference temperature

\( \alpha \)  
 temperature coefficient of resistivity

\( \beta \)  
 temperature coefficient of reactance

\( R \)  
 line series resistance

\( q_s \)  
 solar heat gain

\( q_c \)  
 convection heat loss

\( q_r \)  
 radiated heat loss

\( \varepsilon_o \)  
 permittivity of free space

\( \varepsilon_r \)  
 permittivity of the material surrounding the surface relative to \( \varepsilon_o \)

\( V_S \)  
 sending-end voltage
\( I_S \) sending-end current
\( V_R \) receiving-end voltage
\( I_R \) receiving-end current
\( dx \) length of the differential element
\( \gamma \) propagation constant
\( Z_c \) line characteristic impedance
\( V_{R_{Dist}} \) uniformly distributed parameter model receiving-end voltage
\( I_{R_{Dist}} \) uniformly distributed parameter model receiving-end current
\( Z \) series impedance
\( Y \) shunt admittance
\( f_c \) cutoff frequency of the lumped line model
\( I_{Bus} \) vector of currents injected at each bus
\( V_{Bus} \) vector of bus voltages
\( Y_{Bus} \) bus admittance matrix
\( N \) number of buses in the system
\( n \) bus number
\( N_g \) number of generator buses in the system
\( |Y_{nm}| \) magnitude of the \( nm \) element of the bus admittance matrix
\( \delta_{nm} \) phase of the \( nm \) element of the bus admittance matrix
\( |V_n| \) voltage magnitude at Bus \( n \)
\( \theta_n \) voltage phase angles Bus \( n \)
\( P_n \) real power at Bus \( n \)
\( Q_n \) reactive power at Bus \( n \)
\( f \) operating electrical frequency

\( T_A \) ambient temperature

\( k \) segment number

\( d_k \) length of segment \( k \)

\( Z_k \) series impedance of segment \( k \)

\( Y_k \) shunt admittance of segment \( k \)

\( \mathcal{N}^0 \) size of the original power flow problem, i.e. number of buses

\( \mathcal{N}^1 \) size of the updated power flow problem

\( B \) number of branches in the system

\( b \) branch number

\( K_b \) number of segments in branch \( b \)

\( x \) vector of decision variables representing points of intersection between segments

\( x_S \) point representing the line sending-end

\( x_R \) point representing the line receiving-end

\( \Delta V_{\mathrm{Threshold}} \) threshold value on difference in receiving-end voltage magnitude

\( \Delta \theta_{\mathrm{Threshold}} \) threshold value on difference in receiving-end voltage phase angle

\( \Delta T_{\mathrm{Threshold}} \) allowable temperature difference across a section of the line

\( V_{\mathrm{Bench}}^R \) benchmark model receiving-end voltage

\( P_{\mathrm{max}} \) maximum real power transfer point

\( P_{\mathrm{Bench}}^{\mathrm{max}} \) maximum real power transfer point of the benchmark model

\( pf \) power factor
\( pf^{Bench} \) power factor of the benchmark model

\( \Delta pf_{Threshold} \) threshold value on difference in power factor

\( \Delta z \) portion of the series impedance dependent on \( x \)

\( r_0 \) reference series resistance per unit length

\( x_{L0} \) reference series inductive reactance per unit length

\( T_S \) temperature at the sending-end of the line

\( T_R \) temperature at the receiving-end of the line

\( m_{ab} \) distance between measurement points \( a \) and \( b \)

\( m_{Sa} \) distance between the line sending-end, \( x_S \), and measurement point \( a \)

\( m_{Ra} \) distance between the line receiving-end, \( x_R \), and measurement point \( a \)

\( \Delta x \) distance between temperature checkpoints (Algorithm 1)

\( i, j \) temperature checkpoints (Algorithm 1)

\( n_{ij} \) temperature difference between \( i \) and \( j \) normalized against \( \Delta T_{Threshold} \)

(Algorithm 1)

\( M \) number of temperature checkpoints – 1 (Algorithm 1)

\( q \) point tracking the end of the segmented portion of the line (Algorithm 1)

\( x_i, x_j \) points along the line for evaluation of temp. difference (Algorithm 2)

\( x_{max} \) point along the line tracking a local extremum (Algorithm 2)

\( T_{avg} \) average temperature based on \( T_S \) and \( T_R \)

\( T_{Wavg} \) weighted average temperature based on all temperature measurements

\( T_{Seg} \) average temperature for a segment of the line

\( N \) total # of temperature measurement points

\( a \) temperature measurement point
$T_a$ temperature measured at point $a$

$m_{a,a+1}$ distance between measurement point, $a$, and the subsequent one, $a+1$
By assuming the temperature coefficients $\alpha$ and $\beta$ in (4-3) to equal each other, i.e.

$$\alpha = \beta$$

the following definitions of $z$ and $\Delta z(x)$ hold:

$$z = (r_0 - \alpha r_0 T_0) + j(x_{e0} - \alpha x_{e0} T_0)$$ \hspace{1cm} (B-1)

$$\Delta z(x) = \alpha r_0 T(x) + j\alpha x_{e0} T_A(x) = \alpha z T_A(x)$$ \hspace{1cm} (B-2)

Since,

$$d\Delta z \over dx = \alpha z dT_A \over dx$$ \hspace{1cm} (B-3)

the differential equations in (4-7) can then be expressed as follows:

$$\frac{d^2 V}{dx^2} - \frac{\alpha z}{\gamma^2} \frac{dT_A}{dx} \frac{dV}{dx} - \gamma^2 \left(1 + \alpha T_A(x)\right) V(x) = 0$$ \hspace{1cm} (B-4)

$$\frac{d^2 I}{dx^2} - \gamma^2 \left(1 + \alpha T_A(x)\right) I(x) = 0$$

where

$$\gamma = \sqrt{yz} : \text{ propagation constant}$$

Assuming a temperature profile $T_A(x)$:

$$T_A(x) = Ax + B$$ \hspace{1cm} (B-5)

where $A$ and $B$ are any constant coefficient

$$\frac{dT_A}{dx} = A$$

$$\frac{d\Delta z}{dx} = \alpha z A$$ \hspace{1cm} (B-6)
$z(x)$ becomes

$$z(x) = z + \Delta z(x) = z(1 + \alpha B + \alpha A x)$$  \hspace{1cm} (B-7)

and

$$\frac{d^2 V}{dx^2} - \frac{\alpha A}{1 + \alpha B + \alpha A x} \frac{dV}{dx} - \gamma z (1 + \alpha B + \alpha A x) V = 0$$  \hspace{1cm} (B-8)

$$\frac{d^2 I}{dx^2} - \gamma z (1 + \alpha B + \alpha A x) I = 0$$
For the above example, $K = 2$ segments are created: the 1st between checkpoint 1 ($x_S$) and checkpoint 2, the 2nd between checkpoint 2 and checkpoint 4 ($x_R$).
# Appendix D  Single-line case study

## Tables D.1  Single-line case study 1:

Structure and parameter values for each of the simulated models.

### Distributed model

<table>
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<tr>
<th>Distributed Model</th>
<th>Series</th>
<th>Shunt</th>
</tr>
</thead>
<tbody>
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<td>$x$ (Ω/mi)</td>
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<tr>
<td>$T_{\text{Seas}}$</td>
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<tr>
<td>$T_{\text{avg}}$</td>
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### 1-segment model

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### Multi-segment model with $\Delta T_{\text{threshold}} = 1$

#### Series

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#### Shunt

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## Tables D.1 (cont.) Single-Line Case Study 1:

Structure and Parameter Values for Each of the Simulated Models

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<th>Shunt</th>
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<tr>
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<table>
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<th>Shunt</th>
</tr>
</thead>
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<td>$T_{\text{Wavg}}$</td>
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<td>$X$ ($\Omega$)</td>
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<tr>
<td>Seg 1</td>
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<td>53.1104</td>
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<table>
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<th>Multi-Seg. with $\Delta T_{\text{Threshold}} = 2$</th>
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<th>Shunt</th>
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<th>Shunt</th>
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<tr>
<td>Seg 5</td>
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<td>10.7091</td>
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</table>
### Table D.2 Single-Line Case Study 1 – Simulation Results:
Receiving-End Current Magnitude, $|I_R|$, and phase, $\theta_{IR}$, for Each Line Model

| Model         | $|I_R|$ (kArms) | $\theta_{IR}$ (deg) |
|---------------|----------------|---------------------|
|               | $T_{Seas}$ | $T_{avg}$ | $T_{Wavg}$ | $T_{Seg}$ | $T_{Seas}$ | $T_{avg}$ | $T_{Wavg}$ | $T_{Seg}$ |
| Distributed   | 0.909      | 0.904    | 0.905     | -        | -41.35    | -41.72    | -41.64     | -       |
| 1 Segment     | 0.921      | 0.914    | 0.916     | -        | -42.88    | -43.27    | -43.18     | -       |
| Multi Seg.    |            |          |           |          |          |          |            |         |
| 8 Seg $- \Delta T_{threshold} = 1^\circ$C | 0.912      | 0.901    | 0.903     | 0.903   | -41.68    | -42.06    | -41.97     | -41.97   |
| 5 Seg $- \Delta T_{threshold} = 1.5^\circ$C | 0.914      | 0.907    | 0.908     | 0.908   | -42.05    | -42.44    | -42.35     | -42.37   |
| 4 Seg $- \Delta T_{threshold} = 2^\circ$C | 0.915      | 0.909    | 0.910     | 0.910   | -42.06    | -42.45    | -42.36     | -42.37   |

### Table D.3 Single-Line Case Study 2:
Structure and Parameter Values for Each of the Simulated Models

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<th>Series</th>
<th>Shunt</th>
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### Tables D.3 (cont.) Single-Line Case Study 2: Structure and Parameter Values for Each of the Simulated Models

| Multi-Seg. with $\Delta T_{Threshold} = 1.5$ | $T_{Seg}$ | Series $R$ (Ω) | $X$ (Ω) | C (μF) | Shunt $T_{avg} / T_{Wavg}$ | Multi-Seg. with $\Delta T_{Threshold} = 1.5$ | $T_{seg}$ | Series $R$ (Ω) | $X$ (Ω) | C (μF) |
|---------------------------------------------|----------|-----------------|--------|------|-----------------------------|---------------------------------------------|---------|-----------------|--------|------|}
| Seg 1                                       | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 1                                       | 0.7167  | 7.1218          | 0.2092 |
| Seg 2                                       | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 2                                       | 0.7167  | 7.1218          | 0.2092 |
| Seg 3                                       | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 3                                       | 0.7167  | 7.1218          | 0.2092 |
| Seg 4                                       | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 4                                       | 0.7167  | 7.1218          | 0.2092 |
| Seg 5                                       | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 5                                       | 0.7167  | 7.1218          | 0.2092 |
| Seg 6                                       | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 6                                       | 0.7167  | 7.1218          | 0.2092 |
| Seg 7                                       | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 7                                       | 0.7167  | 7.1218          | 0.2092 |
| Seg 8                                       | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 8                                       | 0.7167  | 7.1218          | 0.2092 |
| Seg 9                                       | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 9                                       | 0.7167  | 7.1218          | 0.2092 |
| Seg 10                                      | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 10                                      | 0.7167  | 7.1218          | 0.2092 |
| Seg 11                                      | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 11                                      | 0.7167  | 7.1218          | 0.2092 |
| Seg 12                                      | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 12                                      | 0.7167  | 7.1218          | 0.2092 |
| Seg 13                                      | 0.7062   | 7.0165          | 0.2092 |      |                             | Seg 13                                      | 0.7167  | 7.1218          | 0.2092 |

### Scenario 1

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<th>$X$ (Ω)</th>
<th>C (μF)</th>
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### Scenario 2

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<th>$X$ (Ω)</th>
<th>C (μF)</th>
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Table D.4  Single-Line Case Study 2 – Simulation Results:

Receiving-End Current Magnitude, $|I_r|$, and phase, $\theta_{IR}$, for Each Line Model

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<th>Scenario2 $T_{Seg}$</th>
<th>$\theta_{IR}$ (deg)</th>
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### APPENDIX E  MULTI-BUS CASE STUDY

#### Table E.1  Select Bus Data for the 9-Bus Test Case – Case File

| Bus | Type | $P_d$ (MW) | $Q_d$ (MVAr) | $|V|$ (pu) | $\theta$ (deg) | $V_{base}$ (kV) | Max $V$ (pu) | Min $V$ (pu) |
|-----|------|------------|-------------|----------|--------------|-----------------|-----------|-----------|
| 1   | 3    | 0          | 0           | 1        | 0            | 345             | 1.1       | 0.9       |
| 2   | 2    | 0          | 0           | 1        | 0            | 345             | 1.1       | 0.9       |
| 3   | 2    | 0          | 0           | 1        | 0            | 345             | 1.1       | 0.9       |
| 4   | 1    | 0          | 0           | 1        | 0            | 345             | 1.1       | 0.9       |
| 5   | 1    | 90         | 30          | 1        | 0            | 345             | 1.1       | 0.9       |
| 6   | 1    | 0          | 0           | 1        | 0            | 345             | 1.1       | 0.9       |
| 7   | 1    | 100        | 35          | 1        | 0            | 345             | 1.1       | 0.9       |
| 8   | 1    | 0          | 0           | 1        | 0            | 345             | 1.1       | 0.9       |
| 9   | 1    | 125        | 50          | 1        | 0            | 345             | 1.1       | 0.9       |

#### Table E.2  Select Generator Data for the 9-Bus Test Case – Case File

| Bus | $P_g$ (MW) | $Q_g$ (MVAr) | $|V|$ (pu) | $S_{base}$ (MVA) | Max $P$ (MW) | Min $P$ (MW) | Max $Q$ (MVAr) | Min $Q$ (MVAr) |
|-----|------------|--------------|-----------|------------------|-------------|-------------|----------------|----------------|
| 1   | 0          | 0            | 1         | 100              | 250         | 10          | 300            | -300           |
| 2   | 163        | 0            | 1         | 100              | 300         | 10          | 300            | -300           |
| 3   | 85         | 0            | 1         | 100              | 270         | 10          | 300            | -300           |

#### Table E.3  Select Branch Data for the 9-Bus Test Case – Case File

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<th>$x$ (pu)</th>
<th>$b$ (pu)</th>
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Table E.4  Select Branch Data for the 9-Bus Test Case:

Mild Temperature Gradients – Multi-Seg at $T_{seg}$

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Table E.5  Select Branch Data for the 9-Bus Test Case:
Severe Temperature Gradients – Multi-Seg at $T_{seg}$

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### Table E.6  Power Flows and Losses for the 9-Bus Test Case – 1-Seg at $T_{Seas}$

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Table E.7  Power Flows and Losses for the 9-Bus Test Case:  
Mild Temperature Gradients – Multi-Seg at $T_{seg}$

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Severe Temperature Gradients – Multi-Seg at $T_{seg}$

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Total: 5.141 53.43
FULL NAME: Valentina Cecchi

DATE AND PLACE OF BIRTH: 10/29/1981, Rome, Italy

EDUCATION: Ph.D. Electrical Engineering Drexel University, PA, USA 2010
M.S. Electrical Engineering Drexel University, PA, USA 2007
B.S. Electrical Engineering Drexel University, PA, USA 2005

SELECT PUBLICATIONS:


