Reconstruction of Cluster Masses using Particle Based Lensing

A thesis
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by
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To my father, Mr. Partha Krishna Deb
Clusters of galaxies are among the richest astrophysical data systems, but to truly understand these systems, we need a detailed study of the relationship between observables and the underlying cluster dark matter distribution. Gravitational lensing is the most direct probe of dark matter, but many mass reconstruction techniques assume that cluster light traces mass, or combine different lensing signals in an ad hoc way. In this talk, we will describe ”Particle Based Lensing” (PBL), a new method for cluster mass reconstruction, that avoids many of the pitfalls of previous techniques. PBL optimally combines lensing information of varying signal-to-noise, and makes no assumptions about the relationship between mass and light.

we will describe mass reconstructions in three very different, but very illuminating cluster systems: the ”Bullet Cluster” (1E 0657-56), A901/902 and A1689. The “Bullet Cluster” is a system of merging clusters made famous by the first unambiguous lensing detection of dark matter. A901/902 is a multi-cluster system with four peaks, and provides an ideal laboratory for studying cluster interaction. we am particularly interested In measuring and correlating the dark matter clump ellipticities. A1689 is one of the richest clusters known, and has significant substructure at the core. It is also my first exercise in optimally combining weak and strong gravitational lensing in a cluster reconstruction. we find that the dark matter distribution is significantly clumpier than indicated by X-ray maps of the gas.we conclude by discussing various potential applications of PBL to existing and future data.
Acknowledgements

The journey to writing my thesis started early on when I got very curious about wound up toy cars. My father explained to me that I provide the energy for the toy car to move. Since then, I have been attracted to the various aspects of Physics with steady encouragement from my school teachers and family. In particular Ms. Gargi Bose and Mr. Basab Bhattacharya my high school physics teachers were very instrumental in sustaining and nurturing my interests in physics.

Joining Presidency College in my hometown, Kolkata, India, to major in Physics was an obvious decision. My undergraduate years were the most formative time in learning Physics. The environment in the Physics department was very invigorating, encouraging us to look beyond our stipulated syllabus. One of my mentors, Professor Dipanjan Rai Chaudhuri, motivated me to read books like “QED” by Richard Feynman and mechanics from “Landau and Lifshitz”. I was also inspired by numerous discussions with my colleagues and friends like Baisakhi Ray, Rahul Biswas and others that made learning particularly enjoyable.

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I did my Masters at Indian Institute of Technology (IIT) at Kanpur, India. The intense environment at IIT taught me the value of meeting deadlines along with enjoying Physics. I also developed the valuable friendship of Devdeep Sarkar during
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Contents

Abstract iv

Acknowledgements v

List of Figures xii

List of Tables xix

1 Introduction 1

1.1 Motivation for Studying Galaxy Clusters . . . . . . . . . . . . . . . . 1

1.2 The Astrophysics of Galaxy Clusters . . . . . . . . . . . . . . . . . 4

1.3 Gravitational Lensing . . . . . . . . . . . . . . . . . . . . . . . . . . 6

1.3.1 Weak Lensing . . . . . . . . . . . . . . . . . . . . . . . . . . . . 8

1.3.2 Strong Lensing . . . . . . . . . . . . . . . . . . . . . . . . . . . 9

1.3.3 Circular Lens Models . . . . . . . . . . . . . . . . . . . . . . . 11

1.3.4 Elliptical Lens Models . . . . . . . . . . . . . . . . . . . . . . 13

1.4 Cluster Lensing . . . . . . . . . . . . . . . . . . . . . . . . . . . . 14

1.4.1 Cluster Observations . . . . . . . . . . . . . . . . . . . . . . . 16

1.4.2 Mass Modeling of Clusters . . . . . . . . . . . . . . . . . . . . 19

1.4.3 Parametric vs Non-parametric techniques . . . . . . . . . . . . . . 22
2 Reconstruction of Cluster Masses using Particle Based Lensing

2.1 Introduction

2.2 Grid-Based Cluster Lensing

2.2.1 Weak Lensing on Grids

2.2.2 Strong Lensing

2.2.3 Regularization

2.2.4 Some Questions

2.3 Particle Based Lensing – PBL

2.3.1 A Particle Description of the fields

2.3.2 PBL vs. Regularization

2.3.3 Estimation of the Potential Field

2.3.4 Interpolated Ellipticities

2.3.5 $\chi^2$ Minimization

2.4 Test Applications

2.4.1 Simulation: Softened Isothermal Sphere

2.4.2 Simulation: A Double Peaked Cluster

2.4.3 Observation: The Bullet Cluster

2.4.4 Summary

3 Covariance analysis of Weak lensing mass reconstructions: Measuring dark matter ellipticity.

3.1 Introduction

3.2 Non-parametric mass Reconstruction Techniques

3.3 Method optimization
3.3.1 Fitting the error ................................................. 64
3.4 Covariance .......................................................... 66
3.5 Where is the information in Cluster Lensing? Cluster Ellipticity .... 69
  3.5.1 Measuring Cluster Shapes Non-parametrically .................. 70
3.6 Data ................................................................. 72
3.7 Results ............................................................... 73
  3.7.1 Parametric Fitting ............................................... 75
  3.7.2 Comparison between dark matter and light distribution ....... 76
  3.7.3 Comparison between parametric and non-parametric results . 78
3.8 Discussion and Future Work ........................................ 78

References ......................................................... 88

4 Measuring substructure in A1689 by combining lensing and X-rays 95
  4.1 Introduction .......................................................... 96
  4.2 Strong+Weak Mass Reconstruction .................................. 97
    4.2.1 Covariance of S+W map ...................................... 99
  4.3 Weak Lensing data ................................................ 100
  4.4 Strong Lensing data ................................................ 101
    4.4.1 X-ray Data .................................................. 102
  4.5 Power Ratios ........................................................ 103
  4.6 Results .............................................................. 105
  4.7 Ratio Weak lensing and X-ray Mass ................................ 106
    4.7.1 Weak Lensing Mass Profile .................................. 106
  4.8 Discussion ......................................................... 107

References ......................................................... 111
5 Future Prospects

5.1 Introduction .................................................. 113
5.2 Additional Sources of Information ........................... 114
  5.2.1 Flux .................................................. 114
  5.2.2 Ellipticity Differences ................................. 115
  5.2.3 Flexion ................................................ 116
5.3 Applications: Multiwavelength mass modelling of galaxy clusters ... 117
  5.3.1 Lensing+Xrays ....................................... 117
5.4 Projection Effects ........................................... 120
5.5 Data .......................................................... 121

References ......................................................... 123

A Details of matrix Inversion and Covariance Calculation ............... 126
  A.1 Inverting the Covariance matrix for $\chi^2$ minimization. ........ 126
  A.2 Generalization to the Non-linear regime ........................ 128
List of Figures

1.1 Cluster formation (Borgani & Kravtsov, 2009) using hydrodynamical simulation carried out by the Tree-SPH GADGET code. The upper panel represents the dark matter distribution, the middle panel shows the gas distribution, and the bottom panel depicts the distribution of stars. Each panel shows a snapshot at three different epochs: \( z = 4 \) (Left), \( z = 2 \) (middle), and \( z = 0 \) (right). The cluster has a mass of \( \sim 10^{15} M_\odot \) and each side in the panels is \( 24h^{-1} \) Mpc.  

1.2 Ray diagram representing gravitational lensing. A ray from a background source like a galaxy gets deflected due to the presence of a massive lens like a galaxy cluster. The ray of light from an angular position \( \beta \) get deflected to the observed position \( \theta \). Due to the close alignment of the source and lens we see multiple images. This image is taken from http://leo.astronomy.cz/grlens/grl0.html.  

1.3 This is a cartoon representation of weak lensing (Williamson et al., 2007). The upper panel represents the effect of cluster lensing for circular galaxies. The lower panel exemplifies the real situation. Unlensed galaxies have intrinsic ellipticities. Hence the lensing signal is not embedded in a single galaxy. Several tens of them have to be averaged to measure the lensing effect.
1.4 A composite image of A1689 in X-ray and optical. The lensing effect is clearly visible in several arcs around the cluster core. The X-ray gas is represented by the extended blue distribution. This image is taken from http://chandra.harvard.edu/photo/2008/a1689/.

1.5 A composite image of the Bullet Cluster in X-ray and optical. Hot gas detected by Chandra in X-rays is seen as two pink clumps in the image and contains most of the baryonic, matter in the two clusters. The bullet-shaped clump on the right is the hot gas from one cluster, which passed through the hot gas from the other larger cluster during the collision. An optical image from Magellan and the Hubble Space Telescope shows the galaxies in orange and white. The blue areas in this image show where astronomers find most of the mass in the clusters. Most of the matter in the clusters (blue) is clearly separate from the baryonic matter (pink), giving direct evidence that nearly all of the matter in the clusters is dark. This image is taken from http://chandra.harvard.edu/photo/2006/1e0657/.

1.6 Distribution of galaxies in the A901/902 field of view. The galaxy sample is shown in green and the cluster sample is shown in black. The y-axis represents the photometric redshifts and the x-axis represents the total R-band magnitude for each galaxy.

2.1 A comparison between PBL and grid based mass reconstruction technique.
2.2 In the left panel, we plot the interpolated Heaviside step function. It is clear from the plot that the function is only approximated by a smooth function near $g=1$, for all other $g$ it behaves like an ordinary step function. Also higher value of the parameter $\eta_0$ increases the accuracy. In the right panel, we plot the resulting ellipticity as a function of reduced shear for the combination, $|\gamma| = \kappa$. 

2.3 A radial plot of the reconstructed convergence ($\kappa$) of a simulated Softened Isothermal Sphere. The circles represent binned reconstructed $\kappa$ and the error bars represent the scatter in each bin. The dots represent the true value of $\kappa$ given by Eq. 2.25. Left Panel: Using PBL. Right panel: Using grid based method. The error bars in the radial plot using PBL is higher. This is because the errors introduced in PBL are dependent on the local signal to noise which are not spherically symmetric. In the grid based method the errors are averaged uniformly on the length scale of a single grid which makes the radial scatter very low.

2.4 The plot of the difference between reconstructed convergence, $\kappa$ and true $\kappa$ for the double peak SIS system. Left Panel: Using PBL. Right panel: Using grid based method described in § 2.2. Both maps are gridded for easy visualization. Also there are empty grid cells with no image galaxies. The value for those grid cells in the above difference map is set to zero for both reconstructions. As we can see the error in the cores of the peaks is much using PBL mass reconstruction.
2.5 A weak-lensing only reconstruction of the bullet cluster using PBL described in § 2.3. Note that both substructure peaks are clearly identified. Left Panel: This the $\kappa$-map using PBL. The cross denotes the centroid of the multiply imaged positions. Right Panel: This a comparison of the $\kappa$ contour derived using PBL(solid) and the publicly available contour plot of $\kappa$(dashed).

3.1 Error vs the smoothing scale for two levels of noise. The triangles and the squares represent the errors measured from mass reconstruction done for different values of noise and smoothing scale. The solid line and the dashed line represent the fitted values for $\frac{\sigma^2}{\langle \kappa^2 \rangle} = 1,5$ given by Equation 3.8, here $\hat{\zeta} = \frac{\zeta}{\lambda}$. The minima in this plot represents the ideal smoothing scale for the given noise. As expected decreasing the signal-to-noise for the mass reconstruction shifts the minimum towards higher smoothing scale.

3.2 Mass reconstruction of dark matter halos in A901/902. Top Left Panel: A901b Top Right Panel: Error Map of A901b. Bottom Left Panel: SouthWest Group. Bottom Right Panel: Error Map for the SouthWest Group. There is an artificial increase of error towards the extremities of the maps where the convolution kernel steps over its hard edges. A901b is a compact dark matter halo, the peak is detected at $7\sigma$ and the Southwest Group has significant substructure with two sub-peaks detected at $4\sigma$ significance level. We measure the ellipticity of these two peaks.
3.3 Mass reconstruction of dark matter halos in A901/902. Top Left Panel: A901a Top Right Panel: Error Map of A901a. Bottom Left Panel: A902. Bottom Right Panel: Error Map A902. A901a has two distinct peaks, the central peak is detected at $5\sigma$ and the secondary peak is detected at $2\sigma$. A902 is reasonably disturbed with the central peak detected at $4\sigma$. These two peaks are not representative of elliptical dark matter halos.

3.4 This is a plot of ellipticity vs radial distance. The dot-dashed line represents the ellipticity of the dark matter halos and the dotted line represents the ellipticity of the light distribution. The upper panel is a plot for A901b and the lower panel is a plot for the Southwest Group. For both sub-clusters the ellipticity of the light distribution decreases with radial distance. For A901b the ellipticity of the dark matter does not vary much with radial distance. The ellipticity of the dark matter also decreases with radial distance for the Southwest Group.

3.5 This is a plot of the cosine of the angle between the major axis of the four sub-clusters with radial distance. A901a, A901b and A902 have major axis pointing in almost the same direction, hence the cosine of the angle between their major axis is very close unity. This is plotted in the upper panel. The major axis of the Southwest Group is misaligned with the other clusters, this effect being most pronounced for A901b, the cosine of angle between A901b and the Southwest Group deviates most from unity. This is seen in the lower panel of the plot.
3.6 The two panels represents the joint 1(2)-σ error probability distribution for A901b and the Southwest Peak derived from the parametric modeling, described in § 1.4.2. The y-axis $f$ is the axis ratio and the x-axis $\alpha$ is the position angle defined in section § 1.4.2 in radians. For both plots we have plotted the 1σ and 2σ contours. Both peaks have non-zero ellipticity at 2σ level.

3.7 A comparison of the light distribution vs the dark matter distribution. The colors represent the light distribution and the over-layed contours represent the dark matter distribution. Left Panel: A901b. Right Panel: Southwest group. The units of the light distribution is $M_\odot$.

3.8 Plot of the cosine of the angle made by the major axis of the light distribution and the dark matter distribution. The light and the dark matter distribution for A901a, A902 and the Southwest group are aligned. Close to the center of the cluster the dark matter and the light distribution are misaligned.

4.1 X-ray Surface Brightness distribution for A1689 analyzed by Riemer-Sørensen et al. (2009). The X-ray distribution is fairly uniform at the center. There is an elongation in the north-east direction.

4.2 Upper Panel: A lensing Strong+Weak mass reconstruction of A1689 using Particle Based Lensing. The X-ray distribution is fairly uniform at the center whereas the (S+W) reconstruction shows significant substructure. Lower Panel: Error map for the same field of view. The contours of the field of view represent values of $\kappa$.

4.3 Ratio between the X-ray and lensing masses. The error bars are correlated since the weak lensing mass in one radial bin depends on the inner bins.
5.1 The magnification as a function of shear and convergence. The right panel is a simple slice through the left, with the choice $\gamma = \kappa$. Neither the magnification nor its derivatives are a continuous function. Moreover, flux ratios are only measurable for systems with at least two images (obviously). One or more of the images will necessarily have negative parity. Thus, a solution to the potential field which is found using standard relaxation methods will not normally converge to a negative parity estimate for any magnification.

5.2 A plot of $P_3/P_0$ vs $P_2/P_0$ (Jeltema et al., 2005). This notation is defined in chapter 4. The X-ray image for 6 clusters have been shown with their power ratios. It is clear that complex clusters have higher power. With the advent of high quality Strong and Weak lensing data, a similar plot can be made from lens mass reconstructions.
List of Tables

1.1 Examples of circularly symmetric lenses. The two-dimesional lensing potential, the deflection angle and the convergence are given as a function of angular co-ordinate $\theta$. $M$ is the mass of the point mass lens, $\sigma$ is the velocity dispersion for the SIS, $\theta_c$ is the core radius for the Softened Isothermal Sphere and $\kappa_0$ is the constant convergence for a mass sheet. .......................................................... 12

2.1 Comparison between PBL and grid based method. ................. 46

3.1 A summary of the relation between various matrices defined in § 3.4. The last column gives the covariance among the observable in the third column. The last row is the final expression for the covariance in the reconstructed $\kappa$. .......................................................... 69

3.2 Measuring ellipticity of dark matter and light distribution. The measurements for the ellipticity and position angle are inferred at a distance of $200h^{-1}$ kpc from the center of each peak for non-parametric measurements. .............................................. 77

4.1 Power ratio measurements of the X-ray gas distribution at 500 kpc from the center. .......................................................... 106
Chapter 1

Introduction

Abstract

In this chapter, we discuss the importance of studying galaxy clusters in the context of the data that will be available from present and future telescopes. We briefly review the basic astrophysics of galaxy clusters, and how they are probed observationally. This thesis primarily deals with using gravitational lensing as a tool for studying galaxy clusters, with a special focus on non-parametric techniques. To set the stage we examine the basic theory of lensing and some well known lens models. We also explain the importance of non-parametric cluster lensing techniques.

1.1 Motivation for Studying Galaxy Clusters

One of the most compelling questions in cosmology today involves the nature of “Dark Matter”, the dominant component of matter in the universe. Evaluating the clustering and evolution of Dark Matter will give insight into its properties. Galaxy clusters are excellent probes of cosmological structure formation and the astrophysics of gas and galaxy formation (Allen et al., 2007; King, 2007; Mannucci et al., 2007).
The deep potential wells of the dark matter halos of the galaxy clusters trap baryons in the form intra cluster gas. Due to their relative dynamical youth, some of the clusters are still in the process of formation, making them excellent snapshots of the non-linear universe.

The mass function of clusters is a sensitive probe of cosmological parameters like $\Omega_m$ and $\sigma_8$ (Rines et al., 2007) and its observed evolution is an important test of theories of structure formation (Gunn & Gott, 1972; Giocoli et al., 2007; Horellou & Berge, 2005; Cooray & Sheth, 2002). The geometrical shape of cluster Dark Matter halos provide valuable information on intra-cluster gas distribution (Flores et al., 2005, 2007). While simulations predict central density distribution of matter in clusters to follow an NFW profile, it is debatable whether observations suggests that clusters have a central core (Sand et al. (2003); Voigt & Fabian (2006)).

$\Lambda$CDM structure formation theories also predict that massive dark matter halos assemble from the hierarchical merging of lower mass subhalos. As noted by several authors (Moore et al. (1999); Klypin et al. (1999)), the number of subhalos that survive in N-body simulations is much greater than the number of dwarf galaxies observed in the Milky Way and the Andromeda. On cluster scales, such discrepancies are not observed. Thus the subhalo mass function in clusters is an important probe of the CDM theory in this mass scale.

High-resolution, accurate measurements of cluster mass maps are thus highly desirable. Gravitational lensing is a powerful tool to probe the projected mass map of the clusters independent of the internal dynamics, and has already been widely applied to mapping mass distribution in clusters (Wittman et al., 2001; Hoekstra et al., 2001; Sheldon et al., 2001; Gray et al., 2002; Taylor et al., 2004; Broadhurst et al., 2005b; Leonard et al., 2007; Okura et al., 2007; Heymans et al., 2008). Some researchers (Natarajan & Springel, 2004; Natarajan et al., 2007) have used the in-
individual galaxy-galaxy lensing signal to estimate individual galaxy masses and thus produce a parametric mass reconstruction of the cluster. Others have used the weak signal to characterize the overall potential from the cluster without recourse to parametric models (Wilson et al., 1996; Hoekstra et al., 1998; Natarajan & Refregier, 2000; Hoekstra et al., 2004).

Given the importance of accurately measuring the mass, shape, and substructure of individual clusters, and given the enormous expense of long time-exposure observations of clusters, it is extremely important to maximize the signal-noise from a particular dataset, and to produce high-resolution maps of substructure within individual clusters. Current mass reconstruction techniques are ill-equipped to handle multi-scale datasets or clusters with significant clumpiness or cuspiness, or are jury-rigged to do so. In this thesis, we propose Particle Based Lensing (PBL; pronounced “pebble”) as an alternative approach to cluster reconstruction. The unique feature of this method is that it allows reconstruction with variable resolution with well understood error covariance. We discuss this method in details in the following chapters.

Development of robust and high fidelity mass reconstruction algorithms such as PBL is essential to interpret the large amount of existing and upcoming data. Optical surveys like SDSS have detected thirteen thousand clusters (Koester et al., 2007) and future all sky surveys like LSST and JDEM will find many more. SUBARU (Miyazaki et al., 2007) has also detected several tens of clusters with lensing. X-ray surveys like ROSAT have also detected hundreds of clusters. Cosmic Microwave Background experiments like ACT (Hincks et al., 2009) and SPT (Vanderlinde et al., 2010) are building their SZ cluster catalogs. Surveys like MACS (Smith et al., 2005) perform pointed observations of individual clusters from HST (optical) and CHANDRA (X-ray). Strong lensing analysis has been done for tens of these clusters (Zitrin et al., 2010).
These observations will help us get insight into triaxiality of galaxy clusters, whether the central region of the cluster profile is cuspy as predicted by ΛCDM simulations or shallower as suggested by observations, and the amount of substructure present in galaxy clusters. With the vast improvement in data and better mass modeling techniques we are shifting paradigms from studying one dimensional mass profiles to understanding two-dimensional mass and gas maps of the clusters.

1.2 The Astrophysics of Galaxy Clusters

Over the last century, cosmological observations have established the presence of a hierarchy of structures on different scales. On scales of kiloparsecs we observe non-linear structure in the form of galaxies. An appreciable fraction (10%) of galaxies are found in gravitationally bound groups forming clusters. Clusters are usually a few Mpc in size and may contain anywhere between a few hundred to a few thousand galaxies. Interestingly, most of the baryonic mass in a cluster is in the form of intracluster gas that is heated up to temperatures of $10^6 - 10^8$ K in the potential well of the dark matter halo of the cluster.

The Cold Dark Matter model of the universe predicts that larger, virialised structures are formed by the accretion of smaller objects. In this scenario the process of cluster formation is violent with strong interplay between three main ingredients - dark matter, gas and stars. This is illustrated in Figure 1 which shows a simulation of cluster formation. The driving force for this process is gravity, causing the accretion of smaller halos onto larger ones. The gas distribution generally follows the dark matter distribution. The stars are formed at an earlier epoch within high density halos making their distribution very clumpy. This is shown in Figure 1.1. At a redshift of $z = 4$ we see a filamentary structure with massive dark matter halos at the nodes. These halos accrete smaller halos that flow along the filaments forming a massive
cluster at the present epoch. When the cluster virialises, it contains galaxies that have survived the merger in a medium of nearly uniform gas distribution as shown by the blue shading in Figure 1.4.

Figure 1.1: Cluster formation (Borgani & Kravtsov, 2009) using hydrodynamical simulation carried out by the Tree-SPH GADGET code. The upper panel represents the dark matter distribution, the middle panel shows the gas distribution, and the bottom panel depicts the distribution of stars. Each panel shows a snapshot at three different epochs: $z=4$ (Left), $z=2$ (middle), and $z=0$ (right). The cluster has a mass of $\sim 10^{15} M_\odot$ and each side in the panels is $24h^{-1}$ Mpc.
Dark matter and gas are accreted at thousands of kilometers per second and this kinetic energy is converted to internal thermal energy of $10^{54} - 10^{58}$ joules via supersonic gas flows and shocks in the resident cold gas. This energy heats up the gas that gets trapped in the potential well of the cluster. Current theories and simulations reproduce the observational properties of clusters beyond the virial radius very well. However, as one approaches the core of the cluster there are discrepancies between observations and simulations (Springel et al., 2001; Tornatore et al., 2007; Nagai et al., 2007) in the temperature and mass profile since the physics of cluster cores is fairly complicated.

### 1.3 Gravitational Lensing

The central theme of this thesis is to map the dark matter distribution using weak and strong gravitational lensing. Gravitational lensing is the deflection of light rays from distant sources due to the gravitational action of the intervening massive objects. According to Einstein’s General Relativity, lensing is a result of light traveling along null geodesics around massive objects. In this thesis, I will be modeling galaxy cluster masses. For such systems we can explain the lensing phenomena using ray diagrams.

Let us consider Figure 1.2. The galaxy cluster at an angular diameter distance $D_d$ is the deflector. The source is located at an angular diameter distance $D_s$. Due to lensing, the path of the light ray from the source S, is deflected by an angle $\hat{\alpha}$, such that the observed angular position $\vec{\theta}$ is related to the true angular position $\beta$ through the lens equation,

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}).$$  \hspace{1cm} (1.1)

where $\vec{\alpha} = \frac{D_d}{D_s} \hat{\alpha}$ The deflection angle is related to the potential via

$$\vec{\alpha} = \vec{\nabla} \psi.$$  \hspace{1cm} (1.2)
For an extended source the image is usually lensed differentially and hence its shape is distorted. Equation 1.1 can have multiple solutions - a situation known as strong lensing.

The dimensionless surface mass density of the lens mass distribution is given by,

$$\kappa = \frac{\Sigma(D_d \vec{\theta})}{\Sigma_{cr}}$$

(1.3)

where $\Sigma$ is the projected mass density and $\Sigma_{cr} = \frac{c^2}{4\pi G D_d D_{ds}}$ is the critical mass density that is dependent on the source redshift and lens redshift.

Figure 1.2: Ray diagram representing gravitational lensing. A ray from a background source like a galaxy gets deflected due to the presence of a massive lens like a galaxy cluster. The ray of light from an angular position $\beta$ get deflected to the observed position $\theta$. Due to the close alignment of the source and lens we see multiple images. This image is taken from http://leo.astronomy.cz/grlens/grl0.html
1.3.1 Weak Lensing

The limit of $\kappa << 1$ represents the “weak lensing” regime. This comprises of subtle distortions of background images due lensing by foreground masses. Lensing changes the shape of the background images in a coherent manner. If the source plane were similar to a wallpaper with circles on it, lensing distorts the circles into ellipses. We could measure the ellipticity of one galaxy and reconstruct the lensing mass locally. This situation is represented in the upper panel of Figure 1.3. In reality galaxies have intrinsic ellipticity. The lower left panel of Figure 1.3 represents a cartoon picture of a field of galaxies and the lower right panel shows the effect of lensing due to massive cluster in the foreground. It is clear that distortion information from one particular image can be misleading but averaging over several tens of images will reveal the coherent lensing signature. When the lensing potential does not vary appreciably across the source, the lens mapping can be linearized. The transformation between the source and the image is given by the Jacobian matrix

$$A(\theta) \equiv \frac{\partial \beta}{\partial \theta} = (\delta_{ij} - \psi_{ij})$$

$$= \begin{pmatrix}
1 - \kappa - \gamma_1 & -\gamma_2 \\
-\gamma_2 & 1 - \kappa + \gamma_1
\end{pmatrix}.$$  \hspace{1cm} (1.4)

The radial eigenvalue for the Jacobian is given by, $\lambda_+ = 1 - \kappa + |\gamma|$ and the tangential eigenvalue is given by, $\lambda_- = 1 - \kappa - |\gamma|$. The matrix is singular where $\lambda_\pm = 0$. These points define the critical curves of the lens.

From this, we see that distortions in shape are well described in terms of shear which is related to the lensing potentials through the relations:

$$\gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22}),$$  \hspace{1cm} (1.5)

$$\gamma_2 = \psi_{,12}$$  \hspace{1cm} (1.6)
using Einstein convention for derivatives. The shear can be written as a complex number defined by
\[ \gamma = |\gamma| e^{(2i\phi)}, \]  
(1.7)
where \( |\gamma| = \sqrt{\gamma_1^2 + \gamma_2^2} \). The shear aligns itself tangentially around the lensing mass distribution.

The convergence, i.e the dimensionless surface mass density introduced in section 1.3.2 can also related to the second derivatives of the potential.
\[ \kappa = \frac{1}{2}(\psi_{,11} + \psi_{,22}). \]  
(1.8)
The weak lensing observables are the measured ellipticity of the background images, is expressed in terms of the shear and convergence.

## 1.3.2 Strong Lensing

Occasionally, a massive galaxy or a cluster of galaxies is closely aligned along the line of sight with a background source like a galaxy or a quasar. This leads to the strong lensing phenomena where multiple images of the source are observed or images of the background galaxies are highly distorted into arcs. Strong lensing occurs when there are multiple solution to Equation 1.1. Strong lensing regions have \( \kappa > 1 \). This phenomena can be used to determine the mass of the lens. The extreme case of strong lensing occurs when the lens and the source lie along the line of sight. In this case the source gets lensed into a circle known as the Einstein Ring.

The strong lensing observable is the angular position \( (\theta) \) of the multiple images. Using Equation 1.1 we can relate the difference of the angular positions to the difference of the deflection angle at those positions. The strong lensing constraint is given by,
\[ \bar{\theta}_A - \bar{\theta}_B = \bar{\alpha}_A\{\psi\} - \bar{\alpha}_B\{\psi\} \]  
(1.9)
Figure 1.3: This is a cartoon representation of weak lensing (Williamson et al., 2007). The upper panel represents the effect of cluster lensing for circular galaxies. The lower panel exemplifies the real situation. Unlensed galaxies have intrinsic ellipticities. Hence the lensing signal is not embedded in a single galaxy. Several tens of them have to be averaged to measure the lensing effect.

We use this constraint in subsequent lens modeling analysis.

Each of the deflection ($\alpha$), the shear, ($\gamma_1, \gamma_2$) and the convergence ($\kappa$), are linear functions of the potential field. When the source is distributed in redshift each of
these terms scales as:

$$\kappa(z_s) = Z(z_d, z_s)\kappa(z_s = \infty).$$ (1.10)

and where

$$Z(z_s) = \frac{D_{ds}}{D_s}.$$ (1.11)

### 1.3.3 Circular Lens Models

In order to understand some basic lensing phenomena we explore circular lens models. In reality clusters or galaxies are more complicated than circularly symmetric models. Nevertheless circular lens models are useful for theoretical understanding. We discuss some general properties of these lenses (Meylan et al., 2006) and particular cases are summarized in Table 1.1. In these lenses deflection of light occurs in the radial direction, the deflection angle is given by,

$$\alpha(\theta) = \frac{2}{\theta} \int_0^\theta d\theta \kappa(\theta) = \frac{4GM(< \xi)}{c^2 \xi} D_s \frac{D_{ds}}{D_s},$$ (1.12)

where $\xi = D_d\theta$ is the proper distance in the lens plane. In this case the lens equation becomes

$$\vec{\beta} = \vec{\theta}[1 - \alpha(\theta)] = \vec{\theta}[1 - \langle \kappa(\theta) \rangle],$$ (1.13)

where

$$\langle \kappa(\theta) \rangle = \frac{2}{\theta^2} \int_0^\theta \theta \kappa(\theta) = \frac{\alpha(\theta)}{\theta},$$ (1.14)

is the average dimensionless surface mass density interior to radius $\theta$.

The convergence of circular lenses is given by,

$$\kappa = \frac{1}{2} \left( \frac{\alpha}{\theta} + \frac{d\alpha}{d\theta} \right)$$ (1.15)

and the shear is given by,
<table>
<thead>
<tr>
<th>Models</th>
<th>$\psi(\theta)$</th>
<th>$\alpha(\theta)$</th>
<th>$\kappa(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point Mass</td>
<td>$\frac{D_{ds}}{D_s} \frac{4GM}{c^2} \ln \theta$</td>
<td>$\frac{D_{ds}}{D_s} \frac{4GM}{c^2D_d[\theta]}$</td>
<td>-</td>
</tr>
<tr>
<td>Singular Isothermal Sphere</td>
<td>$\frac{D_{ds}}{D_s} \frac{4\pi\sigma^2 \theta}{c^2}$</td>
<td>$\frac{D_{ds}}{D_s} \frac{4\pi\sigma^2}{c^2}$</td>
<td>$\frac{D_{ds}}{D_s} \frac{4\pi\sigma^2}{c^2\theta}$</td>
</tr>
<tr>
<td>Softened Isothermal Sphere(SIS)</td>
<td>$\frac{D_{ds}}{D_s} \frac{4\pi\sigma^2}{c^2} \sqrt{\theta^2 + \theta_c^2}$</td>
<td>$\frac{D_{ds}}{D_s} \frac{4\pi\sigma^2}{c^2} \frac{\theta}{\sqrt{\theta^2 + \theta_c^2}}$</td>
<td>$\frac{D_{ds}}{D_s} \frac{4\pi\sigma^2}{c^2} \frac{\theta^2 + 2\theta_c^2}{(\theta^2 + \theta_c^2)^{3/2}}$</td>
</tr>
<tr>
<td>Mass Sheet</td>
<td>$\frac{1}{2} \kappa_0 \theta^2$</td>
<td>$\kappa_0 \theta$</td>
<td>$\kappa_0$</td>
</tr>
</tbody>
</table>

Table 1.1: Examples of circularly symmetric lenses. The two-dimesional lensing potential, the deflection angle and the convergence are given as a function of angular co-ordinate $\theta$. $M$ is the mass of the point mass lens, $\sigma$ is the velocity dispersion for the SIS, $\theta_c$ is the core radius for the Softened Isothermal Sphere and $\kappa_0$ is the constant convergence for a mass sheet.

\[
\gamma = \frac{1}{2} \left( \frac{\alpha}{\theta} - \frac{d\alpha}{d\theta} \right) = \langle \kappa \rangle - \kappa. \tag{1.16}
\]

**Lensing by a Singular Isothermal Sphere**

The Singular Isothermal Sphere is the simplest and most studied model for lensing. We use this model to estimate lensing fields for cluster-sized halos and galaxy sized halos. The shear for the isothermal sphere is given by,

\[
|\gamma| = \left( \frac{\theta_E}{2\theta} \right) \tag{1.17}
\]

where

\[
\theta_E = \frac{D_{ds}}{D_s} \frac{4\pi\sigma^2}{c^2}. \tag{1.18}
\]
When the source is at a very high redshift the ratio \( \frac{D_s}{L_s} \) approaches unity. Here \( \sigma_v \) is the velocity dispersion and \( c \) is the speed of light. For a typical cluster \( \sigma_v \approx 600 \) km/sec

\[
\theta_E = 10.37'' \left( \frac{\sigma_v}{600 \text{km/sec}} \right)^2
\]

\[
| \gamma | = \frac{10.37}{2\theta} \left( \frac{\sigma_v}{600 \text{km/sec}} \right)^2
\]

For a galaxy \( \sigma_v \approx 200 \text{km/sec} \)

\[
\theta_E = 1.4'' \left( \frac{\sigma_v}{200 \text{km/sec}} \right)^2
\]

\[
| \gamma | = \frac{1.4}{2\theta} \left( \frac{\sigma_v}{200 \text{km/sec}} \right)^2
\]

### 1.3.4 Elliptical Lens Models

Circularly symmetric lenses are ideal for pedagogical understanding of the phenomena and should be used as a first approximation to modeling. Observations of clusters in the X-ray and non-parametric lensing mass maps suggests that clusters are not spherical. This is expected as clusters are formed from mergers of smaller halos and most of them are still in the process of formation.

As a next level of complexity in modeling we consider elliptical lenses. The convergence and the shear for this profile (Kormann et al., 1994) is given by,

\[
\kappa = \gamma = \frac{1}{2} \frac{\theta_E}{\theta} f \left[ \cos(\phi - \alpha) + f^2 \sin(\phi - \alpha) \right]^{-\frac{1}{2}},
\]

where \( \theta_E \) is the Einstein radius defined in Equation (1.19). The quantity \( f \) is the axis ratio and \( \alpha \) is the position angle. This relationship is degenerate with \( \alpha = \alpha + \pi/2 \) and \( f = 1/f \). Like shear ellipticity also has a complex representation given by,

\[
e = \left( \frac{1-f}{1+f} \right) e^{2i\alpha}.
\]
This form of the mass model has been investigated by King & Schneider (2001) and applied to a sample of X-ray luminous clusters (Cypriano et al., 2004). This model will be used in chapter 3 to determine the ellipticity parametrically.

Gravitational lensing mass measurement suffer from mass sheet degeneracy when the sources are not distributed in redshift. This implies that \( \kappa \) can be determined up to a degeneracy \( \lambda \kappa + (1 - \lambda) \). This transforms to a degeneracy in the potential of the form,

\[
\psi(\theta) \rightarrow \psi'(\theta) = \frac{1}{2}(1 - \lambda)\theta^2 + \lambda \psi(\theta). \tag{1.25}
\]

### 1.4 Cluster Lensing

Clusters show a wide range of lensing phenomena: the spectacular multiple arcs near the core, flexion measurements at the semi-strong region, and very weak lensing distortions toward the periphery. The most spectacular example of a cluster lens is A1689. We will analyze this cluster in more details in Chapter 4. Figure 1.4 is a composite (X-ray+optical) image of this cluster. The bright yellow galaxies are the cluster members. Centered around the core of the cluster are several arcs produced due to strong lensing. The diffuse blue region represents the X-ray emission.

Lensing analysis is particularly relevant for dynamically active galaxy clusters, like the “Bullet Cluster” where the dark matter peaks are well separated from the gas. This cluster is a merging system located at \( z = 0.296 \). In this system, lensing reconstruction revealed two subclusters that have collided 0.1-0.2 Gyrs ago almost in the plane of the sky. The lower mass sub-clump is leaving the core of the main clump at a velocity of 4500 km/sec as measured from the sudden density jump at bow shock revealed by the X-ray data. The galaxies, being collisionless are coupled to the dark matter clumps and hence separated from gas peaks. We have done a lensing analysis of this cluster in Chapter 2. Another interesting system A520 has
Figure 1.4: A composite image of A1689 in X-ray and optical. The lensing effect is clearly visible in several arcs around the cluster core. The X-ray gas is represented by the extended blue distribution. This image is taken from http://chandra.harvard.edu/photo/2008/a1689/

a huge amount of gas well separated from the two visible dark matter peaks. It is suspected that there is a third structure which left the gas trailing behind. The only way to detect this third sub-cluster is to get lensing data on that field of view. Such clusters are also present in the MACS project Ebeling et al. (2001). For these systems apart from non-parametric lens mass reconstruction techniques there are no other complementary techniques to study the dark matter alone.
Figure 1.5: A composite image of the Bullet Cluster in X-ray and optical. Hot gas detected by Chandra in X-rays is seen as two pink clumps in the image and contains most of the baryonic matter in the two clusters. The bullet-shaped clump on the right is the hot gas from one cluster, which passed through the hot gas from the other larger cluster during the collision. An optical image from Magellan and the Hubble Space Telescope shows the galaxies in orange and white. The blue areas in this image show where astronomers find most of the mass in the clusters. Most of the matter in the clusters (blue) is clearly separate from the baryonic matter (pink), giving direct evidence that nearly all of the matter in the clusters is dark. This image is taken from http://chandra.harvard.edu/photo/2006/1e0657/

1.4.1 Cluster Observations

Clusters of galaxies are clearly visible in optical images. Galaxies belonging to a cluster are usually of similar color (see Figure 1.4). In order to identify the cluster members spectroscopic redshifts of the galaxies are preferred. Galaxies with similar redshifts within a certain radius from the cluster center are deemed as cluster members. Optical images are used for studying cluster member galaxy properties as well
as for lensing analysis. Most of the data used in this thesis is taken using the HST. The object detection for such images is done using SExtractor (Bertin & Arnouts, 1996). In order to do the lensing analysis faint optical galaxies are detected in deep images and the ellipticity of background images are calculated using second order moments of the surface brightness distribution, correcting the images for smearing due to Point Spread Function (PSF) (Kaiser et al., 1995; Clowe et al., 2006b).

Often lensing analysis requires the use of data from multiple instruments, for example in case of the “Bullet Cluster” studied in Chapter 2, the data is taken from three different instruments. The different instruments are- the ESO/MPG Wide Field Imager covering an area of $34' \times 34'$, the 6.5m Magellan Telescope covering a field of view with $8'$ radius and the HST/ACS covers a $3.5' \times 3.5'$ around both peaks (Clowe et al., 2006). This is a large dataset and spectroscopic analysis is not possible. Hence cluster members and background galaxies are separated by using magnitude and color cuts calibrated using photometric redshifts from the Hubble Deep Field (Fontana et al., 1999). The shear measurements for background galaxies were also weighted by the significance of detection.

In Chapter 3 we study the cluster Abell 901/902. This is a multi-cluster system at a redshift of $z = 0.165$. The dataset is very rich with HST coverage for $29.5' \times 29.5'$ field of view. The data were taken in 80 pointings and 90% of the data were collected in 21 days to avoid a time-varying PSF for lensing analysis. This dataset is also too large for spectroscopy. Most galaxies in this field of view were too faint for calculating photometric redshifts, hence a magnitude dependent redshift relation is used to ascertain the median redshift (Schrabback et al., 2007).

$$z_m = 0.29 \ (m_{F606} - 22) + 0.31.$$  \hspace{1cm} (1.26)

The background population is chosen such that $z_m \simeq 1.4$. The cluster galaxies were selected assuming that the photometric redshift of clusters follows a gaussian
Figure 1.6: Distribution of galaxies in the A901/902 field of view. The galaxy sample is shown in green and the cluster sample is shown in black. The y-axis represents the photometric redshifts and the x-axis represents the total R-band magnitude for each galaxy.

and the galaxy number density follows the average number density $n(z,R)$ (R-band magnitude) and varies smoothly with magnitude and redshift. This is shown in Figure 1.6. The detailed data analysis for this field of view is given in (Heymans et al., 2008; Gray et al., 2009).
In chapter 4 we analyze Abell 1689. The HST image of this cluster is the richest dataset on galaxy clusters. It has 525 spectroscopically (Czoske, 2004) identified cluster members in a $6 \times 6h^{-1}\text{Mpc}$ area around the cluster center. Spectroscopic analysis shows that this cluster has substructure along the line-of-sight, and another substructure in the north-east direction. For strong lensing analysis, HST data has been used in four bands. Multiple images have been identified from a composite of the 4 bands since they must have the same color. A statistical study of multiple images has been performed in (Richard et al., 2007). The weak lensing data is obtained form 16 HST WFPC2 pointings covering a central region of $1.8 \times 1.4h^{-1}\text{Mpc}$. This is complemented by ground based data from SUBARU that extends to a $3h^{-1}\text{Mpc}$.

### 1.4.2 Mass Modeling of Clusters

The key to convert observations into physically meaningful quantities describing a system is doing a maximum likelihood analysis between the observables and physical parameters to obtain the best fit result. In case of cluster lensing analysis, the weak lensing observables are ellipticities and strong lensing observables are angular positions of multiple images. These observables are used to constrain the convergence of the cluster. A likelihood fitting the observables with a model is given by,

\[
L = \exp \left[ -\frac{\sum_{mn} (P_n - \bar{P}_n) C^{-1}_{mn} (P_m - \bar{P}_m)}{2} \right].
\] (1.27)

Here P is the observable and C is the error matrix for the observations. If the observations are not correlated then C is a diagonal matrix with the elements given by the noise associated with the data. We will discuss both parametric and non-parametric lens modeling in the following sections.
Parametric Modeling

Weak Lensing
The simplest way to constrain properties of the dark matter halo is to fit the observed constraints to a model (some examples are discussed in § 1.3.3). The model is fitted by minimizing a $\chi^2$ of the form

$$\chi^2 = \sum_{m,i} \frac{(\varepsilon_m^i - g_m^i)^2}{\sigma_\varepsilon^2},$$

where $i = 1, 2$ are the two components of ellipticity, $\varepsilon_m^i$ is the ellipticity of the $m^{th}$ galaxy and $g_m^i$ is the reduced shear and $\sigma_\varepsilon$ is the error in the tangential ellipticity. This process will constrain parameters of this model which can be used to estimate the enclosed mass within a given radius.

Strong Lensing
Traditionally, parametric modeling for strong lensing assumes that light traces mass. Typically, galaxy sized subhalos are placed at the positions of the cluster galaxies and the overall mass distribution is described by one or more halos. This is the approach taken by the software LENSTOOL (Jullo et al., 2007). Given a lens parameterization, this software looks for best fit parameters using Monte Carlo techniques.

Nonparametric Cluster Mass Reconstruction
The main advantage in using nonparametric mass reconstruction techniques is that we do not assume that light traces dark matter and we do not use models with any particular symmetries. The convergence $\kappa$ or the potential $\psi$ are allowed to vary at data locations (in case of PBL) or bins (in case of grid-based techniques) to fit the observed constraints.
**Direct Techniques**

There is a series of mass reconstruction techniques that derive the convergence by convolving the observed measured ellipticity. From Equation 1.8 and 1.5 we get,

$$\gamma(\vec{\theta}) = \frac{1}{\pi} \int d^2\theta' D(\vec{\theta} - \vec{\theta}')\kappa(\theta'),$$

(1.29)

where

$$D(\theta) = \frac{\theta_2^2 - \theta_1^2 - 2\theta_1\theta_2}{\theta^1}.$$

Taking the Fourier Transform (hereafter FT) of this equation we get,

$$\gamma(\vec{l}) = \pi^{-1} \hat{D}(\vec{l})\hat{\kappa}(\vec{l}).$$

(1.30)

Here \(\hat{D}(\vec{l})\) is FT of \(D(\theta)\) and \(\hat{\kappa}(\vec{l})\) is the FT of \(\kappa(\theta)\). Thus we can derive \(\kappa(\theta)\) to be,

$$\kappa(\vec{l}) - \kappa_0 = \int d^2\theta' D^*(\vec{\theta} - \vec{\theta}')\gamma(\theta'),$$

(1.31)

upto an additional constant (Kaiser & Squires, 1993) due to the mass sheet degeneracy. Here it is assumed that the measured ellipticity is equivalent to the \(\gamma\) field. The mass can be derived using two different ways. The ellipticities can be binned and \(\kappa\) can be obtained by replacing the integral with a sum over all bins. The sum can also be done over all ellipticities and kappa is calculated at each image location.

This gives us a flavor of computing \(\kappa\) from shear. This approach suffers from many drawbacks. The integral in the above equation is over all space whereas real fields of views are finite and in the semi-strong region the measured ellipticity is not equal to the shear. These issues are dealt with in great details in the finite field inversion techniques in Seitz & Schneider (1996, 2001).

**Inverse Techniques**

These refer to iterative techniques where a maximum likelihood analysis is done to fit the observables (ellipticities for weak lensing and positions for strong lensing) to \(\kappa\) or the potential \(\psi\) in either bins or image locations. This technique is iterative and
a result is obtained once the likelihood converges to a maximum. PBL is an inverse technique which will be dealt with in great detail in the forthcoming chapters.

There are several nonparametric techniques which use strong lensing only for mass reconstruction. Some of them are PixeLens (Saha et al., 2006), SLAP (Diego et al., 2005) and LensPerfect (Coe et al., 2008). For a strong-lensing only mass reconstruction the metric that decides the accuracy of a mass map is the residuals in fitting the positions of the multiple images. LensPerfect has been used to fit the strong lensing data with minimum residuals (Coe et al., 2010).

Strong+Weak Lensing techniques are generally more complex. Weak lensing is dominated by noise due to intrinsic ellipticities of the galaxies, hence the resolution of the reconstructed mass is lower. When this information is optimized by adding strong lensing simultaneously the expected residuals in fitting the strong lensing positions is higher.

1.4.3 Parametric vs Non-parametric techniques

The advantage of parametric techniques is that the models are physically motivated with very few parameters. Hence fitting to these models gives us a meaningful measure of the dark matter distribution. However, as seen from simulations and observations, galaxy clusters are not circular, as we have pointed out in several examples of highly disturbed clusters in § 1.4. Modeling these clusters as circular or elliptical systems does not give insight into the properties of the dark matter substructure. As a matter of fact, nonparametric mass reconstruction techniques are the only way to map the dark matter distribution of these clusters.

It is important to improve the existing non-parametric techniques. The following chapters in the thesis deal with issues and improvements of non-parametric techniques.
References


Okura, Y., Umetsu, K., & Futamase, T. 2007, ArXiv e-prints, 710


Abstract

We present Particle-Based Lensing (PBL), a new technique for gravitational lensing mass reconstructions of galaxy clusters. Traditionally, most methods have employed either a finite inversion or gridding to turn observational lensed galaxy ellipticities into an estimate of the surface mass density of a galaxy cluster. We approach the problem from a different perspective, motivated by the success of multi-scale analysis in smoothed particle hydrodynamics. In PBL, we treat each of the lensed galaxies as a particle and then reconstruct the potential by smoothing over a local kernel with variable smoothing scale. In this way, we can tune a reconstruction to produce constant signal-noise throughout, and maximally exploit regions of high information density.

PBL is designed to include all lensing observables, including multiple image positions and fluxes from strong lensing, as well as weak lensing signals including shear and flexion. In this chapter, however, we describe a shear-only reconstruction, and
apply the method to several test cases, including simulated lensing clusters, as well as the well-studied “Bullet Cluster” (1E0657-56). In the former cases, we show that PBL is better able to identify cusps and substructures than are grid-based reconstructions, and in the latter case, we show that PBL is able to identify substructure in the Bullet Cluster without even exploiting strong lensing measurements. We also make our codes publicly available.

2.1 Introduction

In the previous chapter we have set the stage for probing cluster masses using gravitational lensing. The primary goal of this chapter is to derive the basic principles of ”Particle Based Lensing” (PBL) and detail the advantages of using PBL for mass reconstructions.

Our outline for this chapter is as follows. In § 2.2 we describe current (grid-based) techniques for reconstructing galaxy clusters, and identify some strengths and complications. In § 2.3 we propose Particle Based Lensing. We then apply this new method to simple simulated clusters of single and double peak softened isothermal spheres and the “bullet cluster” (1E0657-56) in § 2.4.

2.2 Grid-Based Cluster Lensing

Mass reconstruction studies have been very successful on cluster scales (Bartelmann & Schneider (2001); Clowe & Schneider (2002); Hoekstra et al. (2002); Broadhurst et al. (2005a); Okura et al. (2007) and references therein). Because these systems typically contain many lensed images, the shear signal can be extracted with high significance. In this section, we will describe an important class of cluster inversion
techniques which reproduce the convergence field on a grid. Before doing so, we make a couple of important caveats.

First, this discussion is not intended to be exhaustive. For example, there are a number of techniques which utilize finite field inversion (Bartelmann & Schneider, 2001; Seitz & Schneider, 1998), which we don’t discuss directly in this chapter. Our main reason for this omission will be seen clearly in the future work section. That is, we want to set up a foundation for including many different sources of lensing information and finite inversion techniques do not provide a straightforward way of doing so. Our specific choices of grid-based techniques include those which have already been extended to include strong-lensing information with non-parametric models and thus provide a fertile basis for comparison. We focus on non-parametric reconstructions for the simple reason that we do not want to miss dark substructure. Reconstructions which assume that mass traces light (even in a highly biased way), will be unable to identify structure hierarchy besides the cluster galaxies.

Further, there are many variants even within the sub-category of grid-based reconstruction techniques. We focus primarily on their commonalities, as exemplified by those discussed in Bradač et al. (2005a) and Cacciato et al. (2006). We discuss non-parametric mass reconstructions, focusing on those in which various scalar fields \( \{\psi, \kappa\} \) are defined on a Cartesian grid, and minimized according to the criteria described below. In so doing, we note some interesting exceptions: Diego et al. (2005) and Saha et al. (2001), who describe an adaptive mesh technique for refining the field on different resolution scales and Marshall et al. (2002); Marshall (2006) who use a variable smoothing scale for their weak lensing mass reconstruction.

Finally, no single-plane lensing inversion can avoid the mass-sheet degeneracy (Falco et al., 1985; Schneider & Seitz, 1995). However, since our primary goal is the development of a technique to estimate substructure this limitation is not too
grievous.

### 2.2.1 Weak Lensing on Grids

The standard approach to lensing arclet inversion (Luppino et al., 1999) has been to measure the ellipticity of observed images as an unbiased estimator of the reduced shear:

\[
\langle \varepsilon \rangle = g \equiv \frac{\gamma}{1 - \kappa}.
\]  

(2.1)

For relatively weak fields \((\kappa \ll 1)\), this is very nearly a direct estimate of the shear, and can perform a direct finite inversion to estimate the density field.

In recent years, there has been a flurry of work on optimal methods for non-parametric cluster mass reconstructions (Bradač et al., 2005a; Natarajan & Springel, 2004). In general, these papers tend to focus on estimating the potential \(\{\psi\}\) or convergence \(\{\kappa\}\) fields of a cluster by a \(\chi^2\) minimization analysis. Both the shear and the convergence are linear functions of the potential field. Thus, if a model potential field, \(\{\psi\}\), is defined on a grid, then the shear at some grid-cell, \(i\), may be expressed as a linear combination of potential:

\[
\gamma_1i = G_{ij}^{(1)} \psi_j
\]  

(2.2)

with a similar expression for the convergence, \(\kappa\), and the imaginary component of the shear, \(\gamma_2\). We refer to these below simply as \(\gamma_i(\{\psi\})\), since we wish to remind the reader that the estimate of the shear is an explicit function of the test potential field. Because these fields are combinations of second derivatives of the potential field, the \(G^{(1)}\) matrix and the others are easy to compute using finite differencing, and are extremely local. A very good graphical representation of the finite difference operators can be found in Bradač et al. (2005).

In the weak field limit, the complex ellipticity of a lensed galaxy is a linear, albeit noisy, estimator of the complex shear field. The principle component of noise is the
intrinsic ellipticities of galaxies which follow a Gaussian distribution with standard deviation of \( \sigma_\varepsilon \simeq 0.3 \) for each component.

Because the intrinsic variance in ellipticities is so large, it is necessary to average many images together so that the intrinsic noise in each grid cell is averaged to zero or apply an artificial smoothing scale to a more finely gridded mesh. For a weak-lensing only calculation, a \( \chi^2 \) minimization is performed on:

\[
\chi^2_W = \sum_i \left( \frac{\gamma_i(\psi)}{1-\kappa_i\{\\psi\}^{(n-1)}} - \varepsilon_i \right)^2 \sigma^2_i
\]

(2.3)

where the estimate of \( \kappa \) is taken from the previous iteration of the potential field, and thus, the model rapidly converges to a maximum likelihood solution to the potential field.

### 2.2.2 Strong Lensing

When the cluster is massive enough (\( \kappa \geq 1 \)) and the angular position of the source is close to the lens, light takes different paths to reach the observer. This produces multiple images which maybe highly distorted if they form close to the critical curves.

A number of researchers, including Bradać et al. (2005a) have noted that a similar grid-based formalism may be used with strong lensing signals. Strong+weak (S+W) reconstructions use both shear fields and the positions of multiply imaged sources can be used to accurately reconstruct both the cores and halos of clusters.

While our current PBL implementation, described in the next section, does not currently incorporate strong lensing analysis, inclusion of S+W signals has proven especially fertile, and we thus introduce this component of grid-based lensing reconstructions to illustrate how directly a strong-lensing analysis could be incorporated into PBL.
Strong lensing by clusters produces an especially elegant result because if, say, two images are observed at positions, $\theta^A$ and $\theta^B$, then it must be true that both images originated at the same (unknown) position in the sky. Thus, we have a simple relation:

$$\theta^A - \alpha(\theta^A) = \theta^B - \alpha(\theta^B)$$

(2.4)

The appeal of this relationship is that it is fundamentally linear and thus the angular separation between the two images (itself, a measurable quantity), can be directly related to the difference in the first derivatives of the potential at two different points in the field.

As above, the local derivatives can be computed as:

$$\alpha_{xi} = A_{ij}^{(e)} \psi_j$$

with a similar expression for the y component of the displacement. The matrix elements of $A$ are easy to compute as they are simply the 1st derivative in a simple grid-based 2nd order difference scheme. More generally we can express this as $\alpha_i(\{\psi\})$.

Thus, an additional $\chi^2$ term can be added:

$$\chi^2_S = \sum_{i,\text{pairs}} \frac{((\alpha^A(\{\psi\}) - \alpha^B(\{\psi\})) - (\theta^A - \theta^B))^2}{\sigma_i^2}$$

(2.5)

and minimized either independently, or simultaneously with the weak lensing component.

### 2.2.3 Regularization

Several issues may complicate grid-based mass-reconstructions. For example, because the real and imaginary components of the reduced shear have largely independent noise, a convergence field with a checker-board pattern can easily be produced.
Moreover, because the reconstruction is subject to the mass-sheet degeneracy, solutions with arbitrary mass-sheets may result. Finally, researchers will often wish to generate a reconstruction which yields hierarchical structure. Indeed, that is the main focus of the current study. As a result, a reconstruction is often done on one scale and then used as a prior in subsequent scales in order to regularize their result.

This is especially true, but not limited to, cases in which strong+weak lensing signals are combined. To make this argument concrete consider a toy isothermal sphere model of a cluster with a 1-d velocity dispersion of 600 km/s. Each multiply imaged pair will be separated by twice the Einstein radius, about 20 arc-seconds in this case. This represents the minimum necessary resolution in the reconstruction to say anything about strong lensing.

On the other hand, even very efficient space-based weak lensing analysis of clusters seldom yield more than approximately 100 images/square arc-minute. Using a simple Poisson noise estimate, we may achieve uncertainty of $\sigma_\gamma = 0.06$ only with images binned on scales larger than 30 arc-seconds on a side. Smaller binnings will naturally yield larger uncertainties.

Simple grid based method cannot capture both the weak-lensing signal to high accuracy as well as resolve the strong lensing regime. In order to deal with this issue, different investigators have used different regularization techniques.

One method is to use a series of finer and finer griddings, and at each successive level of refinement the convergence field from the previous level is matched as closely as possible. The Bradač et al. (2005) S+W technique uses this method, with the weighting parameter selected to provide a $\chi^2$ per degree of freedom equal to 1, such that:

$$R = \eta \sum_i (\kappa_i^{(n)} - \kappa_i^{(n-1)})^2.$$  \hfill (2.6)

Where $\kappa_i^{(n-1)}$ represents the estimated convergence on the previous, coarser, gridding,
and where $\kappa_i^{(0)} = 0$. We use this form explicitly in §2.4 where we test the PBL method and contrast it to grid-based reconstruction methods.

Seitz et al. (1998) have discussed that regularizations of the form $R = \sum_{x,y} |\nabla \kappa|^2$ tend to flatten the mass profile and suggested a moving prior maximum entropy regularization of the form $R = \eta \sum_{x,y} \hat{\kappa}_{ij} \ln \left( \frac{\hat{\kappa}_{ij}}{b_{ij}} \right)$. All these regularizations add a quadratic term to the $\chi^2$ which smooths the small scale noise peaks in weak lensing and resolves strong lensing structure.

### 2.2.4 Some Questions

Grid-based reconstructions have produced some excellent measurements. However, there remain a number of complications. First, grid-based techniques are really optimized to measure a single scale, the grid-spacing. However, as we discuss above, in many interesting systems, both the structure and information are hierarchical. An optimal technique should provide higher resolution in regions of greater information content.

Moreover, the smoothing and weighting of the strong lensing, weak lensing, and regularization are created in an ad hoc basis. The ideal smoothing scale should be variable, and such that the signal/noise ratio of the reconstructed field is similar in every smoothed cell.

Third, the information from the image ellipticity can only be inverted outside the critical curves of the lenses. Inside the (tangential) critical curve (Schneider et al., 1992; Petters et al., 2001; Schneider & Weiss, 1992; Hoekstra et al., 2004) there is an abrupt switch in parity of the induced ellipticity of an image. More plainly, in the regime $|\gamma| > |1 - \kappa|$, the ellipticity is related to the shear via:

$$\langle \epsilon \rangle_{\text{strong}} = \frac{1}{g^*} \quad (2.7)$$
As discussed in §2.3.4, this produces a discontinuity in the ellipticity as a function of $\kappa$ and $\gamma$. No simple linear minimization scheme, even an iterative one, will converge to the “strong lens” solution if one starts with a “weak lens” initial guess for the local potential field.

### 2.3 Particle Based Lensing – PBL

In this section, we introduce a new technique called Particle Based Lensing (PBL) which has the ability to combine the disparate lensing scales in a coherent way without requiring a regularization scheme. Several of the concerns discussed in the previous sections have to do with the method of discretizing the data for the reconstruction of the lens potential. In order to address this, we turn to a technique which is widely used in another area of astrophysics in which information must be analyzed on a wide range of physical scales – numerical N-body simulations. Smoothed Particle Hydrodynamics (SPH; see, e.g. Monaghan (2005), for a recent review) is used in the modeling of a wide range of physical systems including planets (Woolfson, 2007), star formation (Springel & Hernquist, 2003; Nagamine et al., 2004) and galaxy Formation (Kaufmann et al., 2007). The mathematical details of PBL can be complicated, hence we have made our codes for the method public\(^1\) through our website.

Before getting into the details, however, it is important to emphasize what PBL is and is not. PBL is a new way of discretizing and describing a reconstructed field. Moreover, it includes a metric for comparing a reconstructed model to the observed data. Everything we describe below is aimed at demonstrating why this model and metric are ideal for lensing systems with uneven information content. While the current code, and the worked examples are based on weak-lensing data only, PBL is

\(^1\)http://www.physics.drexel.edu/~deb/PBL.htm
based on the idea that other probes of the potential field: strong-lensing positions, flux ratios, and flexion, can be added to the metric with little complication.

PBL is not, however, a minimization scheme. That is, much like grid-based reconstruction methods, PBL fundamentally consists of a list of dimensionless potentials and a metric to describe the goodness of fit. It does not describe how that minimization criterion is to be met, however. In our model section, we describe a number of approaches to efficient model convergence. The major argument in favor of PBL, however, is not that $\chi^2$ minimizes efficiently, but rather that a low $\chi^2$ in PBL actually corresponds to a model which closely matches the true underlying system.

### 2.3.1 A Particle Description of the fields

The fundamental description of the PBL field lies in the a list of potentials, $\{\psi\}$, one each at the positions of each observed lensed image. In order to make the field as continuous as possible, we may expand the local potential field around the position of any lensed image, ($\psi_n$, in this case) to arbitrary order:

$$ \psi(\theta) = \psi_n + \theta_j \psi_{n,j} + \frac{1}{2} \theta_j \theta_k \psi_{n,jk} + ... $$

(2.8)

where $\theta$ is the relative offset of the test-point from galaxy $n$.

As with grid-based lensing, the local derivatives are composed of a linear combination of the potentials at each grid point. That is:

$$ \psi_{n,j} = D^{(j)}_{nm} \psi_m $$

(2.9)

$$ \psi_{n,jk} = D^{(jk)}_{nm} \psi_m $$

(2.10)

and so on for arbitrarily higher derivatives. In reality, we typically extend the $D^{(\nu)}$ matrices up to 3rd order, where $\nu$ corresponds to 2 matrices for 1st derivatives (displacement field), 3 for second derivatives (shear and convergence), 4 for 3rd derivatives
(flexion). Here we use Einstein summation convention, the sum over $m$ runs from 1 to $N_g$.

In terms of the $D^{(\nu)}$ matrices, equation (2.8) may be rewritten as:

$$\psi(\theta_m) = \psi_n + \sum_\nu D^{(\nu)}_{nl} X^{(\nu)}_{nm} \psi_l$$  \hspace{1cm} (2.11)

where we are explicitly estimating the potential at the $m$-th galaxy from the local derivatives defined at the $n$-th. We also compactify equation (2.8) by defining:

$$X^{(1)}_{nm} = \theta_{nx} - \theta_{mx}$$  \hspace{1cm} (2.12)
$$X^{(2)}_{nm} = \theta_{ny} - \theta_{my}$$  \hspace{1cm} (2.13)
$$X^{(3)}_{nm} = \frac{1}{2} (\theta_{nx} - \theta_{mx})^2$$  \hspace{1cm} (2.14)

and so on.

In order to estimate the derivatives of the potential field near each galaxy, we need to first compute the $D^{(\nu)}$ matrices. Since this problem is under-constrained, we solve for these matrices via a $\chi^2$ minimization:

$$\chi^2 = \sum_m \left( \psi_m - \psi_n - \sum_\nu D^{(\nu)}_{nl} X^{(\nu)}_{nm} \psi_l \right)^2 w_{nm}$$  \hspace{1cm} (2.16)

where $w_{nm}$ is a window function, guaranteeing that only neighboring galaxies effect the potentials of one another. We use a window function of the form:

$$w_{nm} = w \left( \frac{|\theta_n - \theta_m|}{h_n} \right)$$  \hspace{1cm} (2.17)

where $h_n$ is inversely proportional to the signal-noise at the $n$-th image positions. The smoothing scale can also be chosen to be of the form $h_{nm}$, i.e symmetric between the points $n$ and $m$.

The signal-noise is a function of the local density of background images and type of constraint (e.g. ellipticity, positions of multiple images, etc). A similar approach
of using signal to noise dependent smoothing scale has been used in image analysis of X-ray data Ebeling et al. (2006). In regions where there is a high density of information, the smoothing scale $h_n$ may be set much lower than in regions of low information density.

This function must be minimized for every matrix element such that:

$$\frac{\partial \chi^2}{\partial D_{nl}^{(\nu)}} = 2\psi_l \sum_m X_{nm}^{(\nu)} w_{nm} \left( \psi_m - \psi_n - \sum_{\mu,p} D_{np}^{(\mu)} X_{nm}^{(\mu)} \psi_p \right) = 0 \quad (2.18)$$

Solving this equation we get,

$$\sum_{\mu} X_{nm}^{(\nu)} X_{nm}^{(\mu)} w_{nm} D_{nm}^{(\mu)} = X_{nm}^{(\nu)} w_{nm} \quad (2.19)$$

for all $n, m$ and $\nu$. This can be solved with a simple matrix inversion, yielding the desired elements for $D^{(\nu)}$. Of course, since the elements are a function only of the positions and weightings of the galaxy images, these elements need only be computed once. The method potentially incorporates higher-order derivatives of the potential, thus, combination of strong, weak and flexion information becomes a relatively straightforward minimization problem.

### 2.3.2 PBL vs. Regularization

One of the major advantages of PBL is that we no longer need to introduce an explicit regularization in order to resolve multi-scale structure in a reconstruction. The various regularization schemes discussed in § 2.2.3 are not motivated from the associated observations, but are rather derived from assumptions about the mass profile of a cluster motivated by theory and simulations.

However, one of the motivations behind using gravitational lensing is to be able to measure the projected mass without making any assumptions about the physical state of the system. The advantage of using PBL is that we do not need to make any
assumptions that go into choosing the regularization term. The smoothing scale of a “pebble” is controlled by $h_n$ which is determined by the local signal to noise. This means that the position representing weak lensing measurement will have a low signal to noise and correspondingly a high $h_n$. This is similar to the typical weak lensing measurement which is done by averaging over a bin size larger than $\sim 30''$. In case of strong lensing we know the positions of the multiple images for certain, implying high signal to noise and correspondingly low $h_n$. This can be a few arc-seconds which is the scale at which the strong lensing structure can be resolved from multiple images. Thus scales of a few arc-seconds can be combined with scales greater than $\sim 30''$ without making any assumptions about the mass profile, rather by taking input from the data.

### 2.3.3 Estimation of the Potential Field

As with grid-based lensing analysis, in PBL, we use a $\chi^2$ minimization to estimate a maximum-likelihood potential field. In this case, however, we sample the potential at every point, and use the local derivatives of the potentials as defined in equation (2.10) to minimize:

$$
\chi^2 = \sum_{i, n} \left[ \frac{\gamma_n(i) \{\psi\} - \varepsilon_n^{(i)}}{1 - \kappa_n(\{\psi\})} \right]^2 \frac{1}{\sigma_n^2} \quad (2.20)
$$

where $i$ ranges from 1,2, and indicates the real or imaginary component of the shear, reduced shear, or ellipticity. We shall henceforth refer to the first term in the parentheses as $g_n^{(i)}(\{\psi\})$, the estimate of the reduced shear of a model, and the weighting term outside the parentheses as $w_n$, yielding:

$$
\chi^2 = \sum_{i,n} \left[ g_n^{(i)}(\{\psi\}) - \varepsilon_n^{(i)} \right]^2 w_n \quad (2.21)
$$

which is the form we will refer to from now on.
This is a weak lensing only expression. Replacing $g_n^i(\{\psi\})$ with $1/g_n^{i*}(\{\psi\})$ gives the strong lensing counterpart of Eq. 2.21. In the next section we discuss how we include this strong lensing version of the equation.

2.3.4 Interpolated Ellipticities

Linear inversion techniques require that the function to be minimized is smoothly varying over the domain of interest. The ellipticities are given by two functions in the weak and strong lensing regimes by Eqs. (2.1,2.7). The boundary of the two regimes define the critical curves where $|g| = 1$ making ellipticities continuous but not differentiable.

The transition between the two regimes can be facilitated if the sources are distributed in redshift, but minimization functions will be much easier if we allow a smoothing of the discontinuities. This is a two step process, first we need to write Eq. (2.1,2.7) in terms of a step function,

$$\tilde{\epsilon} \simeq [1 - \mathcal{H}(g)] g + \frac{\mathcal{H}(g)}{g^*}$$

(2.22)

where the function $\mathcal{H}(g)$ is a step function at $g = 1$. We may replace the step function by an approximate smooth function. We define:

$$u = \eta_0 \left( g^2 - \frac{1}{g^2} \right)$$

(2.23)

Here $\eta_0$ is the free parameter that controls the accuracy of the step function. A higher value of $\eta_0$ makes the step function more accurate. The step function is approximated as (Fig. 2.2),

$$\mathcal{H}(u) = \frac{1}{1 + e^{-2u}}$$

(2.24)

This approximation replaces the ellipticities only in the neighborhood of the critical curves (discontinuity) by a continuously differentiable function. The problem can
now be solved by standard minimization techniques. The interpolated ellipticity function is shown by a dotted line in the second panel of Fig. 2.2, showing the derivative discontinuity explicitly.

2.3.5 $\chi^2$ Minimization

When we first introduced PBL above, we remarked that it was primarily a way of describing a lens reconstruction in such a way that a small $\chi^2$ would necessarily correspond to a good representation of the underlying field. In practical terms, though, for a reconstruction code to be useful, we need to describe a means of minimizing (or nearly minimizing) the $\chi^2$. Below, we describe our pipeline for fast convergence of a maximum likelihood solution.

While PBL is a non-parametric reconstruction scheme, it has the useful property that we may start a minimization with any assumed model we like. However, no extra weight is given to our a priori assumptions. At the end of a minimization we may simply use the standard techniques to estimate the likelihood of a particular value of $\chi^2$.

That said, even with the caveat above regarding smoothing of critical curves, it is very difficult to smoothly vary a solution such that strongly lensed regions are produced. As pointed out by Bradač et al. (2006) a $\chi^2$ minimization process does not ensure reaching a global minimum.

To that end, our initial configuration of $\{\psi\}$ is generated by laying down a small number of Singular Isothermal Spheres (SIS’s). Since there are a low number of parameters (3 for each model sphere), a global minimum may be reached through a combination of trial and error, simulated annealing, or even (for small numbers of spheres), finite sampling. Indeed, one may even use an interpolation of a reconstruction recommended by a grid-based solution. For systems with strong lenses, one
may apply the reconstructed field generated by “LensPerfect” (Coe et al., 2008), for example as a starting point.

We hasten to remind the reader that while this technique will produce the optimum parametric fit, it will not, in general, produce the overall best fit. As a result, further iteration is required.

We have found that by starting with an initial model with well-identified strong-lensing regions, convergence to $\chi^2/DOF \simeq 1$ may be achieved relatively quickly, even if the strong lensing regions are only approximate. For the current implementation of our code, we use Newton’s method to reach a local minimum. We have found satisfactory, fast, convergence for several thousand background sources.

## 2.4 Test Applications

In this section, we apply PBL to three systems as a proof of concept. In the first, we model a Softened Isothermal Sphere, and examine the relative abilities of PBL and grid-based inversion to reconstruct the a relatively peaked core. In the second, we model a superposition of two softened isothermal spheres at a given separation as a simple model of a system with substructure. Finally, we reconstruct the “Bullet Cluster” (1E0657-56) (Markevitch et al., 2002, 2004; Clowe et al., 2004; Bradač et al., 2006; Clowe et al., 2006), an observed multi-peak system of considerable interest. We show that using weak lensing alone, we are able to reconstruct both Dark Matter peaks.
2.4.1 Simulation: Softened Isothermal Sphere

Model

We begin by generating a softened isothermal sphere with a potential:

\[ \psi = \theta_E \sqrt{\theta^2 + \theta_c^2}, \]  

(2.25)

and convergence:

\[ \kappa = \theta_E \frac{(\theta^2 + 2\theta_c^2)}{(\theta^2 + \theta_c^2)^{3/2}}. \]  

(2.26)

The data is simulated on a unit square field of view. For simplicity we have assumed all sources to be at \( z = \infty \), with \( \theta_E = 0.2 \) and \( \theta_c = 0.08 \). We lens 607 background galaxies, and apply an intrinsic ellipticity (noise) with \( \sigma_{\epsilon_s} = 0.1 \) in each of the principle directions.

PBL and Grid-Based Reconstructions

For the single peak and the double peak simulation (see below), we perform both a grid-based reconstruction as well as PBL. We use the regularization suggested by Bradač et al. (2005), and described in detail in § 2.2.3 for the grid based method. In case of the single peak the reconstruction is initially performed on a coarse grid \( (n_x = 6 \text{ gridcells}) \), and is refined up to \( n_x = 24 \), using the \( \kappa \) estimated at each previous step as the prior. For the double peak system we start with \( n_x = 10 \) and refine up to \( n_x=40 \). For both systems the final reconstruction contains less than one particle per grid cell.

For the PBL reconstruction, we use a smoothing scale of the form:

\[ h_n = \frac{c}{(\rho_n)^{\xi}} \]  

(2.27)

where \( \rho_n \) is the local number density of points, \( c \) is a constant, and \( \xi \) is a tunable parameter to maximize signal-to-noise. For our simulation \( \xi = 1 \) is an optimal choice.
and for the observational case we have used $\xi = 0.5$, which is the obvious choice for equalizing signal to noise for every smoothing length. We select $c$ such that the integrated S-N is greater than unity. The PBL reconstructions are gridded to the same resolution as the grid-based reconstruction to aid visualization.

For both reconstructions, we begin our iterations with a best-fit SIS. We do not, however, use this in the regularization for the grid-based reconstruction.

**Results**

In Table 2.1 we compare the $\chi^2$ for the best fits of both the grid-based reconstruction along with PBL for a variety of smoothing normalization parameters, $c$. In each case, the ostensible $\chi^2$/DOF is of order unity. However, one needs to be careful with simply asserting that the lower $\chi^2$ produces the best result, since the regularization in grid-based reconstruction adds a penalty function, and the smoothing scale in PBL lowers the effective degrees of freedom.

So while both models produce small values of $\chi^2$, the real question is whether these good fits correspond to an accurate reconstruction of the underlying density field. In Table 2.1, we do several comparisons which relate the reconstructed $\kappa$ at each galaxy (or grid-point) with the true $\kappa$ modeled by the simulation. The comparisons are done with a range of values for both $\eta$ (the regularization weight in grid based method) and $c$ (the proportionality constant in PBL).

In the 2nd, 3rd, and 4th data-column, we weight the results uniformly by galaxy, by local density within gridcells, and uniformly by gridcells. In each of the 3 comparisons, PBL reproduces the original reconstruction with the highest fidelity.

In Fig. 2.3, we show the radial reconstruction of the softened isothermal sphere using the two different techniques. The bulk of the penalty associated with the grid-based reconstruction relative to PBL occurs near the core. By construction, PBL is
2.4.2 Simulation: A Double Peaked Cluster

Model

While PBL has been shown to perform well modeling a single Softened Isothermal Sphere in the previous section, the other major goal of this method is to reconstruct small-scale substructure in a system. To that end, we model a doubly-peaked system with 814 lensed background galaxies. As before, they are placed on a unity grid,
and are modeled as 2 Softened Isothermal Spheres, with: \( x_1 = 0.65, y_1 = 0.35, \)  
\( x_2 = 0.35, y_2 = 0.65, \theta_{E1} = 0.2, \theta_{c1} = 0.1, \theta_{E2} = 0.2, \) and \( \theta_{c2} = 0.1. \) The simulated noise, and reconstruction technique for the double peaked system are identical to the single peak system.

**Results**

As with a single sphere, both PBL and grid-based reconstructions produce \( \chi^2/\text{DOF} \approx 1, \) as illustrated in Table 2.1. However, as with the single sphere reconstruction, PBL produces smaller errors with regards to the underlying model than does the grid-based reconstruction.

In Fig. 2.4, we show a grey-scale plot of the residuals between the underlying model and each of the reconstructions. Unsurprisingly, both models have the greatest difficulty reproducing highly peaked cores, though PBL is more responsive to high local gradients in \( \kappa. \) We describe the general quality of the fit in Table 2.1.

**2.4.3 Observation: The Bullet Cluster**

**Observations**

Finally we perform a mass reconstruction of the bullet cluster (1E0657-56). This galaxy cluster is a rare supersonic merger in the plane of the sky. Its distinctive structure and orientation makes it an ideal cluster for observing dark matter using gravitational lensing. It consists of two sub-clusters separated by 0.72 Mpc, which have just undergone a merger and are moving away from each other. The western sub-cluster is less massive and the eastern main cluster is more massive. The line-of-sight velocity difference suggest that their cores passed each other 100 Myr ago. The collisionless dark matter in each of the sub-clusters have crossed each other but the fluid-like intracluster plasma is in the process of electromagnetic and thermal...
interaction producing high X-ray luminosity far removed from lensing mass peaks (Clowe et al., 2006; Bradač et al., 2006).

For the Bullet Cluster, we perform a PBL reconstruction only, since it has been well-studied with grid-based methods (using Schneider (1995); Kaiser (1995)) and the $\kappa$-contours are publicly available. We use publicly available weak lensing data from the Bullet Cluster Project Page\(^2\). The catalog was constructed using data from three different instruments: the ESO/MPG Wide Field Imager, IMACS on Magellan, and two pointings of ACS on HST. The shapes of the galaxies were measured independently on each of the image sets averaging for the common galaxies. The weighting for each galaxy is based on its significance of detection in every image set and normalized appropriately (Clowe et al., 2006).

The catalogs were combined using weighted average reduced shear measurements and the weights of individual galaxies were increased when they occurred in several catalogs. This weighting is listed in the shear catalog. We include this weighting in our reconstructions as well and choose only those images with a weighting greater than 1. As we have already illustrated in the simulations PBL is most effective when the information density is variable, i.e close to the core of the clusters. In case of the bullet cluster we zoom into a region bounded by $104.53^\circ$ to $104.69^\circ$ in right ascension and $55.92^\circ$ to $55.97^\circ$ in declination. Following this cut, our sample includes 1259 weak lensing background galaxies.

Reconstruction

While the Bullet Cluster was made famous through the Strong+Weak reconstruction of Bradac and collaborators, it remains interesting even when applying a weak-only lensing analysis. Indeed, since one of the major findings of this group is that the dark matter appears offset from X-ray emissions, we do not include any prior model

\(^2\)http://flamingos.astro.ufl.edu/1e0657/public.html
when reconstructing the system, but are able to achieve fast convergence with two clearly visible peaks. This reconstruction guides us in choosing an initial condition for subsequent $\chi^2$ minimization. We have calculated the integrated mass within 250 kpc of each peak. The main peak has a mass of $2.7 \times 10^{14} M_\odot$ and the subcluster has a mass of $1.7 \times 10^{14} M_\odot$. Bradač et al. (2006) report a value of $(2.8 \pm 0.2) \times 10^{14} M_\odot$ for the main peak and $(2.3 \pm 0.2) \times 10^{14} M_\odot$ for the subcluster within 250 kpc of the each peak.

In Fig. 2.5, we show our PBL reconstruction of the bullet cluster. Note that, despite using weak lensing signals only, we are able to identify both density peaks and using initial conditions we are able to get $\kappa > 1$ for the main peak. We also do a comparison of the publicly available $\kappa$-contour with the $\kappa$-contours reconstructed using PBL. The location of the main peak coincides for both reconstruction. The subcluster contours for PBL are slightly removed from the publicly available $\kappa$-contours.

It is important to examine the errors associated with the PBL reconstruction of the bullet cluster. Table 2.1 summarizes the typical errors associated with PBL reconstruction of a double peak system. However, the peaks in our simulation are circularly symmetric smooth analytical models and we have circularly symmetric initial condition which aids the minimization process for both the grid based method and PBL. As evident from Fig. 2.5 the peaks in the bullet cluster are not circularly symmetric and suggests presence of substructure. Thus Table 2.1 can be used to put a lower bound on the errors. In reality the errors would be higher.

### 2.4.4 Summary

We have developed PBL, a new particle based technique of mass reconstruction of clusters. The distinguishing feature of PBL is its ability to adjust its smoothing scale depending on the local signal to noise or the type of constraint and thus not require
any regularization. PBL has the scope of calculating derivatives upto any order. Hence, lensing constraints that are a function of the derivatives of the potential can be easily included in the reconstruction. In this chapter we have successfully applied PBL to do weak lensing only mass reconstruction for a single peak and a double peak system. We have made the codes for PBL publicly available for application weak lensing measurements through our website (see § 2.3).

As already explained PBL is a method of discretizing data and not a minimization method. A $\chi^2$ minimization does not necessarily ensure reaching a global minimum. In many cases the global minimum is guarded by steep walls surrounded by shallow valleys. Without any prior knowledge of the mass distribution it is very easy to get trapped in a shallow valley and not reach the global minimum. We have started with an initial condition and interpolated the ellipticity function to aid us in this regard.

In future work we will be including the additional constraints, like the flux ratios, ellipticity differences and flexion along with measured ellipticities and strong lensing positions. We will also be exploring different minimization schemes to facilitate convergence to a global minimum.
Figure 2.1: A comparison between PBL and grid based mass reconstruction technique.
Figure 2.2: In the left panel, we plot the interpolated Heaviside step function. It is clear from the plot that the function is only approximated by a smooth function near $g=1$, for all other $g$ it behaves like an ordinary step function. Also higher value of the parameter $\eta_0$ increases the accuracy. In the right panel, we plot the resulting ellipticity as a function of reduced shear for the combination, $|\gamma| = \kappa$. 
Figure 2.3: A radial plot of the reconstructed convergence ($\kappa$) of a simulated Softened Isothermal Sphere. The circles represent binned reconstructed $\kappa$ and the error bars represent the scatter in each bin. The dots represent the true value of $\kappa$ given by Eq. 2.25. Left Panel: Using PBL. Right panel: Using grid based method. The error bars in the radial plot using PBL is higher. This is because the errors introduced in PBL are dependent on the local signal to noise which are not spherically symmetric. In the grid based method the errors are averaged uniformly on the length scale of a single grid which makes the radial scatter very low.
Figure 2.4: The plot of the difference between reconstructed convergence, $\kappa$ and true $\kappa$ for the double peak SIS system. Left Panel: Using PBL. Right Panel: Using grid based method described in §2.2. Both maps are gridded for easy visualization. Also there are empty grid cells with no image galaxies. The value for those grid cells in the above difference map is set to zero for both reconstructions. As we can see the error in the cores of the peaks is much using PBL mass reconstruction.

Figure 2.5: A weak-lensing only reconstruction of the bullet cluster using PBL described in §2.3. Note that both substructure peaks are clearly identified. Left Panel: This the $\kappa$-map using PBL. The cross denotes the centroid of the multiply imaged positions. Right Panel: This a comparison of the $\kappa$ contour derived using PBL(solid) and the publicly available contour plot of $\kappa$(dashed).
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Chapter 3

Covariance analysis of Weak lensing mass reconstructions:
Measuring dark matter ellipticity.

Abstract

We present a non-parametric measure of the ellipticity and the alignment of the dark matter halos in Abell 901/902 supercluster. This super-cluster is a system of four separate peaks in a $0.5^\circ \times 0.5^\circ$ field of view. We map the mass distribution of each individual peak using an improved version of Particle Based Lensing (PBL) and measure the ellipticity of the dark matter halos associated with two of the peaks directly from the mass map and by fitting them to a singular isothermal ellipse. The parametric and non-parametric measurements are consistent for A901b while the position angle for the Southwest Group is different for the two techniques. We account for this discrepancy to substructure present in the Southwest Peak. We estimate an axis ratio of $0.37 \pm 0.1$ for A901b and $0.54^{+0.08}_{-0.09}$ for the Southwest Group.
3.1 Introduction

Weak lensing inversion techniques study the distortion pattern in background galaxies to infer properties of the mass distribution of the cluster independent of its physical/dynamical state (Clowe et al., 2006; Okabe & Umetsu, 2008). Weak lensing has been used to measure dark matter density profiles in clusters (Broadhurst et al., 2008, 2005a; Umetsu & Broadhurst, 2008; Mandelbaum et al., 2008) to test predictions of N-body simulations of the standard ΛCDM model (Spergel et al., 2007; Komatsu et al., 2009). However, weak lensing has systematic and statistical errors. We have to understand and reduce these errors in order constrain the mass, size and shape of galaxy clusters.

Gravitational Lensing mass reconstruction of clusters have been studied for two decades (Okabe et al., 2009; King & Corless, 2007; Pedersen & Dahle, 2007; Cypriano et al., 2001; Williams et al., 1999; Allen, 1998; Wu & Hammer, 1995; Tyson et al., 1990; Oguri et al., 2009; Bardeau et al., 2007; Bradač et al., 2006; Broadhurst et al., 2005a; Clowe & Schneider, 2001; Bacon et al., 2006) and many different methods have been developed in the process (Kaiser, 1995; Seitz & Schneider, 1995; Bartelmann, 1995; Schneider & Bartelmann, 1997; Seitz & Schneider, 2001). Almost all of them perform reasonably well in detecting massive peaks (primarily associated with the Brightest Cluster Galaxy) in the field of view. This is because massive peaks produce significant lensing signal which can be detected by the simplest weak lensing technique. The rapid improvement in the quality of observations has lead to extensive research on lens mass reconstruction techniques (Seitz et al., 1998; Geiger & Schneider, 1999; Marshall et al., 2002; Lombardi & Bertin, 1999; Diego et al., 2007; Bradač et al., 2005; Merten et al., 2009; Bradač et al., 2009; Mandelbaum et al., 2009).

In this chapter we have improved Particle Based Lensing (PBL) (Deb et al., 2008) by smoothing the ellipticities prior to mass reconstruction and evaluated the covari-
ance of the resulting mass distribution. PBL is a mass reconstruction technique where the mass (or potential) is calculated at the location of each image. There is a weight function associated with each image with a kernel having a width that varies according to the local number density.

We aim to measure the ellipticity of the dark matter halos of the super cluster Abell901/902 using weak gravitational lensing. Abell 901/902 is a complicated system with several sub-clusters spread around a field of view of $0.5^\circ \times 0.5^\circ (\sim 5 \times 5 \text{ Mpc}^2)$ at a redshift of $z = 0.165$. This field was surveyed to study how galaxy evolution gets affected by the density of the environment (Gray et al., 2009). The weak lensing analysis (Heymans et al. (2008), hereafter H08) of this field reconstructs the large scale dark matter distribution for this super cluster. The mass estimates of each peak were made using NFW profiles and non-parametric maximum likelihood methods. Comparison with the light profile has shown that the substructure in the dark matter peaks are closely followed by the substructure in galaxy groups.

The mass and morphology of the dark matter halos of these galaxies play an important role in their response to surrounding environment. Since this field of view has three clusters and a group of galaxies it is a very good laboratory to study large scale structure and its influence on galaxy transformation.

The chapter is organized in the following way. In 3.3, 3.4 we discuss the technical aspects of mass reconstruction and noise covariance analysis using PBL. In § 3.5 we describe measuring ellipticity of the dark matter halo using parametric and non-parametric techniques. In § 3.6 we give a brief description of the data and in § 3.7 we report the estimated ellipticity for the individual peaks of the super cluster Abell901/902. In the last section we discuss our results and possible directions of future work.
3.2 Non-parametric mass Reconstruction Techniques

Non parametric weak lensing mass reconstruction techniques are broadly divided into two categories. The convergence reconstruction techniques like KS93 (Kaiser & Squires, 1993) where the $\kappa$ is recovered using a direct convolution on the measured ellipticities and the potential reconstruction techniques (Bartelmann et al., 1996; Deb et al., 2008) where the potential is reconstructed from the observables and $\kappa$ is recovered from the potential. Traditionally the convergence techniques measure $\kappa$ directly by applying a convolution on the measured ellipticity (with the exception of (Diego et al., 2007)) whereas potential techniques do a $\chi^2$ minimization. The advantage of convergence methods is that they are very fast and useful for testing the data. The disadvantage is that these techniques reconstruct the semi-strong regimes less accurately. Hence the potential techniques gain importance as we approach the strong lensing regime. The super cluster A901/902 is subcritical with $\kappa << 1$. In this case the advantage of using PBL, a potential technique, is having an estimate of the covariance matrix. In potential techniques a likelihood function is written between the observed ellipticities and the reduced shear and maximized iteratively to obtain the best solution

$$
\mathcal{L} = \exp \left[ -\sum_{mn} \left( \epsilon_{nm}^{i} - \frac{\gamma_{nm}(\psi_{m})}{1-\kappa_{nm}(\psi_{m})} \right) C^{-1}_{mn} \left( \epsilon_{n}^{i} - \frac{\gamma_{n}(\psi_{n})}{1-\kappa_{n}(\psi_{n})} \right) \right],
$$

where $\epsilon^{(i)}_{m}$, are the observables. Using finite differencing or Particle Based Lensing (PBL) the derivatives of the potential can be written as,

$$
\psi^{(\nu)}_{n} = D^{(\nu)}_{nm} \psi_{m}
$$

where $\psi^{(\nu)}$ represents derivatives of the potential, $\nu$ is the order of the derivative. For first derivatives $\nu = (x, y)$, for second derivatives $\nu = (xx, xy$ and $yy$) and so on. The likelihood function given by Equation 3.1 is linearized and written as a function
of $D_{mn}^{(\nu)}$, the data correlation, the potential and the constraints at each step of the minimization and the potential is solved iteratively at the maximum of the likelihood.

### 3.3 Method optimization

Weak lensing is a statistical measure of small distortions in background galaxies caused by a cluster. In order to extract a lensing signal from the weak distortion of image galaxies several of them have to be averaged to smooth out the intrinsic noise. Smoothing is an integral part of any problem with noisy data and low signal-to-noise ratio. The error budget of weak lensing is controlled by the scale at which the data is smoothed. Optimizing this scale is necessary independent of the technique that is used. In this section we will introduce smoothing, describe the inversion technique used to create mass maps from measured ellipticities and lay down a foundation for the choice of the optimal smoothing scale that will produce minimum reconstruction errors in the recovered mass.

In order to give equal weight to all ellipticity measurements around a single image we use an azimuthally symmetric smoothing function. We choose a normalized gaussian for this purpose.

$$\hat{\varepsilon}_m^i = Q_{mn}\varepsilon_n^i$$  \hspace{1cm} (3.3)

where $\hat{\varepsilon}_m^i$ represents smoothed ellipticity field and $Q$ is the smoothing function. Here $m, n$ represent the background image index. In this chapter we have smoothed the data with a normalized gaussian given by,

$$Q_{mn} = \frac{exp(-r_{mn}^2/2\zeta^2)}{\sum_{mn} exp(-r_{mn}^2/2\zeta^2)}$$  \hspace{1cm} (3.4)

Smoothing of data prior to mass reconstruction has been done by several groups (Bartelmann, 1995; Seitz & Schneider, 1996, 1998, 2001), the covariance due to this
smoothing is given by,

\[ C_{mn} = Q_{km}Q_{kn}\sigma_n^2 \]  

(3.5)

Here \( \sigma_n \) is the noise due to intrinsic ellipticities. This covariance between the constraints is used in the likelihood analysis in Equation 3.1.

### 3.3.1 Fitting the error

We have defined a smoothing function and described the inversion procedure to create mass maps. The input parameter to this method is the smoothing scale. In this section we fit the weak lensing reconstruction error as a function of the smoothing scale, measurement error, number density and the length scale at which structure can be resolved.

**Toy Problem: A Sine Wave**

In order to understand how errors propagate into the reconstructed mass we will study a simple problem. The convergence \( \kappa \) for this field is defined by,

\[ \kappa = A \sin \left( \frac{2\pi x}{\lambda} \right) \]  

(3.6)

where \( \lambda \) is the wavelength, it represents the scale at which structure can be resolved. We do a weak lensing reconstruction of this wave and determine the scale at which the reconstruction error is minimal. We generate the mock data by adding noise to the shear (given by Equation 3.6) drawn from a Gaussian of width \( \sigma_e \). This noise is varied over a wide range to determine a fitting formula for the error in the reconstructed \( \kappa \).

We define the error that we are trying to measure as the difference between the model (Equation 3.6) and the reconstructed mass at every image location (for particle based approaches like PBL) averaged over the entire field of view. This error is given by,

\[ \sigma_{\kappa}^2 = \langle (\kappa_{\text{true}} - \kappa_{\text{reconstructed}})^2 \rangle \]  

(3.7)
Here it is important to remember that while reconstructing $\kappa$ from data we will not be able to use Equation 3.7 as our goodness of fit since we do not know the underlying distribution of matter. In that case, the minimum $\chi^2$ will determine the best reconstructed mass.

We smooth the ellipticity field using a normalised gaussian of width $\zeta$, The covariance introduced by smoothing data has been thoroughly studied by Lombardi & Schneider (2003, 2002, 2001). We will construct an empirical form that fits Equation (3.7) by considering the effects of smoothing on mass reconstruction. The aim of smoothing is to average out the noise in the data, however, in this process we also wash out any structure smaller than the smoothing scale. We take into account these two effects and fit them to the error in the reconstructed convergence from the mock data.

$$\frac{\sigma^2_\kappa}{\langle \kappa^2 \rangle} = A_0 \left( \frac{\zeta}{\lambda} \right)^4 + B_0 \frac{\sigma^2_\varepsilon/\langle \kappa^2 \rangle n}{(\zeta/\lambda)^2} + C_0$$  \hspace{1cm} (3.8)

Here $n$ is the number density per unit wavelength of background galaxy images. The first term represents the second order bias due to smoothing of small scale structure. The error introduced by this term is proportional to the square of the smoothing length. The second term is the contribution from external error, in case of weak lensing this is the error due to the intrinsic ellipticities of the background galaxies. $\sigma^2_\varepsilon/\langle \kappa^2 \rangle$ represents the noise-to-signal ratio. The contribution from this term is inversely proportional to $(\zeta/\lambda)^2 n$, implying that the external errors get washed out as the number density of images increases or as the area of smoothing increases.

Figure 3.1 shows a fit for equation 3.8 to the observed error for increasing amount $\sigma^2_\varepsilon$. We have plotted two cases, the solid (dashed) line represents the fitted value and the triangles (squares) represent error for $\frac{\sigma^2_\kappa}{\langle \kappa^2 \rangle} = 1(5)$ measured from the reconstruction. Each data point (triangles and squares) on the graph represents a different mass reconstruction corresponding to a different smoothing scale and noise. As it is evident
from the plot Equation 3.8 is valid around the minima of the curves which defines the optimal smoothing scale for mass reconstruction. This happens because Equation 3.8 was constructed by considering the first term in the Taylor expansion with non-zero contribution to errors. As we move away from the minima higher order terms start dominating. Since we are interested in computing the smoothing scale at which the error is minimum we do not need the higher order terms. This is a demonstration with a toy model, we generalize this form and use Equation 3.8 to determine the scale at which we smooth ellipticities prior to the $\chi^2$ minimization.

### 3.4 Covariance

In this section we will compute the covariance due to smoothing in the reconstructed mass, which will be used to compute the ellipticity of the mass distribution. The covariance in the likelihood function has been studied extensively in the context of cosmic shear, Eifler et al. (2008b) have studied the covariance between cosmological parameters to determine its effect on the likelihood analysis for cosmological parameter estimation. Eifler et al. (2008a) uses the covariance among the data when comparing the information content in aperture mass and two-point correlation function. However, not much importance has been given to the role of the covariance in cluster mass reconstructions with some exceptions, that of Bridle et al. (1998) who derive an analytic expression for the covariance and compare it with the covariances from Monte-Carlo simulations. Also van Waerbeke (2000) has shown that a maximum likelihood lensing mass reconstruction has a noise that follows a gaussian random field. Merten et al. (2009) have also used covariance between the mass bins for computing mass maps.

We have defined the covariance due to smoothing of external error in § 3.3 in the ellipticities. The covariance of any linear function $f(\theta)$ sampled at positions $\theta_n$
between any two positions \( \theta_n \) and \( \theta_m \) is given by,

\[
\text{Cov}(f; \theta_n, \theta_m) = \langle (f_n - \langle f \rangle)(f_m - \langle f \rangle) \rangle.
\]

For the sake of simplicity of notation in the rest of the chapter we will denote
\( \text{Cov}(f; \theta_n, \theta_m) \equiv \text{Cov}_{nm}^f \), where \( n, m \) represent either the total number of image galaxies in case of PBL or the total number of grid cells in case of a grid based method.

We will derive the expression for the covariance matrix in the linear regime. This calculation is applicable to any technique where the derivatives of the potential can be expressed as a matrix times a potential. Hence it is applicable to PBL and finite-differencing techniques. In the very weak lensing regime where \( \kappa \ll 1 \) the ellipticity can be approximated by the shear, \( \gamma \),

\[
\langle \varepsilon^i_n \rangle = \gamma^i_n
\]

where \( i=1, 2 \) represents the two components of ellipticity and shear. In this case we can write a likelihood between the observed ellipticity and the measured shear. Using Equation 3.1 we can write the likelihood as,

\[
\mathcal{L} = \exp \left[ -\sum_{mn} \left( \varepsilon^i_m - G^i_{np} \psi_p \right) C^{-1}_{mn} \left( \varepsilon^i_n - G^i_{nq} \psi_q \right) \right]
\]

where \( G^i \) is matrix that relates the potential to \( \gamma^i \), \( C \) is the covariance due to smoothing and \( \hat{\varepsilon}^i \) is the smoothed ellipticity field. Using Equation 3.2

\[
K = \frac{D^{xx} + D^{yy}}{2}
\]

\[
G^1 = \frac{D^{xx} - D^{yy}}{2}
\]

\[
G^2 = D^{xy}
\]

Setting the first derivative of the log of the likelihood to zero we get,

\[
\sum_i (G^i)^T C^{-1} G^i \psi = \sum_i (G^i)^T C^{-1} \hat{\varepsilon}^i
\]
From this equation we can solve for the potential, once we know the potential there is a linear relationship between the measured ellipticities and $\kappa$. In this case the estimated $\hat{\kappa}$ is given by,

$$\hat{\kappa} = KM^{-1} \sum_i (G^i)^T C^{-1} \hat{\varepsilon}^i = \sum_i V^i \hat{\varepsilon}^i$$  \hspace{1cm} (3.16)

where $K$ is matrix that relates the potential to $\kappa$, $M = \sum_i (G^i)^T C^{-1} G^i$ and $V^i = KM^{-1} (G^i)^T C^{-1}$

$$V^i = KM^{-1} (G^i)^T C^{-1}$$  \hspace{1cm} (3.17)

The covariance in $\hat{\kappa}$ follows Equation 3.9. This can be re-written as,

$$\text{Cov}^{\hat{\kappa}} = V \text{Cov}^{\hat{\varepsilon}} V^T$$  \hspace{1cm} (3.18)

where $\text{Cov}^{\hat{\varepsilon}}$ is the covariance in the ellipticity. There are several effects that contribute to this covariance. As discussed in § 3.3.1 one term is due to smoothing of small scale structure and the other is due to averaging the error. We have derived the effect of smoothing on errors due to intrinsic alignment of galaxies and used it for the maximum likelihood analysis. Here we are going to derive a more general expression for the covariance in the final reconstructed mass. The covariance in measured ellipticity is given by,

$$\text{Cov}^{\varepsilon}_{mn} = \langle (\varepsilon_m - \bar{\gamma}_m) (\varepsilon_n - \bar{\gamma}_n) \rangle = \delta_{mn} \sigma_n^2$$  \hspace{1cm} (3.19)

where $\bar{\gamma}_p$ is the true shear for the lens. After smoothing this becomes,

$$\text{Cov}^{\hat{\varepsilon}}_{mn} = \langle (\hat{\varepsilon}_m - \hat{\gamma}_m) (\hat{\varepsilon}_n - \hat{\gamma}_n) \rangle$$  \hspace{1cm} (3.20)

$$= Q_{mp} Q_{np} (\sigma_p^2 + \bar{\gamma}_m \bar{\gamma}_n) + \bar{\gamma}_n \bar{\gamma}_m - \gamma_m Q_{np} \bar{\gamma}_p - \gamma_n Q_{mp} \bar{\gamma}_p$$

Let us define,

$$\hat{\gamma}_m = Q_{mp} \bar{\gamma}_p$$  \hspace{1cm} (3.21)

Using this notation we can write the above equations as,

$$\text{Cov}^{\hat{\varepsilon}}_{mn} = Q_{mp} Q_{np} \sigma_p^2 + \langle (\bar{\gamma}_m - \hat{\gamma}_m) (\bar{\gamma}_n - \hat{\gamma}_n) \rangle$$  \hspace{1cm} (3.22)
The first term is same as Equation 3.5. Since we do not know the true shear we replace $\tilde{\gamma}$ with the reconstructed shear. Replacing this in Equation 3.18 we have an expression for the covariance in the linear regime. These relations are summarized in Table 3.1.

<table>
<thead>
<tr>
<th>$\mathcal{M}$</th>
<th>$\mathcal{X}$</th>
<th>$\mathcal{Y} = \mathcal{M}\mathcal{X}$</th>
<th>Cov$^J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G^1$</td>
<td>$\psi$</td>
<td>$\gamma_1$</td>
<td>$G^{(1)}\text{Cov}^\psi G^{(1)T}$</td>
</tr>
<tr>
<td>$G^2$</td>
<td>$\psi$</td>
<td>$\gamma_2$</td>
<td>$G^{(2)}\text{Cov}^\psi G^{(2)T}$</td>
</tr>
<tr>
<td>$K$</td>
<td>$\psi$</td>
<td>$\kappa$</td>
<td>$K\text{Cov}^\psi K^T$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$\gamma$</td>
<td>$\tilde{\gamma}$</td>
<td>$Q\sigma\sigma^T Q^T + \langle(\tilde{\gamma}_m - \gamma_m)(\tilde{\gamma}_n - \gamma_n)\rangle$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$\epsilon$</td>
<td>$\tilde{\epsilon}$</td>
<td>$V\text{Cov}^\epsilon V^T$</td>
</tr>
<tr>
<td>$V$</td>
<td>$\epsilon$</td>
<td>$\kappa$</td>
<td>$V\text{Cov}^\epsilon V^T$</td>
</tr>
</tbody>
</table>

Table 3.1: A summary of the relation between various matrices defined in § 3.4. The last column gives the covariance among the observable in the third column. The last row is the final expression for the covariance in the reconstructed $\kappa$.

### 3.5 Where is the information in Cluster Lensing?

#### Cluster Ellipticity

It has long been established from simulations (Jing & Suto, 2002; Rahman et al., 2006) that galaxy clusters are triaxial. Oguri et al. (2005) have used triaxial dark matter halos to study the steep mass profile of A1689 and arc statistics using semi-analytic models (Oguri et al., 2003) for triaxial dark matter halos have explained the abundance of gravitationally lensed arcs for a sample of clusters. Cypriano et al. (2004) have measured the ellipticity and the position angle of a sample of X-ray selected clusters parametrically using gravitational lensing and found that the dark matter halos were aligned with the brightest cluster galaxies. In this chapter we will compute the ellipticities of dark matter halos non-parametrically. This model-free
estimation is done by calculating the quadrupole moments of the mass map using the noise matrix derived in the previous section.

There is a strong dependence of ellipticity on amplitude of mass fluctuations $\sigma_8$ (Ho et al., 2006). A higher value of $\sigma_8$ indicates that cluster formation has started earlier and hence the measured ellipticity of clusters in the local universe would be lower. Clusters are formed through hierarchical merging of smaller dark matter halos. Thus at their infancy they have more infalling matter and are more elliptical. As they virialise they become more and more spherical. Thus we expect clusters at higher redshift to be more elliptical than low redshift clusters. This has been confirmed by measuring higher order moments of the X-ray gas distribution (Jeltema et al., 2005; Buote & Tsai, 1995). Following the procedure that we lay down in this chapter we can measure the ellipticity of cluster halos from lensing for a large sample of clusters distributed in redshift. This will make the contribution of the errors due to projection smaller. Since gravitational lensing probes the projected mass of the lens it is very difficult to constrain whether the halo is oblate or prolate.

We measure the shape of the lens by calculating the second order moments of the mass distribution, this will give us the eccentricity and the position angle for an elliptical mass distribution. We will also fit the ellipticity of the lens parametrically, for a truly elliptical lens this will give a very good description of the shape of the underlying dark matter halo.

### 3.5.1 Measuring Cluster Shapes Non-parametrically

In the previous sections we have derived the covariance of a mass map in the linear regime. The super cluster Abell 901/902 is a sub-critical cluster, hence for the error analysis we assume linearity. It is important to note that linearity is not assumed for doing the mass reconstruction. Since we have a correlated mass map with a covariance
that is singular, we will perform a singular value decomposition as explained in § A.1 and consider a few eigenmodes that are significant. We then transform to a basis where the eigenmodes are independent. This is done by the transformation,

\[ \kappa' = \langle \kappa \rangle + U^T (\kappa - \langle \kappa \rangle) \]  

(3.23)

where \( U \) is the orthogonal matrix from singular value decomposition defined in Equation A.1. The \( \kappa' \)’s are independent. We will express all quantities in terms of \( \kappa' \). Here \( \langle \kappa \rangle \) is the mean density of the field, it is calculated as follows

\[ \langle \kappa \rangle = \frac{\sum_{mn} C_{mn}^{-1} \kappa_m}{\sum_{mn} C_{mn}^{-1}} \]  

(3.24)

The ellipticity of the dark matter is defined in terms of the quadrupole moments. The simple definition for quadrupole moments for the \( \kappa \) field is given by,

\[ I_{ij} = \frac{\sum_m \kappa_m w_{mx(i)} x(j)}{\sum_m \kappa_m} \]  

(3.25)

We know that \( \kappa \) is correlated and hence we write the above expression in terms of \( \kappa' \) and inverse weight it by \( s \), where \( s \) is defined in Equation A.1. The errors in the Quadrupole moments are given by the Octopole moments and are calculated in a similar fashion. Here \( w \) is the weight function, we vary the width of the weight function to evaluate the moments at different radius.

The ellipticities of the lens are defined by,

\[ e_1 = \frac{(I_{xx} - I_{yy})}{I_{xx} + I_{yy} + 2(I_{xx}I_{yy} - I_{xy}^2)\frac{1}{2}} \]  

(3.26)

\[ e_2 = \frac{2I_{xy}}{I_{xx} + I_{yy} + 2(I_{xx}I_{yy} - I_{xy}^2)\frac{1}{2}} \]  

(3.27)
3.6 Data

The shear catalog for this cluster is generated following algorithms described in Kaiser et al. (1995) and Heymans et al. (2006). The modeling of the temporal variation of the PSF is outlined in Heymans et al. (2005). The Abell 901/902 field of view has several tens of good stars for stellar modelling of the PSF, the stars were chosen to maximize the signal-to-noise of the stellar ellipticity function and minimize the temporal variation. The charge transfer efficiency of the ACS has degraded over the years, this is corrected using the methods proposed by Rhodes et al. (2007). Furthermore magnitude cuts are applied to the galaxy catalog to eliminate cluster members and foreground galaxies. The magnitudes are chosen such that $23 < m_{F606W} < 27.5$ and the signal to noise for each galaxy is chosen to be $> 5$ with a size greater than 3 pixels. The total sample size is $\sim 60000$ galaxies with 65 galaxies per square arcminute. For this sample the contamination due to cluster member galaxies is low since the redshifts of the background galaxies are chosen such that the median redshift $z_m > 0.6$. This redshift is calculated using a magnitude dependent redshift relation given by Equation. 9 in H08. As shown by Oguri et al. (2010) the weak lensing dilution should have negligible effect on cluster ellipticity. The ellipticity of the dark matter halo depends on the ellipticity of the shear pattern and not on its amplitude. For Abell901/902 the redshift for 90% of the source galaxies were not known. However, the redshift weight function does not have a very strong dependence on the redshift for $z_s > 1$ and $z_l = 0.165$. Hence we assume the sources to be at a single redshift of 1.4 which is the estimated median redshift of the background sources. A more detailed description of the data and tests for systematics can be found in H08.
3.7 Results

We have outlined a recipe for choosing the smoothing scale and using PBL for mass reconstruction. We apply this technique to the Abell 901/902 field of view. The dark matter peaks in this field of view have been detected by H08. We zoom into each of the peaks and reconstruct a mass map and an error map for the peaks. It is evident from the mass maps that these galaxy clusters are sub-critical, typically around the peaks the average $\kappa = 0.1$ and the noise from the intrinsic ellipticity is $\sim 0.3$, hence the signal-to-noise around the mass peaks is $\sim 0.3$. Using this value in Equation (3.8) we get an approximate smoothing scale of $0.5'$, below which we will not be able to measure any significant structure. Because the number density of source varies across the field the ellipticities are smoothed with a gaussian with a width of $\frac{0.5'}{\sqrt{n/\langle n \rangle}}$ where $n$ is the number density of the images.

**A901b**

First we discuss A901b, the most compact peak with a single halo. The reconstruction parameters are reported in Table 3.2. The ellipticity of the peak is $0.45^{+0.11}_{-0.10}$ and the position angle is $90^\circ$ suggesting that it is elongated in the North-South direction. The top left panel of Figure 3.2 is a map of A901b and the right panel shows the error map for the same field of view. We have plotted the ellipticity of the dark matter halo vs radial distance in the upper panel of Figure 3.4. The squares connected by dot-dashed line represents the ellipticity of the dark matter halo. The ellipticity does not change significantly with radius.

**SouthWest Group**

The bottom left panel of Figure 3.2 is the dark matter reconstruction of the Southwest Group. This peak clearly shows that it is not spherically symmetric, it is elongated in the north west direction with significant substructure suggesting that the group is not completely virialized. We measure an ellipticity of $0.3 \pm 0.07$ and a position angle
of 120° ± 4.8°. Since the dark matter distribution has multiple maxima an elliptical
dark matter halo may not be the best description for the Southwest Group. We have
plotted the ellipticity vs radial distance for the southwest group in the lower panel of
Figure 3.4. The ellipticity decreases as we move away from the center.

**A901a and A902**

A901a has two distinct peaks. This is clear from the dark matter map of the top left
panel of Figure 3.3. It has non-zero quadrupole moments, however, it is not possible
to describe it using an ellipse. If we compare Figure 3.2, 3.3 with H08 we see that
the dark matter distribution is very similar. In this analysis we have smoothed the
data prior to reconstruction and included the covariance due to smoothing in the $\chi^2$
minimization and in calculation of covariance of the final mass reconstruction. This
makes the errors of the reconstruction well understood. The error maps in the right
hand panels of Figure 3.2, 3.3 is computed by taking the square root of the diagonal of
the covariance matrix of $\kappa$. The central peak of A901b is detected at 7σ significance,
the two sub-peaks of the Southwest Group is detected at 4σ. The central peak of
A901a is a 5σ detection the secondary peak is a 2σ detection. The peak of A902 is
detected at 4σ. The mass measured within one arcminute of the center of each peak
is listed in Table 3.2. We measure the ellipticity of A901b and the Southwest peak
since these two peaks are relatively less disturbed. A901a has two distinct peaks and
A902 is very disturbed even in the outer regions. Hence it is difficult to represent
these peaks with ellipses.

In order to understand the errors of our reconstruction we used a Monte-Carlo
simulation with no intrinsic signal for a hundred realizations for the same field of view
as one of the sub-peaks, and did a noise reconstruction using the same smoothing
scale. Only 9.2% of these reconstructions had peaks detected at 2σ and 24.7% of
them had 1.5σ peaks. We did not detect any peaks with higher significance. In the
reconstructed mass maps of A901/902 we have detected the most significant peak at 7σ which is a robust detection. The least significant peak is detected at 2σ which has a 90% probability of being real.

We have four distinct sub-clusters in this field-of-view separated by less than $3h^{-1}Mpc$. Hence we also investigate whether the clusters to be aligned to each other (Hopkins et al., 2005). We test the alignment by looking at the cosine of the angle between the major axis of the dark matter halos. We plot the value of this angle as a function of distance in Figure 3.5. A901a, A901b and A902 are very closely aligned. They are plotted in the upper panel of the figure. The cosine of the angle between the major axis of these three peaks are very close to unity. The lower panel represents the alignment between the Southwest Group and the other peaks. The angle made by the major axis of the Southwest Group deviates most from A901b. This is seen in the dotted line. The misalignment is less obvious for A901a and A902, especially when we go out radially. Even though the Southwest Peak shows some misalignment it is well within expectations from simulations (Hopkins et al., 2005).

### 3.7.1 Parametric Fitting

From the non-parametric reconstruction it is clear that A901a and A902 are double peaked systems, while A901b and the SouthWest Peak are closer to a single halo with substructure. We have fitted this data to a singular isothermal ellipse described in § 1.4.2. We fit a single halo centered on the BCG in each cluster. Since the aim of this study is to measure the ellipticity of the lens we do this fit for the SouthWest Peak and A901b only because A901a and A902 have irregular structure and hence cannot be modeled as an ellipse. The field of view contains four distinct peaks, so we consider a patch of 178 square arcminutes around each peak. This ensures that the shear signal is not contaminated by the other dark matter halos. The constraints
on the velocity dispersion, ellipticity and position angle are listed in Table 3.2. The constraints on the velocity dispersion are consistent with Gray et al. (2002). We estimate the ellipticity of A901b to be $0.39 \pm 0.09$, and a position angle to be $90^\circ$ implying that the dark matter halo is elliptical in the vertical direction. The dark matter map of the southwest peak is clearly not spherical. This peak is well fitted by the singular isothermal ellipse and the ellipticity of this peak is $0.4^{+0.13}_{-0.16}$. In Figure 3.6 we have plotted the joint $1(2)-\sigma$ error probability distribution between the axis ratio and the position angle. It is not possible to constrain the ellipticity of A901a and A902 with an elliptical mass model. In fact a parametric fit is consistent with the spherical model with high error bars. It is clear from the mass reconstruction that the peaks are disturbed. As a matter of fact A901a has two distinct peaks, the second peak coincides with an infalling X-ray group. A902 also has another galaxy group in the background at a redshift $z = 0.46$.

### 3.7.2 Comparison between dark matter and light distribution

We have measured the ellipticity of the light distribution by measuring quadrupole moments of the cluster member galaxies weighted by their stellar masses. The results are listed in Table 3.2. The light distribution and the dark matter distribution is coincident for A901b. In the Southwest peak the light distribution is coincident with one of the sub-peaks. The light distribution is less elliptical compared to the mass distribution.

The results from the non-parametric reconstruction of A901b indicates that the major axis of the light distribution and the dark matter distribution are not coincident. Figure 3.7 shows a comparison of the light and dark matter distribution. The background map represents the stellar mass of the cluster member in units of $M_\odot$. 


Table 3.2: Measuring ellipticity of dark matter and light distribution. The measurements for the ellipticity and position angle are inferred at a distance of 200 $h^{-1}$ kpc from the center of each peak for non-parametric measurements.

The contours represent the dark matter distribution.

We also compare the ellipticity of the dark matter and light distributions in Figure 3.8. The light is distribution becomes more circular with increasing radius and the dark matter ellipticity does not change significantly with radial distance. For the Southwest Group the ellipticity of both the dark matter and the light distribution decreases with radial distance. In Figure 3.8 we plot the alignment between the light and the dark matter ellipticity as a function of radius. For A901a, A902 and the Southwest Group the light and the dark matter are well aligned. However, A901b the
orientation of the light distribution differs from that of the dark matter distribution.

3.7.3 Comparison between parametric and non-parametric results

We have computed the ellipticity and the position angle for A901b and the Southwest Peak parametrically and non-parametrically. The results for A901b using both methods are consistent. The position angle for the Southwest Peak is inconsistent between the measurements. This peak has an irregular mass distribution, hence an elliptical mass model does not provide a full description of its shape. The values of the measured ellipticity are very close to that expected from simulations (Ho et al., 2006; Bailin & Steinmetz, 2005) and previous observations of dark matter galaxy halo ellipticities from the Red Sequence Cluster Survey (RCS) Hoekstra et al. (2004) survey. The measured velocity dispersion for the peaks is consistent with the results from ground based investigations (Gray et al., 2002). The error in the minor to major axis ratio is quite high since weak shear is a noisy estimator of the dark matter distribution, however it is the only way to uniquely measure the dark matter halo shape.

3.8 Discussion and Future Work

In this chapter, we have measured the projected ellipticity of dark matter halos non-parametrically. This is done using an improved PBL by including smoothing of the ellipticity field. We have also calculated the covariance of the resulting mass distribution making the errors of the reconstruction well understood. We have applied this technique to the super cluster A901/902 and reconstructed each of the peaks individually and measured the ellipticity of A901b and Southwest Peak from the PBL
reconstruction and using parametric models. The other two peaks A901a and A902 have a lot of substructure and cannot be modeled as ellipses. We have not considered the line-of-sight ellipticity of the dark matter halos of Abell 901/902. This is because lensing probes the projected mass distribution. Corless et al. (2009) have fitted triaxial NFW to galaxy clusters with high errors on the concentration and mass suggesting that more information from strong lensing, X-rays and Sunyaev Zeldovich effect is required to constrain parameters pertaining to the line-of-sight. A joint analysis using X-rays and lensing has been done by Morandi et al. (2010) to constrain the ellipticity along the line of sight.

In future we will measure the ellipticity of the projected dark matter halos for a large sample of clusters and compare them to simulations (Hopkins et al., 2005). The error introduced due to the line-of-sight ellipticity for a large sample will be insignificant. A similar study has been done in X-rays by Jeltema et al. (2005) by studying the ratio of the higher order moments to the zeroth order moments.
Figure 3.1: Error vs the smoothing scale for two levels of noise. The triangles and the squares represent the errors measured from mass reconstruction done for different values of noise and smoothing scale. The solid line and the dashed line represent the fitted values for \( \sigma^2 = \langle \kappa^2 \rangle \), here \( \hat{\zeta} = \zeta \lambda \). The minima in this plot represents the ideal smoothing scale for the given noise. As expected decreasing the signal-to-noise for the mass reconstruction shifts the minimum towards higher smoothing scale.
Figure 3.2: Mass reconstruction of dark matter halos in A901/902. Top Left Panel: A901b Top Right Panel: Error Map of A901b. Bottom Left Panel: SouthWest Group. Bottom Right Panel: Error Map for the SouthWest Group. There is an artificial increase of error towards the extremities of the maps where the convolution kernel steps over its hard edges. A901b is a compact dark matter halo, the peak is detected at $7\sigma$ and the Southwest Group has significant substructure with two sub-peaks detected at $4\sigma$ significance level. We measure the ellipticity of these two peaks.
Figure 3.3: Mass reconstruction of dark matter halos in A901/902. Top Left Panel: A901a Top Right Panel: Error Map of A901a. Bottom Left Panel: A902. Bottom Right Panel: Error Map A902. A901a has two distinct peaks, the central peak is detected at 5σ and the secondary peak is detected at 2σ. A902 is reasonably disturbed with the central peak detected at 4σ. These two peaks are not representative of elliptical dark matter halos.
Figure 3.4: This is a plot of ellipticity vs radial distance. The dot-dashed line represents the ellipticity of the dark matter halos and the dotted line represents the ellipticity of the light distribution. The upper panel is a plot for A901b and the lower panel is a plot for the Southwest Group. For both sub-clusters the ellipticity of the light distribution decreases with radial distance. For A901b the ellipticity of the dark matter does not vary much with radial distance. The ellipticity of the dark matter also decreases with radial distance for the Southwest Group.
Figure 3.5: This is a plot of the cosine of the angle between the major axis of the four sub-clusters with radial distance. A901a, A901b and A902 have major axis pointing in almost the same direction, hence the cosine of the angle between their major axis is very close unity. This is plotted in the upper panel. The major axis of the Southwest Group is misaligned with the other clusters, this effect being most pronounced for A901b, the cosine of angle between A901b and the Southwest Group deviates most from unity. This is seen in the lower panel of the plot.
Figure 3.6: The two panels represent the joint 1(2)-\(\sigma\) error probability distribution for A901b and the Southwest Peak derived from the parametric modeling, described in § 1.4.2. The y-axis \(f\) is the axis ratio and the x-axis \(\alpha\) is the position angle defined in section § 1.4.2 in radians. For both plots we have plotted the 1\(\sigma\) and 2\(\sigma\) contours. Both peaks have non-zero ellipticity at 2\(\sigma\) level.
Figure 3.7: A comparison of the light distribution vs the dark matter distribution. The colors represent the light distribution and the overlayed contours represent the dark matter distribution. Left Panel: A901b. Right Panel: Southwest group. The units of the light distribution is $M_{\odot}$. 
Figure 3.8: Plot of the cosine of the angle made by the major axis of the light distribution and the dark matter distribution. The light and the dark matter distribution for A901a, A902 and the Southwest group are aligned. Close to the center of the cluster the dark matter and the light distribution are misaligned.
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Chapter 4

Measuring substructure in A1689 by combining lensing and X-rays

Abstract

We report a mass reconstruction of A1689 using a new technique for Strong+Weak lensing based on Particle Based Lensing (PBL) and we compute the noise Covariance matrix for the mass map. The mass map produced using this technique has a variable resolution depending on the data density and the signal-to-noise. We have applied this technique to reconstruct the mass distribution of A1689. The reconstruction shows a secondary mass peak in the north-east direction confirming previous X-ray and optical observations. This shows that the core of the cluster is still undergoing a weak merger. This is the first Strong+Weak lensing mass map of A1689 with well understood covariance matrix. We have used this mass map to measure power ratios of the dark matter distribution and compared it to the X-ray distribution. We find that the power in the X-ray distribution is lower suggesting a smoother gas distribution compared to the dark matter distribution.
4.1 Introduction

A1689 is an X-ray luminous cluster with a very high velocity dispersion (Lenze et al., 2009) at a redshift of 0.18 and it is one of the richest clusters observed to date. It is well known for its spectacular arcs and many multiple images. Velocity dispersion analysis of more than 500 cluster members suggests the existence of secondary structure that coincides with a group of galaxies with north-east to the central region (Czoske, 2004). Recent SUZAKU observations in the X-rays (Kawaharada et al., 2010) suggests anisotropic gas and temperature distributions in the cluster outskirts correlated with the presence of the large scale structure galaxies in the photometric redshift slice centered around the cluster. Some studies have revealed a discrepancy between lensing and X-ray masses (Peng et al., 2009), which have been addressed by triaxial modeling of X-ray and strong lensing (Morandi et al., 2010) and by non-thermal gas pressure (Molnar et al., 2010). The NFW (Navarro et al., 1997) fit for the lensing data for this cluster suggests a very high value of the concentration parameter compared to ΛCDM simulations. Corless et al. (2009) have modeled the weak lensing data with a triaxial NFW profile. They have computed a lower value for concentration with much larger errors. These studies suggest a fairly complicated structure of A1689.

There have been several mass reconstruction studies for A1689. Broadhurst et al. (2005a) and Umetsu & Broadhurst (2008) have found a very high concentration using weak lensing SUBARU data. Strong lensing reconstruction using both parametric (Limousin et al., 2007, hereafter L07) and non-parametric (Coe et al., 2010, hereafter C10) have estimated a lower concentration for an NFW profile fitting. Diego et al. (2005) have done a multi-resolution non-parametric mass reconstruction for A1689 using strong lensing only for the core of the cluster.

In this chapter, we study A1689 by comparing the X-ray distribution with a non-parametric Strong+Weak lensing mass reconstruction using “Particle Based Lensing”
(PBL) (Deb et al., 2008), and quantify substructure by measuring higher order moments of the mass and gas distribution. The advantage of using PBL is the variable resolution that can be obtained in the strong (high resolution) and weak (low resolution) lensing regions. Additionally, the errors in this technique are well understood. This makes calculation of moments from the mass map possible.

4.2 Strong+Weak Mass Reconstruction

The primary challenge in combining strong and weak lensing data is the difference in scales at which the various signals dominate. As discussed in the earlier chapters, the resolution of weak lensing mass reconstructions vary from 1’ for ground based data to 0.5’ for space based data. This happens because ground based observations are shallow having less than 20 galaxies per square arcminute whereas space observations are deep with as high as 60 galaxies per square arcminute making the poisson errors lower. The strong lensing information, such as the positions of multiply imaged galaxies have very low errors (less than an arcsecond) are concentrated within an arcminute from the cluster center. Thus strong lensing constrains mass distribution at the core of the cluster very precisely but it is unable to produce a mass map towards the outskirts of the cluster.

In this section we develop the Strong+Weak (S+W) lensing procedure using PBL. A combined likelihood function is given by,

\[
\mathcal{L} = \exp \left[ -\sum_{mn} \left( \frac{\varepsilon_m^i - \frac{\gamma_m}{1-\kappa_m(\psi_m)}}{2} \right) \frac{1}{C_{mn}^{-1}} \left( \frac{\varepsilon_n^i - \frac{\gamma_n}{1-\kappa_n(\psi_n)}}{2} \right) \right] - \sum_{i,pairs} \left( \frac{(\alpha^A(\{\psi\}) - \alpha^B(\{\psi\}))Z(z_i) - (\theta^A - \theta^B)^2}{\sigma_i^2} \right).
\]

The first term of the equation is due to weak lensing only. The covariance in the data arises because of the smoothing procedure described in Chapter 3 (Deb et al., 2009).
The second term represents the fit between the multiple images and the deflection field at the location of the multiple images. Maximizing the likelihood for anyone of these terms is simple. The weak lensing mass map has a resolution of $0.5''$, and the strong lensing mass map can have a resolution as low as $10''$ (Coe et al., 2010) and the positions of the multiple images are fit exactly. The S+W reconstruction has a variable resolution, the outskirts have a resolution determined by the smoothing scale of the weak lensing data and the core has a higher resolution since we fit to the high signal-to-noise strong lensing data. The simultaneous fitting of the strong and weak lensing data is complicated since there is some ambiguity in the choice of relative weighting between the strong and the weak lensing measurements. The advantage of an S+W reconstruction is that it ensures the mass map at the core of the cluster is consistent with the outskirts.

The contribution to the error of the $i^{th}$ pair is given by a combination of the error in redshift and the astrometric error in measuring the positions of the multiple images. The error in measuring the positions of the images is $\sigma_{\theta} = 0.2'$. The error $\sigma$ is given by,

$$\sigma^2 = \left( \frac{\Delta \theta}{z} \right)^2 \left( \frac{\sigma_{\theta}}{\Delta \theta} \right)^2 + \left( \frac{\Delta \theta}{z} \right)^2 \left( \frac{\sigma_z}{z} \right)^2.$$ (4.2)

where $\Delta \theta$ is the difference in positions for multiple images, $\sigma_{\theta}$ are the astrometric errors and $\sigma_z$ are the errors in redshift.

The astrometric errors associated with the positions of the multiple images is low. However the error in the strong lensing mass reconstruction is dominated by Poisson error. Multiple images sample the cluster potential at discrete locations at finite number of points. Thus, even if we fit the mass at those finite locations very accurately the overall mass distribution will have higher error since most of the positions are fitted to weak lensing data.
4.2.1 Covariance of S+W map

We have calculated the errors for weak lensing in (Deb et al., 2009) and we now calculate the error in the reconstruction in a similar fashion. Before going into further details we present a brief description of the notation used. As discussed in Chapter 2 and 3 the potential is related to the convergence and shear via the following linear relationships,

\begin{align*}
\kappa &= K \psi, \quad (4.3) \\
\gamma_1 &= G^1 \psi, \quad (4.4) \\
\gamma_2 &= G^2 \psi, \quad (4.5) \\
\end{align*}

Similarly, the deflection field is also related to the potential via linear matrix operators given by,

\begin{align*}
\alpha_1 &= A^1 \psi, \quad (4.7) \\
\alpha_2 &= A^2 \psi. \quad (4.8) \\
\end{align*}

Here the shear and the convergence are dimensionless and the deflection field has the dimension of length.

Taking the first derivative of Equation 4.1, we have,

\begin{equation}
\sum_i (G^i)^T C^{-1} G^i \psi + w_s \sum_i A_i^\dagger A_i^\psi = \sum_i (G^i)^T C^{-1} \hat{\varepsilon}^i + w_s \sum_i A_i^\dagger \delta^i. \quad (4.9)
\end{equation}

Here \( \hat{\varepsilon} \) is the smoothed ellipticity defined in Chapter 3, and \( \delta_x \) is difference are the residuals to the fit of the strong lensing observables.

The solution for the potential is obtained by inverting the above equation,

\begin{equation}
\psi = V_w \hat{\varepsilon} + V_s \delta_x, \quad (4.10)
\end{equation}
where
\[
V_w = \left( \sum_i (G^i)^T C^{-1} G^i + w_s \sum_i A^{i,T} A^i \right) \sum_i (G^i)^T C^{-1},
\]
(4.11)
and
\[
V_s = \left( \sum_i (G^i)^T C^{-1} G^i + w_s \sum_i A^{i,T} A^i \right) w_s \sum_i A^{i,T}.
\]
(4.12)

The covariance in the potential is given by,
\[
\Cov^\psi = \psi\psi^T = V_w \Cov^\epsilon V_w^T + V_s \Cov^\delta V_s^T.
\]
(4.13)

Here we have assumed that the terms \( V_w \hat{\epsilon} \delta_x^T V_x^T \) and \( V_s \delta_x \hat{\epsilon}^T V_w^T \) go to zero since
\[
\langle \gamma \alpha \rangle = 0
\]
(4.14)

The above relationship holds since the shear has \( m = 2 \) symmetry and the deflection field has an \( m = 1 \) symmetry. The covariance in the has been calculated in Chapter 3, it consists of a noise term and a signal term.

The noise covariance matrix for the reconstructed \( \kappa \) map is given by,
\[
\Cov^\kappa(\zeta) = K \Cov^\psi K^T,
\]
(4.15)
where the matrix \( K \) is the linear operator that converts the potential \( \psi \) into \( \kappa \). Here the reconstructed mass is dependent on the scale \( \zeta \) used to smooth the ellipticities. It is important to note that the resolution scale in the strong lensing regime is dictated by the separation between the multiple images.

### 4.3 Weak Lensing data

The weak lensing shear catalog has been provided by Dr. Hakon Dahle using a composite of the SUBARU telescope for a large field of view and HST/ACS to obtain
high resolution data around the central region using HST/ACS camera. The SUBARU archive SMOKA contains the images for A1689 in the V(1920 sec exposure) and SDSS $i'$ band. The image analysis was done using software developed by Yagi et al. (2002). The limiting magnitude used for object detection are $V=26.5$ and $i' = 25.9$ for a $3\sigma$ detection within a $2''$ aperture. For lensing distortion measurements, all galaxies with colors 0.22 magnitude redder than the color magnitude diagram of E/S0 galaxies are chosen to avoid contamination due to cluster members. The mean redshift of the background galaxies is $z_s \simeq 1 \pm 0.1$. The distortion analysis was done using IMCAT software and the KSB formalism with modifications described in Erben et al. (2001).

The central region of the cluster is dominated by extended bright halos of luminous central galaxies. In order to obtain a better resolution, this region is imaged by HST in four bands ($g', r', i', z'$). Ellipticities of the background galaxies are done using IM2SHAPE (Bridle et al., 2002). This is a software for measuring ellipticities of background galaxies and is better equipped than KSB techniques to deal with arclike images in the strong lensing regime. The entire data set consists of 11000 images, we use 3000 images in the central region.

### 4.4 Strong Lensing data

The strong lensing data used in this thesis is tabulated in L07. In order to identify all the strong lensing images a composite image of HST/ACS data the four bands: F475W, F625W, F775W and F850LP are necessary. The accuracy of the modeling of the strong lensing constraints depends on the knowledge of the redshifts of the background sources. The first strong lensing analysis for this cluster was done by (Broadhurst et al., 2005b, hereafter B05). In B05 about half of the multiply imaged galaxies were identified by eye to create a mass model which was then used to detect more multiple images. There was misidentification of a few of the multiple images.
which were corrected in L07 and C10 and additional multiple image systems have been identified. More than half (67%) of the multiple images are have spectroscopic redshift. Typically one (or two in some cases) of the multiple images have spectroscopically confirmed redshifts, the other members of that particular system are obtained by fitting a model and ensuring that they have the same color.

4.4.1 X-ray Data

Clusters of galaxies show very strong X-ray emission. Due to the low efficiency of galaxy formation, 90% of the baryons are in the form of intergalactic gas. The deep potential well of galaxy clusters traps this gas and heats it to X-ray emitting temperatures. The X-ray temperature serves as a proxy for the depth of the potential well and hence the mass of the cluster.

The X-ray observable is the X-ray surface brightness distribution due to free-free emission. It is proportional to the $n_e^2$ where $n_e$ is the number density of the gas. In presence of substructure clumps can be visible at varying energies and radii. The details of the data analysis done in this Riemer-Sørensen et al. (2009) have found clumps of gas in the northeastern part at a temperature of $9.3 \pm 0.9$ kev than the central region at a temperature of $10.5 \pm 0.1$ kev. The Chandra observations for A1689 is available at NASA HEASARC archive with a total exposure of 180 ks. The temperature of the X-ray gas is calculated in radial bins using an isothermal model. This temperature is used to calculate the mass profile using the hydrostatic mass equation (Sarazin, 1988). The resulting mass profile is commonly fitted to the universal NFW profile (Navarro et al., 1997) to constrain dark matter halo properties. Figure 4.1 gives the surface brightness distribution of the X-ray emission for A1689.
4.5 Power Ratios

In order to quantify the substructure from non-parametric mass maps we use the simplest form of parameterization. We do a multipole expansion of the potential (Buote & Tsai, 1995) given by,

$$\psi = \psi_0 + \psi_1 \sum_m \frac{1}{mr^m} (a_m \cos(m\phi) + b_m \sin(m\phi))$$  \hspace{1cm} (4.16)
where $\psi_0$ and $\psi_1$ are constants and $(r,\phi)$ are conventional polar co-ordinates. Here $a_m$ and $b_m$ are moments calculated within a circular aperture. The $m^{th}$ moment in the $x$ and the $y$ direction is given by,

$$a_m(r) = \int_{r' < r} \Sigma(\vec{r}') (r')^m \cos(m\phi') d^2\vec{r}' ,$$  \hfill (4.17) \\
$$b_m(r) = \int_{r' < r} \Sigma(\vec{r}') (r')^m \sin(m\phi') d^2\vec{r}' .$$  \hfill (4.18)

This technique of multipole expansions is directly related to the dynamical state that results from fluctuations in the cluster potential. These moments have the following properties. A circularly symmetric mass distribution produces a monopole only term. The dipole term vanishes if the co-ordinate system is set to be at the center of the mass distribution. An ellipse contributes to even terms only, thus significant contribution to odd terms would indicate presence of substructure. These moments are calculated in a circular aperture. This makes sure that the shape of the aperture does not produce any bias.

The powers are given by,

$$P_0 = [a_0 \ln(R)]^2 ,$$  \hfill (4.19) \\
$$P_m = \frac{1}{2m^2 r^{2m}} (a_m^2 + b_m^2) .$$  \hfill (4.20)

We calculate these powers and calculate their ratio in the form $P_2/P_0, P_3/P_0$ and $P_4/P_0$. These ratios are very sensitive to substructure and describe a wide range of cluster morphologies.
4.6 Results

In this section we have applied PBL as described in § 4.2 to the strong and weak lensing data for A1689. Figure 4.2 shows the mass and the error map for the central region of the cluster. The error map is computed from the square root of the diagonal of the covariance matrix. The advantage of computing the covariance matrix is that we are able to calculate physical parameters (with errors bars) related to the mass distribution. The mass map in the upper panel of the figure shows a secondary peak in the north-east direction to the cluster. Compared to previous strong lensing only mass reconstruction we are able to probe the mass distribution out to a larger radius. The mass reconstruction shows the presence of a secondary structure in the north east direction (Figure 4.2). The presence of the secondary structure is revealed on addition of the strong lensing data. The optical image shows some second group of galaxies at that location. This is also consistent with findings of Riemer-Sørensen et al. (2009). They performed a hardness ratio test to recover a cool core and found some substructure in the north eastern part.

In order to quantify these secondary structures, we use the power ratio formalism. We find that the power in the dark matter is higher than the power in the gas distribution. The lower value in the X-ray distribution (Table 4.1) is expected because it is proportional to the square of the gas density. This result implies that the gas is much more smoothly distributed than the dark matter. This becomes clear when the we compare the X-ray distribution (Figure 4.1) and the dark matter distribution(Figure 4.2) visually. This behavior is also seen in simulations (Suwa et al., 2003). We have measured the power ratios at 500 kpc for the X-ray distribution.
<table>
<thead>
<tr>
<th>Power</th>
<th>Value (X-ray)</th>
<th>Value (lensing)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_2/P_0$</td>
<td>$(6.68 \pm 0.27) \times 10^{-6}$</td>
<td>$(1.6 \pm 0.25) \times 10^{-9}$</td>
</tr>
<tr>
<td>$P_3/P_0$</td>
<td>$(3.71 \pm 1.12) \times 10^{-7}$</td>
<td>$(0.9 \pm 0.14) \times 10^{-5}$</td>
</tr>
<tr>
<td>$P_4/P_0$</td>
<td>$(6.42 \pm 2.65) \times 10^{-8}$</td>
<td>$(8.6 \pm 0.3) \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 4.1: Power ratio measurements of the X-ray gas distribution at 500 kpc from the center.

### 4.7 Ratio Weak lensing and X-ray Mass

For many clusters that are not isothermal the predicted X-ray mass and lensing mass differ significantly. The temperature profile has been calculated in radial bins non-parametrically in Riemer-Sørensen et al. (2009). The weak lensing reconstruction suggests that A1689 is approximately spherically symmetric. In this section we compare the radial profile of the X-ray and Lensing mass.

#### 4.7.1 Weak Lensing Mass Profile

In order to measure a radial mass profile from the weak lensing data we assume circular symmetry. For a circular lens the magnitude of the shear is related to the projected mass density via,

$$\gamma(r) \times \Sigma_{\text{crit}} = \langle \Sigma(<r) \rangle - \Sigma(r) = \Delta \Sigma(r) \quad (4.21)$$

where

$$\gamma(r)^2 = \gamma_1^2(r) + \gamma_2^2(r) \quad (4.22)$$

and the critical mass density at this redshift is given by,

$$\Sigma_{\text{crit}}^{-1} = \frac{4\pi G D_{LS} D_S}{c^2 D_S} \quad (4.23)$$

We bin the data in annular sections and calculate $\gamma(r) \times \Sigma_{\text{crit}}$ for each bin. In order to calculate $\Sigma(r)$ for each annulus, we need to know the $\langle \Sigma(<r) \rangle$ for the innermost
annulus. We use the weak lensing result for this purpose. We calculate the cumulative mass in radial bins and compute the ratio between the X-ray mass and lensing mass at three different radius namely at 360 kpc, 510 kpc, 733 kpc (Figure 4.3). We find that the ratio is consistent with unity.

4.8 Discussion

In this chapter we have reconstructed the mass map of A1689 using Strong and Weak lensing data. This is the first non-parametric S+W mass map with a covariance matrix. The unique property of a PBL S+W reconstruction is that we combine the high signal-to-noise positions of multiple images with HST data having 50 galaxies per square arcminute and SUBARU data with 20 galaxies per square arcminute. This is possible because the weak lensing data is smoothed on a scale that optimizes averaging of both signal and noise and varies with number density of background galaxies. The strong lensing data is added to the likelihood function with a weight that creates a high resolution mass map in the cluster core (strong lensing regime) and lower resolution map (dictated by the weak lensing smoothing scale) in outer regions of the cluster (weak lensing regime).

This gives us the advantage of calculating physical properties like the power ratios from the mass map. We have compared the power ratios calculated from lensing as well as the X-ray distribution. The power in the X-ray is lower than the power in the lensing distribution implying a smoother gas distribution.

Currently there are several tens of clusters for which there is strong and weak lensing data. Future observations like the CLASH Multi-Cycle treasury program will image 25 clusters using 20 orbits to produce deep images similar to A1689. This analysis can be applied to such data sets to yield a quantitative measure of the amount of substructure in observed galaxy clusters. This can be compared to ΛCDM
simulations to test cosmology in the local universe. In the next chapter we will provide details of future directions for both mass modeling and its applications.
Figure 4.2: Upper Panel: A lensing Strong+Weak mass reconstruction of A1689 using Particle Based Lensing. The X-ray distribution is fairly uniform at the center whereas the (S+W) reconstruction shows significant substructure. Lower Panel: Error map for the same field of view. The contours of the field of view represent values of \( \kappa \).
Figure 4.3: Ratio between the X-ray and lensing masses. The error bars are correlated since the weak lensing mass in one radial bin depends on the inner bins.
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Chapter 5

Future Prospects

Abstract

We explore the possibilities of future research seeded in this thesis. The central theme of this thesis has been cluster lens modeling by combining weak lensing ellipticities with multiple positions from strong lensing. We discuss additional lensing observables that can be used for mass reconstruction. We also discuss astrophysical and cosmological application of lens modeling combined with multiwavelength analysis for a large sample of clusters.

5.1 Introduction

In the previous chapters, we have developed a Strong+Weak lensing mass measurement technique and computed a noise covariance matrix for the mass distribution. We have also computed physical parameters like the ellipticity of the dark matter distribution for individual clusters. This study opens up the possibility of future research in several directions. In the next few sections we will discuss these possibilities in more detail.
5.2 Additional Sources of Information

Several groups have already shown how multiple image positions may be added to the information yielded by lens ellipticities to produce very high quality mass-maps of clusters. It was our desire to maximally exploit the different information scales of the strong and weak lensing signals which motivated the development of PBL in the first place.

However, there is yet more information potentially available that may be utilized in a reconstruction. Consider that in addition to the two constraints generated by the positional difference between two images, we also can measure a flux ratio, and (2) ellipticity differences. Thus, in principle, we have 5 measurable, model parameters per strong lensing pair rather than 2, and in an idealized case, this improves potential resolution of a system in the strong lensing regime by a factor of $\sqrt{\frac{5}{2}} \simeq 1.6$.

As a way of guiding the future development of PBL, we discuss possible future avenues of investigation below.

5.2.1 Flux

Apart from the centroid position, the Petrosian flux of an image is the most straightforward to measure. The relationship between magnification of a lens and the convergence and shear is simply the inverse of the determinant of the projection matrix:

$$\mu = \frac{1}{(1 - \kappa)^2 - |\gamma|^2}$$  \hspace{1cm} (5.1)

Unlike the displacement vectors ($\alpha$), which are simple linear operators of the potential field (the gradient), or the weak-lensing shear field which is nearly so (since in the limit of $\kappa << 1$, the image ellipticity is an unbiased estimator of the shear field), the flux is a highly nonlinear function of the shear and convergence fields. This accounts, in part, for the reason that it has not been used previously in cluster reconstructions. Here
we note that Saha et al. (2007) show that the image positions themselves constrain the fluxes for a source with three non-collinear components. This is a special case, for cluster lensing three component sources for strong lensing may not always be available. Also, Natarajan et al. (2007) use magnification information in their parametric mass modelling of clusters.

The other major consideration is that magnification is not a smoothly varying function of the potential fields. It is well-known that on the critical curves, magnification goes to infinity (Figure ??see, e.g. Schneider et al., 1992, for an extensive discussion). The issue is that the parity of the image reverses as an image crosses the critical curve.

Negative magnification stands for the reversal of image parity, and thus cannot be easily detected. Thus, we are much more interested in computing terms which scale like $\mu$. Indeed, since we cannot measure the magnification directly, but only the flux, we propose that the combination:

$$\frac{\mu_A - \mu_B}{\mu_A + \mu_B} = \frac{f_A - f_B}{f_A + f_B}$$

is directly measurable, and has no poles.

### 5.2.2 Ellipticity Differences

While most measurements of the shear are based on an assumption that any given image is randomly oriented, two images of the same source are not. The difference in their measured ellipticity can be wholly modeled by the relative lensing fields at their respective locations. If both images were in the weak regime, we would be able to use the simple estimator

$$\varepsilon_A - \varepsilon_B = \gamma_A - \gamma_B$$

where all terms in the equation are complex, and thus provide two constraints with high signal to noise per image pair.
Figure 5.1: The magnification as a function of shear and convergence. The right panel is a simple slice through the left, with the choice $\gamma = \kappa$. Neither the magnification nor its derivatives are a continuous function. Moreover, flux ratios are only measurable for systems with at least two images (obviously). One or more of the images will necessarily have negative parity. Thus, a solution to the potential field which is found using standard relaxation methods will not normally converge to a negative parity estimate for any magnification.

In general, however, a more likely configuration is that one image may be in the strong regime, and the other in the weak. If we can determine from the configuration of lenses which is which, we might imagine a better estimator as:

$$\varepsilon_A - \varepsilon_B = \frac{1}{g_A^*} - g_B,$$

with the only associated noise corresponding to photon noise rather than random variance in the intrinsic ellipticity of the images.

### 5.2.3 Flexion

Until recently, the analysis of clusters in the weak or semi-weak regime has primarily relied only on shear. However, Okura et al. (2007), and Leonard et al. (2007) have
reconstructed A1689 using flexion. In particular, the Okura et al. used a Fourier inversion suggested by Schneider & Er (2007). However, the advantage of our proposed PBL is that flexion (and, in principle, any higher-order derivative of the potential) may be explicitly included as additional constraints in the cluster reconstruction. Unlike Fourier techniques, which rely on binning of the data, the PBL method will allow us to exploit the natural small-scale signal probed by flexion.

5.3 Applications: Multiwavelength mass modelling of galaxy clusters

In the previous chapters we have developed non-parametric mass reconstruction techniques with well understood errors and measured the higher order moments of the mass distribution using gravitational lensing. This gives us information about the dark matter distribution. With a large sample of clusters distributed in redshift and mass we can measure the dependence of substructure on these parameters and compare them to simulations.

5.3.1 Lensing+Xrays

While PBL can be used to measure ellipticity of dark matter halos (Deb et al., 2009) similar attempts have been made with non-parametric algorithms for measuring moments of the X-ray distribution (Jeltema et al., 2005). In the following sections, we will outline a systematic approach to measuring the moments of the dark matter and the gas distribution non-parametrically.
Measuring Shapes: Lensing vs X-ray

In future we intend to study the physical properties of galaxy clusters by comparing the distribution of dark matter (from lensing), gas (from X-ray) and light (from member galaxies). We will apply PBL to obtain reliable mass reconstructions of clusters using weak lensing and strong+weak lensing. PBL has well understood covariance matrices, the effectiveness of adaptive methods, and the simplicity of uniform gridding (Deb et al., 2008, 2009). This makes a PBL reconstruction ideal for comparison with X-ray brightness and the optical luminosity distribution.

These studies will help us to investigate the relation between dark matter and baryons in individual systems, which we will compare with predictions from hierarchical structure formation simulations (Cen & Ostriker, 1994) and hydrodynamical simulations (Bode et al., 2009).

Apart from studying individual systems, we will carry out a statistical study of morphology of the different components of clusters by measuring the quadrupole moments of their dark matter from lensing (Deb et al., 2009) and gas using X-ray data. A systematic study of these moments for increasing aperture sizes, and the calculation of the ellipticity (using isodensity contours) in each wavelength as a function of radius will help us understand the asymmetry in each wavelength (Buote & Canizares, 1994; Schneider & Bartelmann, 1997; Schneider, 2006). We will study the dependence of cluster ellipticity with mass, redshift and radius and test theoretical predictions from N-body ΛCDM simulations (Hopkins et al., 2005). Since ellipticity of dark matter halos is a strong function of the amplitude of mass fluctuations $\sigma_8$ (Ho et al., 2006) reliable measurement of cluster ellipticity can be used to constrain cosmology. Many cluster systems are reasonably clumpy and measuring their higher order moments should describe them more accurately. We will measure power ratios (introduced in Chapter 4) (Buote & Tsai, 1995; Schneider & Bartelmann, 1997) for these clusters to
quantify the substructure. This ratio is dependent on the amount of substructure and the size of the aperture chosen. For a very large aperture the dominant term will be the zeroth order term. Calculating the power ratios for increasing apertures will let us investigate the scales at which the substructure is dominant. Jeltema et al. (2005) have done this calculation for 40 X-ray clusters. Figure 5.2 shows a plot from this paper, the plot clearly shows that the power ratios trace the clumps in a distribution.

Figure 5.2: A plot of P3/P0 vs P2/P0 (Jeltema et al., 2005). This notation is defined in chapter 4. The X-ray image for 6 clusters have been shown with their power ratios. It is clear that complex clusters have higher power. With the advent of high quality Strong and Weak lensing data, a similar plot can be made from lens mass reconstructions.
Studying High redshift galaxies

One of the key applications of galaxy clusters is using them as telescopes to study high redshift galaxies responsible for cosmic reionization. The high magnification close to the critical regions of the cluster amplifies faint sources that lie at or beyond the limits of exposures similar to the Ultra Deep Field (UDF). Theoretical studies have suggested that the redshift window during which reionization has occurred is $7 < z < 15$. Galaxies in this redshift range will be very faint making their identification extremely challenging. The high redshift Lyman Alpha Emitters, for example the one at $z \sim 6.96$ (Iye et al., 2006), suggest that the brightest galaxies detectable at $z > 6$ are not responsible for reionization since bright end of the rest frame UV luminosity function (LF) fades rapidly from $z \sim 4$ to $z \sim 7 - 9$ (Bouwens et al., 2008, 2007). Kneib et al. (2004) and Egami et al. (2005) have used clusters as gravitational telescopes to characterize the stellar continuum slope, star formation rate and stellar mass ($\sim 10^9 M_\odot$) of a $z \sim 6.8$ galaxy with very little telescope time. Using mass models (that will be generated using PBL) we will generate magnification maps that predicts the depth of the field of view. Similar attempts (Stark et al., 2007) have shown that it is possible to detect galaxies with Star Formation Rate $< 1 M_\odot yr^{-1}$. We will use magnification maps to constrain the properties of these very faint galaxies.

5.4 Projection Effects

With a large sample of data we will be able to quantify substructure statistically. One caveat of this approach is the error introduced due to projection effects. A sub-sample of the clusters will have secondary structure along the line of sight. Both lensing and X-rays are susceptible to projection effects. This will lead to over-estimate of masses and under-estimation of cluster ellipticities. The redshift distribution of member
galaxies will give some estimate of the line-of-sight substructure. However both dark matter halos and the gas are extended much beyond the cluster member distribution, and cluster members only sample the dark matter potential at finite positions. Thus even if the redshift for all cluster members were known spectroscopically it will not give a complete picture for the line-of-sight substructure of clusters. It is important to quantify these effects from simulations. The Millenium Simulation (Hilbert & White, 2010) is well suited for this purpose with prescriptions to add gas (Dolag et al., 2005). Systematics for some these clusters have been studied by several research groups (Borgani et al., 2004; Rasia et al., 2006).

5.5 Data

The sample of 10 clusters proposed for Cycle 17 on WFC3 and ACS in conjunction with IRAC data and the sample of 25 clusters proposed by Postman et al. are ideal for the above analysis. The Postman et al. sample will be observed in 14 bands making it ideal for photometry. Photometric redshifts (Ilbert et al., 2009) are essential for identification of multiple images necessary for modeling the mass distribution of the clusters. There is archival Chandra data on clusters like A2218, MACSJ2248.7-4431, MS2137.3-2353 and RXJ1347.5-1145. Furthermore additional clusters observed with Hershel Open-Time Lensing Cluster Survey will also be useful for this study. Part of this data will also have complementary ground based imaging using SUBARU. Due to the large field of view of the SUBARU telescope weak lensing measurements can be obtained for a few Mpc around the cluster center.

The dataset from the 400 Square Degree Galaxy Cluster Survey will be also ideal for this research. This sample consists of serendipitously detected galaxy clusters from the ROSAT PSPC pointings. There optical confirmation for the cluster redshifts. They lie in the range $0.3 < z < 0.8$ and are chosen to represent a typical population.
The aim of this survey is to constrain the mass function accurately at a redshift of $z = 0.5$. Some of these clusters have been observed by (Reiprich, 2007) using the Chandra and the XMM Telescope. There is archival HST data on part of this cluster sample and data from the MEGACAM at MMT (Israel et al., 2009). The availability of optical, X-ray and lensing data for this cluster sample makes it very suitable for joint analysis of multiwavelength data. This sample has a well understood selection function (Vikhlinin et al., 2009). This will be used to extract clusters from simulation for comparison.
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123


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Appendix A

Details of matrix Inversion and Covariance Calculation

A.1 Inverting the Covariance matrix for $\chi^2$ minimization.

In order to do a $\chi^2$ minimization the inverse of the covariance matrix is important. Before doing the inverse it is important to understand the properties of the covariance matrix. It is essential to remember that every image location is not independent. This is because we are smoothing the data, this makes the number of image positions larger than the number of 'resolution units' for a given smoothing scale. This situation has been encountered before (Pan & Szapudi, 2005; Gaztañaga & Scoccimarro, 2005) when the data vector was larger than the number of independent information content. This makes input data degenerate, i.e for a smoothing scale considerably higher than the interparticle separation two positions spatially close to each other essentially represent the same information. This makes the covariance matrix singular. The number of independent components of the covariance matrix is inversely proportional
to the area under the smoothing kernel.

We invert this singular matrix by using the Singular Value Decomposition (SVD) implementation of Tikhonov regularization. The traditional SVD matrix inversion is given by,

$$M^{-1} = VSU^T$$  \hspace{1cm} (A.1)

Here U and V are orthogonal matrices, since C is symmetric U=V. S is a diagonal matrix, the diagonal elements are given by \{1/s\}. Here s represents the eigenvalues of C. In case of a singular matrix some of the eigenvalues are zero. Usually s is written in descending order and the first \(l\) non-zero eigenvalues are used for the matrix inversion. For the \(n - l\) eigenvalues 1/s is replaced by zero. In real cases it is commonly seen that the eigenvalues are not zero, rather they are numerically very small, and hence dominated by round-off error. These problems are termed as ill-conditioned problems. In order to deal with this situation a truncated SVD is used to do the matrix inversion, i.e eigenvalues below a certain threshold are considered to be zero. Choice of this threshold is dependent on the particular problem. If the singular values of a matrix can be distinguished from the non-singular ones in a well-defined fashion then the choice of the threshold becomes simple. This is the case when there is a substantial gap between the largest singular eigenvalue and the smallest non-singular eigenvalue. However, in this particular problem where the covariance is due to gaussian smoothing of the data there is no such distinction. As a matter of fact the eigenvalues smoothly asymptote toward zero. Hence the problem does not direct us toward any obvious choice of the threshold value. Since the problem is severely ill-posed we use Tikhonov regularization(ref) to invert the covariance matrix. This is equivalent to
\[ M^{-1} = V f U^T \]  
\[ \text{(A.2)} \]

where \( f_m = \frac{s_m}{s_m^2 + \alpha^2} \) are the filter factors. For \( s_m \gg \alpha, f_m = 1/s_m \) and for \( s_m \ll \alpha, f_m = 0 \). The presence of the regularization ensures that there is a smooth transition between \( f_m = 1/s_m \) and \( f_m = 0 \) instead of an abrupt cutoff threshold. The regularization parameter \( \alpha \) is given by,

\[ \alpha = C \zeta^2 \]  
\[ \text{(A.3)} \]

As \( \alpha \) decreases more and more eigenvalues are included in matrix inversion. As the smoothing scale is increased the area becomes higher, \( \alpha \) becomes higher and the number of modes used in the matrix inversion becomes lower.

### A.2 Generalization to the Non-linear regime

In the previous section we have evaluated the covariance for the reconstructed \( \kappa \) in the linear regime. Inverse techniques like PBL are often able to reconstruct the semi-strong regime of weak lensing clusters within a couple of iterations. Hence we write down a general formalism in the semi-strong region.

In the second iteration the potential is given by,

\[ \psi^{(1)} = (G)^T C^{-1} \hat{\epsilon} = L(\varepsilon^{(0)} - \hat{\epsilon}) \]  
\[ \text{(A.4)} \]

where \( \varepsilon^{(0)} \) is the modelled ellipticity from the first step of the minimization. In order to get the correct solution for the potential both components of the ellipticity will be summed, for the sake of simplicity of notation we have not included it in Equation A.4. We assume that we are in the semi-strong region and \( \kappa < 1 \), hence the modelled ellipticity id given by,
\[ \langle \varepsilon_m^{(0)} \rangle = \frac{\gamma_{m}^{(0)}}{(1 - \kappa_{m}^{(0)})} = \gamma_{m}^{(0)} (1 + \kappa_{m}^{(0)}) \]  

(A.5)

The reconstructed \( \kappa \) in the second iteration is given by,

\[ \kappa^{(1)} = K\psi^{(0)} + K\psi^{(1)} \]  

(A.6)

The expectation value for the ellipticity is given by,

\[ \langle \varepsilon_m \rangle = \gamma_{m}^{(1)} (1 + \kappa_{m}^{(1)}) \]  

(A.7)

The covariance in \( \kappa \) after the first iteration is given by,

\[ \text{Cov}^{\kappa^{(1)}} = \langle K\psi^{(0)}\psi^{(0)^T} K^T + 2K\psi^{(1)}\psi^{(0)^T} K^T + K\psi^{(1)}\psi^{(1)^T} K^T \rangle \]  

(A.8)

We have already calculated the covariance due to the first term in the previous section. We have to evaluate the second term and the third term. The second term is given by

\[ \text{term2} = 2KL(\langle \hat{\varepsilon}\varepsilon^{(0)} \rangle - \langle \hat{\varepsilon}\hat{\varepsilon} \rangle) L^T K^T \]  

(A.9)

and the third term is given by,

\[ \text{term3} = KL(\langle \varepsilon^{(0)}\varepsilon^{(0)} \rangle + \langle \hat{\varepsilon}\hat{\varepsilon} \rangle - 2\langle \hat{\varepsilon}\varepsilon^{(0)} \rangle) L^T K^T \]  

(A.10)

Now we will evaluate the covariance the terms \( \langle \varepsilon^{(0)}\varepsilon^{(0)} \rangle, \langle \hat{\varepsilon}\hat{\varepsilon} \rangle, \langle \hat{\varepsilon}\varepsilon^{(0)} \rangle \).
Using these expressions for term2 and term3 we can evaluate the covariance in the reconstructed mass after two iterations. We can do a similar calculation for the next steps iteratively and keep higher order terms in the expansion of Equation A.5 and obtain an expression for the covariance of the reconstructed mass.