INVESTIGATION OF 1-G SIMILITUDE LAWS BY “MODELING-OF-MODELS” EXERCISE

F. Ozkahriman¹, Member, ASCE and J. Wartman², P.E., Member, ASCE

ABSTRACT: This paper presents the results of a “modeling-of-models” laboratory exercise conducted in a 1-g environment to investigate the applicability and validity of established similitude laws. The study, which was performed under static condition, considers the behavior of cohesive model slopes at small (stiffness) and large strains (failure). The test program involved three small-scale model slopes tested under different geometry scaling factors. The models were comprised of an artificial, fully saturated "model" clay (3 parts kaolinite to 1 part bentonite). The results indicate that for fully saturated cohesive soils tested under undrained conditions, similitude laws are valid for static conditions provided that three dimensional effects and boundary conditions are properly accounted for.

INTRODUCTION

Physical modeling is seeing growing use in the geotechnical community for the studying complex earth systems and the effects of extreme events. With this technique, small-scale physical models of larger prototype geotechnical structures (e.g. foundations, dams, retaining walls) are built in a laboratory and tested under controlled and carefully monitored conditions. Physical modeling has many benefits for engineers analyzing geotechnical systems: boundary conditions, model parameters, and loading can be controlled; complex, unusual, or highly 3-dimensional problems that are not amenable to closed-form analysis can be studied; rare or extreme events (e.g. earthquakes, blast loadings), for which there are few fully documented case histories, can be evaluated; the performance of novel geotechnical systems and products (e.g. new foundation systems, geosynthetics) can be rapidly tested for relatively little cost. Modeling also has limitations, which principally involve issues related to boundary conditions and similitude (e.g. simultaneous

¹ Graduate Student Researcher, Dept. of Civil, Architectural and Environmental Engineering, Drexel Univ., 3141 Chestnut St., Philadelphia, PA, 19104, USA; PH (215) 895 2272; FAX–(215) 895 1363; email:fo25@drexel.edu
² Associate Professor, Dept. of Civil, Architectural and Environmental Engineering, Drexel Univ., 3141 Chestnut St., Philadelphia, PA, 19104, USA; PH (215) 895 2087; FAX–(215) 895 1363; email:jw64@drexel.edu
scaling of multiple phenomena, scale effects of instrumentation, side boundary friction).

For model tests to be meaningful, similitude must be established between stresses and strains in the model and prototype. For soils that exhibit stress-dependency (e.g. sands), this commonly requires that testing be performed in a high-gravity centrifuge environment. For cohesive materials (e.g. clays), it is possible to establish similitude in a normal gravitational environment (1-g) by reducing the undrained shear strength of the model soil. Established laws of similitude are based on the similarity between the model and prototype with respect to geometry, force and time [e.g. Clough and Pirtz (1956), Roscoe (1968), Iai (1989)]. While there is a sound theoretical basis for these scaling factors, experimental verification is nevertheless required to fully validate established similitude relationships. Although this has been performed for high gravity (i.e. geotechnical centrifuge) scaling relationships (e.g. Law et al. 1994), the authors are unaware of similar comprehensive experimental verification of similitude laws under 1-g conditions.

The primary objective of this work is to investigate the established similitude laws for 1-g environments (i.e., Iai 1989, Table 1). Owing to the difficulties associated with obtaining similitude for granular soils in a 1-g environment, this study focused on cohesive soils subjected to rapid, undrained loading. The experiments considered the behavior of a simple earth system, a slope subjected to an applied load at its crest, over both small (pre-failure deformations) and moderate-to-large (failure) strains. The work was conducted as a “modeling-of-models” laboratory exercise using three small scale models having different geometric scaling factors (λ).

<table>
<thead>
<tr>
<th>Engineering Properties</th>
<th>Scaling Factor (Prototype/Model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length and displacement</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Soil density</td>
<td>1</td>
</tr>
<tr>
<td>Mass</td>
<td>$\lambda^3$</td>
</tr>
<tr>
<td>Undrained shear strength</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Soil shear wave velocity</td>
<td>$\lambda^{0.5}$</td>
</tr>
<tr>
<td>Time</td>
<td>$\lambda^{0.5}$</td>
</tr>
</tbody>
</table>

**Table 1. Select scaling factors for 1-g physical modeling (after Iai 1989)**

**EXPERIMENTAL PROGRAM**

**Test Schedule**

The experimental program involved three small-scale model slopes tested under different geometric scaling factors. The largest slope model was taken as the “prototype” earth structure. The other two smaller models’ geometry, strength and low-strain properties (shear modulus) were adjusted by applying the similitude relationships shown in Table 1 to reflect the reduced-scale of models. The selected scaling factors for smallest and medium model were 2.5 and 1.43, respectively. These scaling factors were selected considering the dimensions of the model container and the workability limit (related to water content) of the “model clay” soil used in the experiments. The geometry and instrumentation plan for the largest model are presented in Figure 1. When constructing the smaller models, instrumentation was
positioned to be consistent with the selected geometric scaling factor to facilitate direct comparison of the experimental results. As shown in the figure, the model includes stiff and soft zones. The purpose of the stiff zone was to minimize the boundary effects by precluding development of shear surfaces near the rigid model container. Displacement potentiometers were placed at predefined positions shown in the figure to record the deformation response of the model to the applied load at the crest, while accelerometers were placed near the rear of the model so that the shear wave velocity of the soft clay could be measured prior to the test. A surface load was applied through the center point of a 1.90 cm thick Plexiglas "footing" and measured by load cell.

FIG. 1. Geometry and instrumentation plan of the largest model

Experimental Facilities
The study was performed in 203 cm long by 122 cm wide Plexiglas box, with an inside dimension of 168 cm in length and 118 cm in width. Two 63.5 cm high Plexiglas side walls spaced at a distance of 51 cm were placed in model container for all three of the model tests.

Surface displacements occurring in response to the load application were measured using 50 mm or 100 mm range displacement potentiometers. The potentiometers were connected to a hinge assembly to allow for two-dimensional surface displacements (i.e., combined vertical and horizontal deformation). Additional potentiometers were including in fixed positions so that horizontal sliding on the slope face and the vertical displacement at top of the model could be recorded.

The accelerometers used in the study were hermetically sealed, low-mass, critically damped, dc-response accelerometers having a flat response up to 350 Hz. The diameter and height of the accelerometers were 21.5 mm and 10 mm, respectively, and included 8 mm threaded protrusion stub for mounting purposes. The accelerometers were placed at predefined coordinates during the model building process.
To measure the applied external load to the system, a 1134 kg capacity “S”-type load cell was used in the study. The static “footing” loads were applied using a combined steel reaction beam and screw jack. The time to failure was on the order of 10-12 minutes for each of the models. Experiment data was recorded using an electronic data acquisition system.

Model Material
The “model clay” consisted of including 3 parts kaolinite to 1 part bentonite. This material has been used in several previous 1-g model studies (e.g. Wartman and Riemer 2005). The model clay had liquid and plastic limits of LL = 120% and PL = 25%, and a plasticity index of PI= 95%. Primarily due to its bentonite content, the clay exhibits moderate thixotropy and low to moderate sensitivity (sensitivity = 1.6 to 2.1). As indicated in Table 2, the strength of the model clay was "scaled" for each of the tests to meet the similitude requirements presented in Table 1.

Model Construction and Test Procedures
The container’s Plexiglas sidewalls were cleaned and canola oil was applied to minimize friction between the model and sidewalls. The clay was then remixed and remolded. Because this remolding temporarily lowered the strength of the clay (owing to thixotropic effects), considerably less effort was required to handle and place the soil. The stiff layer was constructed first, followed by placement of the upper, softer clay layer. During construction, care was taken to preclude development of air voids within the models. Once the model was built, a vertical track-mounted survey device was used to measure the model along three sections. The constructed model was covered with a thin sheet of plastic to preserve its water content, and then cured for approximately 36 hours before testing to allow for thixotropic strength gain. Shortly before testing, uncooked durum semolina spaghetti strands were placed vertically into the model along two profiles to capture internal displacements within the models.

RESULTS
The validity of the 1-g similitude laws was assessed by considering consistency in pre-failure response, ultimate capacity, and failure mechanism between the three models.

Model Properties
The properties of the models are summarized in Table 2, which includes both "target" (design) and actual (measured) values. The reported water content values are based on approximately twenty samples from each model. The undrained shear strength of the models, which is a function of the water content (Wartman and Riemer 2005), was measured with a portable laboratory-scale mechanized vane shear having a 5.0 cm high by 2.52 cm diameter rectangular vane blade. Two to four vane shear tests were conducted for each model slope. Results of a typical vane shear test for each model slopes are shown in Figure 2 and summarized in Table 2.
FIG. 2. Results of vane shear tests

Table 2. Undrained shear strength of “model clay” used in each model test

<table>
<thead>
<tr>
<th>Geometric Scaling Factor (λ)</th>
<th>Target Shear Strength (kPa)</th>
<th>Measured Shear Strength (kPa)</th>
<th>Range of Measured Water Content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Model</td>
<td>2.5</td>
<td>1.40</td>
<td>1.45</td>
</tr>
<tr>
<td>Medium Model</td>
<td>1.43</td>
<td>2.45</td>
<td>2.4-2.5</td>
</tr>
<tr>
<td>Largest Model</td>
<td>1.00</td>
<td>3.5</td>
<td>3.4-3.6</td>
</tr>
</tbody>
</table>

Note 1. The largest model is taken as the prototype, and hence its "target" strength = measured strength. The target shear strength of the smaller models was based on λ and the similitude relationship in Table 1.

Model Responses to Applied Load

Pre-Failure Response and Ultimate Capacity

Figure 3 presents the response of the models to the applied crest load. The response is represented as the applied load versus displacement of the Plexiglas "footing." In this and the other figures that follow, the largest model test will be presented in full scale as the prototype. To facilitate direct comparison, the smaller two tests will be scaled using the similitude relationships of Table 2 and presented at the prototype scale. Reflecting the strain-softening nature of the model clay (Figure 2), a peak resistance (considered here as failure) is observed for all tests followed by a gradual reduction in load with continued displacement. The pre-failure load-deformation response is similar, but shown to be "softer" for smaller models. This suggests that the lower strain stiffness of the model clay does not scale in direct proportion to its undrained shear. Similar observations were noted by Wartman and Riemer (2005) based on bender element testing of the model soil. Generally good agreement in model response is shown for the large and medium models. However, the small model, while exhibiting similar stress-strain characteristic as the prototype, is
nevertheless shown to have failed at a lower load than would have been expected based on the similitude relationships. Both the large and medium models failed under an applied load of approximately 88 kg, while the small model failed at a lower load level of about 67 kg. The footing deflection at failure for all models was approximately 2 cm.

**FIG. 3. Relationship between vertical displacement measured by potentiometer P6 and applied loads**

Figure 4 shows again the response of the models, this time as the measured horizontal deformation response at potentiometer P5 (along lower slope face, Figure 1) as a function of the applied crest load. The response approximates more rigid-perfectly plastic relationships, and suggests that the rapid onset slope movement may coincide with localization of a shear surface near the lower portion of the model. Such a response would be expected in a strain softening material. Though the pre-failure response of the model is similar for all three tests, significant movement (i.e., failure) in the smallest model occurs at a lower then predicted applied load. These observations are commensurate with the trends presented earlier in Figure 3.

**Failure Mechanisms**

Figure 5 presents the post-test profile along the center of each model. The profiles were obtained by surveying each model with track-mounted survey device along several longitudinal sections. The profiles are generally similar for all three models, though the behavior near the toe of the largest model differs somewhat from that of the others. Figure 5 also shows the location of the localized shear surface that developed in the models; which was determined by processing digital photographs of deformed spaghetti "inclinometer" strands (Wartman et al, 2005). The internal shear surface extends toward the toe of the slope for the medium and small models, and is somewhat shallower for the large model.
FIG. 4. Relationship between horizontal displacements at lower part of the slope faces of models between applied loads

ANALYSIS OF RESULTS

The data presented above indicates that small model reaches failure at a relatively low applied level. A three-dimensional slope stability analyses was conducted to study what role, if any, side boundaries had on the model behavior. A closed form solution developed by Gens et al. (1988) was adopted for this purpose. The solution is assumes that models' side boundaries are vertical surfaces, and thus accurately represents the test conditions. The solution presents the three-dimensional factor of safety (F₃) as a function of the conventional two-dimensional factor of safety (F₂):

\[ F_3 = F_2 \left(1 + \frac{2M_E}{(AQC)RL}\right) \]

Where \( M_E \) is the first moment of area of each end plane about the center of cylindrical failure surface and \( L \) is the overall length of the slide and \( R \) is the radius of the circle and \( (AQC) \) is the length of failure surface. Basic calculation were performed using Eqn. 1 and considering the side friction parameters determined from "sliding block" tests with the model soil and container material. The results indicate that due to relatively large three dimensional effects the medium and large models have a load capacity that is in the range of 20% to 25% greater than the small model. This generally agrees with the 33% difference in load capacity between the large/medium and small models (Figure 3). It is noted that the width of the models was constant, and thus end or three dimensional effects contribute a larger portion of ultimate resistance for the larger models. Thus the discrepancy between the capacities of the models is more likely related to boundary effects rather than inconsistencies in scaling relations.
Conventional two-dimensional limit equilibrium analyses indicate that the FOS for
the toe (small and medium models) and shallow failure surfaces (large model) are
within several percent of each other (i.e. the failure surfaces were close to being
equally likely). Therefore, it is believed that the difference in location of failure
surfaces (Figure 5) may be related to minor spatial variations in the water content
and/or shear strength, rather than similitude.

FIG. 5. Pre-test and post-test profiles of models in the large model scale

CONCLUSION
A “modeling-of-models” laboratory exercise was conducted in a 1-g environment to
investigate the applicability and validity of established similitude laws. The study,
which was performed under static condition, considers the behavior of cohesive
model slopes at small (stiffness) and large strains (failure). While the response of the
models was generally consistent, differences were noted in (i) ultimate capacity
between the small and medium/large models, and (ii) failure surface between the
small/medium and large models. These differences, however, were attributed to
boundary effects and minor spatial variation in model properties, rather than the
similitude relationships. Based on this research, it is believed that the laws governing
similitude of cohesive models under 1-g conditions are valid.

When developing a 1-g physical modeling experimental program, it is important to
take boundary conditions into account, especially when the model is relatively
narrow, or when the model is large enough to have significant interaction with the
sidewalls. It is also important to recognize that spatial variability is an inherent part of many model programs, and can in some instance effect the experimental results.

ACKNOWLEDGMENT

Financial support for this research was provided by the National Science Foundation under grant CMS-0134370. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

REFERENCES


