Dynamic Response of Cantilevered Rail Guns Attributed to Projectile/Gun Interaction—Theory

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An analytic approach is proposed to investigate the dynamic behavior of laboratory rail guns resulting from launching a projectile. The rail gun is modeled as a beam of finite length sitting on an elastic foundation with cantilevered support at the breech end of rails. The structural response of the rail is governed by a transient fourth-order differential equation with an extra term of elastic support (containment and insulator) subjected to a transient forcing function (a moving magnetic pressure). The complete solution of the governing equation is derived and illustrated in details. The displacement solution can be further derived to obtain strain and stress profile as well as dynamic response of the rail gun. This paper mainly reports the theoretic solution which provides a step forward to predict the dynamic behavior of rail guns.

Index Terms—Critical velocity, dynamics, electromagnetic gun, railgun.

I. INTRODUCTION

The progress of rail gun technology in recent years has resulted in the possibility of tactical electrical gun applications. One of the major differences between a tactical and a laboratory rail gun is maneuverability and service life requirement. A tactical gun has to be robust enough while constructed within a reasonable weight [1], [2]. It is important that the gun be cantilever supported, as a conventional cannon, to be maneuverable. The gun barrel construction and supports need to be able to sustain a significant dynamic loading condition as the projectile is accelerated from at rest to the muzzle velocity within a few thousandth of a second.

Projectile and gun interaction can be modeled with numerical-structural codes coupling with electromagnetic codes [3], [4]. This approach generally involves an extensive effort in detailed construction of finite-element modeling, complex electromagnetic analysis, and structural dynamics. A complete analysis is extremely time-consuming in computation and may be impractical with current electromagnetic and structural codes. Furthermore, the result of computation might not be accurate upon the convergence of the lengthy computation. A closed-form solution with reasonable assumptions can be used to establish upper or lower bounds solution and guide the numerical modeling efforts.

Fig. 1 shows a schematic of a rail gun composed of rails, insulators, containment, and a projectile traveling down to the tube. Current flows through one rail and crosses the armature (part of the projectile) to the other rail. Accordingly, magnetic force is developed in the rail behind the projectile and the armature. It is highly transient in nature and is determined by the travel of the projectile. The magnetic force can be modeled as a step function of magnetic pressure acting on the rail surface. Since the magnetic force only develops in the rail and projectile, it is reasonable to assume all structural response and projectile interaction occurring in the rails. The mechanical support of the containment and insulator can be then modeled as an elastic foundation. This approach was previously proposed in [4] and [5] to determine the critical velocity of rail and conventional guns previously. In this study, the field equation is solved for a rail gun with cantilevered support in closed form. Accordingly, the transition deformation and stress profiles of the rails are obtained in detail. The dynamic characteristics of the rails can be understood in response to the magnetic force.

II. THEORY DERIVATION

As discussed, a cantilever-supported rail was modeled as an elastic beam subjected to concentrated magnetic force propagating along the rail surface, as shown in Fig. 2. The containment and insulators, which provide support to the rails, are modeled as an elastic support to the rails. A schematic of the rail gun...
model is illustrated in Fig. 3. Accordingly, the governing equation can be derived as a transient fourth-order beam with elastic support along the entire rails as follows:

\[
m \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + kw = q[1 - H(x - vt)],
\]

(1)

Here, \(w\) is the lateral displacement dependent upon time, \(t\), and axial position coordinate, \(x\); the symbol, \(m\) is the mass per unit length and is equal to \(\rho Bl\); \(\rho\) is the density of rail material; and \(B\) and \(h\) are the width and thickness of the rail, respectively; \(E\) is the modulus of rail material, and \(I\) is the moment of inertia of the rail cross section. The elastic constant \(k\), from the elastic foundation, will be derived from either a closed-form or numerical solution such as finite element analysis. The loading function, \(q(1 - H(x - Vt))\) in (1), represents the magnetic pressure front traveling along the rail with a constant velocity \(V\) represented by a Heaviside step function \(H(x - Vt)\). The magnetic pressure \(q\) is assumed to be constant also. Accordingly

\[
H(x - vt) = \begin{cases} 
0, & \text{for } x < vt \\
1, & \text{for } x \geq vt \end{cases}
\]

(2)

III. HOMOGENEOUS SOLUTION

The homogeneous part of the solution can be derived from the method of separation variables. In this investigation, the particular part of solution is obtained by the approach Lagrange’s equation. The derivation of the completed solution is derived in details. The homogeneous solution \(w(x, t)\) can be expressed as follows:

\[
w(x, t) = \phi(t) \theta(x).
\]

(3)

Substituting (3) into the homogeneous equation of (1) yields

\[
m \frac{\partial^2 \phi}{\partial t^2} + EI \frac{\partial^4 \theta}{\partial x^4} + k \phi = 0.
\]

(4)

That can be expressed as follows:

\[
- \frac{(\frac{\partial^2 \phi}{\partial t^2})}{\phi} = \frac{EI}{m} \frac{\theta''''}{\theta} + \frac{k}{m}.
\]

(5)

From (5), we assume

\[
\frac{(\frac{\partial^2 \phi}{\partial t^2})}{\phi} = \lambda^2
\]

(6)

and

\[
\frac{EI}{m} \frac{\theta''''}{\theta} + \frac{k}{m} = \lambda^2
\]

(7)

with

\[
\beta^4 = \left(\lambda^2 - \frac{k}{m}\right) \frac{m}{EI}.
\]

(8)

Solutions of (6) and (7) can be expressed as

\[
\phi(t) = A \cos \lambda t + B \sin \lambda t
\]

(9)

and

\[
\theta(x) = C \cosh \beta x + D \sinh \beta x + E \cos \beta x + F \sin \beta x
\]

(10)

where the coefficients \(A, B, C, D, E, \) and \(F\) are constants and will be determined either from the boundary conditions or from the initial conditions.

In the case of a cantilever beam as shown in Fig. 1, the boundary condition can be represented by

\[
\text{At } x = 0 \left\{ \begin{array}{l} \theta = 0 \\
\theta = 0 \\
\frac{\partial \theta}{\partial x} = 0 \\
\frac{\partial \theta}{\partial x} = 0 \end{array} \right.
\]

(11)

and

\[
\text{At } x = L \left\{ \begin{array}{l} M = 0 \text{ or } \frac{\partial^2 \theta}{\partial x^2} = 0 \\
V = 0 \text{ or } \frac{\partial \theta}{\partial x} = 0 \end{array} \right.
\]

(12)

where \(M\) represents the moment and \(V\) represents the shear force of the beam.

Applying the boundary conditions (11) and (12) to the general solution expressed by Equation (10), we obtain

\[
\theta(0) = C + E = 0, \quad \therefore \quad C = -E
\]

(13)

\[
\left(\frac{\partial \theta}{\partial x}\right)_{x=0} = \beta(D + F) = 0, \quad \therefore \quad D = -F
\]

(14)
\[
\left( \frac{\partial^2 \theta}{\partial x^2} \right)_{x=L} = \beta^2 C \cosh \beta x + D \sinh \beta x - F \cos \beta x - F \sin \beta x, \quad x=L = 0 \tag{15}
\]

\[
\therefore \quad C (\cosh \beta L + \cos \beta L) + D (\sinh \beta L + \sin \beta L) = 0
\tag{16}
\]

and

\[
\left( \frac{\partial^3 \theta}{\partial x^3} \right)_{x=L} = \beta^3 C \sinh \beta x + D \cosh \beta x + E \sin \beta x - \cos \beta x, \quad x=L = 0
\]

\[
\therefore \quad C (\sinh \beta L - \sin \beta L) + D (\cosh \beta L + \cos \beta L) = 0.
\tag{17}
\]

From (15) and (17), we obtain

\[
\frac{\cosh \beta L + \cos \beta L}{\sinh \beta L - \sin \beta L} = \frac{\sinh \beta L + \sin \beta L}{\cosh \beta L + \cos \beta L}
\]

which reduces to

\[
\cosh \beta L \cos \beta L + 1 = 0. \tag{18}
\]

Equation (18) is satisfied by a number of values of \( \beta \), corresponding to each normal mode of oscillation. For example, the consecutive roots of this equation are listed as follows [6]:

\[
\beta_1 L, \beta_2 L, \beta_3 L, \beta_4 L, \beta_5 L, \beta_6 L, \beta_7 L, \beta_8 L, \beta_9 L
\]

1.875, 4.694, 7.855, 10.996, 14.137, 17.279

For such defined \( \beta_i \), (8) gives

\[
\beta_i^2 = \left( \lambda_i^2 - \frac{k}{m} \right) \frac{m}{EI}
\]

or

\[
\lambda_i^2 = \frac{EI}{m} \beta_i^4 + \frac{k}{m}
\tag{19}
\]

In terms of thus defined \( \lambda_i \), the solution of (9) can be expressed as

\[
\phi(t) = A_i \cos \lambda_i t + B_i \sin \lambda_i t.
\]  \tag{20}

The frequency of vibration of the beam at any mode will be

\[
f_i = \frac{\lambda_i}{2\pi} = \frac{1}{2\pi} \left( \frac{EI}{m} \beta_i^4 + \frac{k}{m} \right)^{\frac{1}{2}}. \tag{22}
\]

We observe that the frequencies of the natural vibrations depend not only on the rigidity of the beam but also on the rigidity of the foundation.

The fundamental mode of vibration will be

\[
f_1 = \frac{1}{2\pi} \left[ \frac{EI}{m} \left( \frac{1.875}{L} \right)^4 + \frac{k}{m} \right]^{\frac{1}{2}}. \tag{23}
\]

Solutions for different end supports could also be found in Temoshenko and Young [6].

For the \( \beta_i \) defined by (18), and the constants relations defined by (13), (14), and (16), (10) can be expressed as

\[
\theta_i(x) = \left[ \cosh \beta_i x - \cos \beta_i x \right] \frac{(1) (17)}{(2) (24)}
\]

\[
\therefore \quad \frac{\theta_i(x)}{\theta_j(x)} = \begin{cases} 0, & \text{for } i \neq j \\ 1, & \text{for } i = j \end{cases} \tag{25}
\]

From (21) and (24), solution of (3) can be expressed in terms of \( \beta_i \) and \( \lambda_i \) by

\[
u_i(x, t) = \theta_i(x) \phi(t) = \left[ \cosh \beta_i x - \cos \beta_i x \right] \frac{(1) (17)}{(2) (24)}
\]

\[
\times \left( \frac{1}{\sinh \beta_i L + \sin \beta_i L} \right) \times (\sinh \beta_i x - \sin \beta_i x) \frac{(1) (17)}{(2) (24)}
\]

\[
\times [A_i \cos \lambda_i t + B_i \sin \lambda_i t]. \tag{26}
\]

Using the summation-mode procedure, we obtain the general solution

\[
w(x, t) = \sum_i \theta_i(x) \phi_i(t) = \sum_i \left[ \cosh \beta_i x - \cos \beta_i x \right] \frac{(1) (17)}{(2) (24)}
\]

\[
- \frac{(\cosh \beta_i L - \cos \beta_i L)}{(\sinh \beta_i L + \sin \beta_i L)} \times (\sinh \beta_i x - \sin \beta_i x) \frac{(1) (17)}{(2) (24)}
\]

\[
\times [A_i \cos \lambda_i t + B_i \sin \lambda_i t]. \tag{27}
\]

where constants \( A_i \) and \( B_i \) can be determined from the initial conditions. For example, at \( t = 0 \)

\[
w(x, 0) = \sum_i \theta_i(x) \phi_i(0) = \sum_i \theta_i(x) A_i = \varphi(x) \tag{28}
\]

and

\[
\frac{\partial w(x, 0)}{\partial t} = \sum_i \theta_i(x) \frac{\partial \phi(0)}{\partial t} = \sum_i \theta_i(x) B_i \lambda_i = \psi(x). \tag{29}
\]

By the orthogonality, we could determine the constants \( A_i \) and \( B_i \) from

\[
A_i = \int_0^L \varphi(x) \theta_i(x) dx \tag{30}
\]

\[
B_i = \frac{1}{\lambda_i} \int_0^L \psi(x) \theta_i(x) dx \tag{31}
\]

where \( \varphi(x) \) and \( \psi(x) \) are defined by (28) and (29), and \( \theta_i(x) \) by (24), respectively.
IV. PARTICULAR SOLUTION

We use the Lagrange’s equation to determine the particular solution \( \phi(t) \) defined by the applied force \( p(x,t) = q[1 - H(x - vt)] \).

First, we express the kinetic energy of the beam in terms of \( \phi_i(t) \)

\[
T = \frac{1}{2} \int_0^L m \left( \frac{\partial w}{\partial t} \right)^2 dx
\]

\[
= \frac{1}{2} \sum_i \sum_j \frac{\partial \phi_i}{\partial t} \frac{\partial \phi_j}{\partial t} \int_0^L m \theta_i \theta_j dx.
\]

Recall the orthogonal relation defined in Equation (25), we can rewrite \( T \) as

\[
T = \frac{1}{2} \sum_i \left( \frac{\partial \phi_i}{\partial t} \right)^2
\]

(33)

where

\[
M_i = \int_0^L m \theta_i^2 (x) dx
\]

(34)

represents the general mass of the beam, and \( \theta_i(x) \) is expressed by Equation (24).

The total potential (strain energy) of the beam consists of \( U_b \), the strain energy of the beam, and \( U_f \), the strain energy of the elastic foundation

\[
U = U_b + U_f
\]

(35)

where

\[
U_b = \frac{1}{2} \int_0^L EI \left( \frac{\partial^2 w(x,t)}{\partial x^2} \right)^2 dx
\]

\[
= \frac{1}{2} \sum_i \sum_j \phi_i \phi_j \int_0^L EI \frac{\partial^2 \theta_j}{\partial x^2} \frac{\partial^2 \theta_j}{\partial x^2} dx
\]

\[
= \frac{1}{2} \sum_i \beta_i^4 M_i \phi_i^2
\]

(36)

and

\[
U_f = \frac{1}{2} \int_0^L k w^2 dx = \frac{k}{2} \int_0^L \left( \sum_i \phi_i \phi_i \right)^2 dx
\]

\[
= \sum_i \phi_i \phi_j \int_0^L \theta_i \theta_j dx
\]

\[
= \frac{k}{2m} M_i \sum_i \phi_i^2 \phi_i^2
\]

(37)

So, the total potential energy \( U \) is obtained as

\[
U = U_b + U_f
\]

\[
= \frac{1}{2} \sum_i \beta_i^4 M_i \phi_i^2 + \frac{k}{2m} \sum_i \phi_i^2 \phi_i^2
\]

\[
= \frac{1}{2} \sum_i \left[ \frac{M_i \phi_i^2}{m} \left( \frac{EI}{m} \beta_i^4 + \frac{k}{m} \right) \right]
\]

\[
= \frac{1}{2} \sum_i \left[ M_i \phi_i^2 \beta_i^2 \right].
\]

(38)

For a given virtual displacement \( \delta \phi_i \), the virtual work done by the applied external force \( p(x,t) = q[1 - H(x - vt)] \) can be found as

\[
\partial W = \int_0^L p(x,t) \delta w_i dx
\]

\[
= \int_0^L p(x,t) \sum_i \theta_i(x) \delta \phi_i \ dx
\]

\[
= \sum_i \delta \phi_i \int_0^L p(x,t) \theta_i(x) dx = \sum_i \delta \phi_i Q_i
\]

(39)

where we define \( Q_i \) as the generalized force

\[
Q_i = \int_0^L p(x,t) \theta_i(x) dx
\]

\[
= \int_0^L q[1 - H(x - vt)] \theta_i(x) dx.
\]

(40)

Considering the definition of the force \( p(x,t) \) in (2) at \( x < vt \), and after the integration for \( \theta_i(x) \), the generalized force \( Q_i \) can be expressed by

\[
Q_i(t) = \int_0^L q[1 - H(x - vt)] \theta_i(x) dx
\]

\[
= \int_0^\infty \left[ (\cosh \beta_i x - \cos \beta_i x) - (\cosh \beta_i L + \cos \beta_i L) \right. \left. (\sinh \beta_i L + \sin \beta_i L) \right. \times (\sinh \beta_i x - \sin \beta_i x) \ dx
\]

\[
= \frac{q}{\beta_i} \left[ \left( \sinh \beta_i x - \sin \beta_i x \right) - (\cosh \beta_i L + \cos \beta_i L) \left( \sinh \beta_i L + \sin \beta_i L \right) \times (\cosh \beta_i x - \cos \beta_i x) \right] \right. \left. \left|_0^\infty \right.
\]

\[
= \frac{q}{\beta_i} \left[ \left( \sinh \beta_i x - \sin \beta_i x \right) - (\cosh \beta_i L + \cos \beta_i L) \left( \sinh \beta_i L + \sin \beta_i L \right) \right. \times (\cosh \beta_i x - \cos \beta_i x) - 2 \right].
\]

(41)
Substituting $T$, $U$, and $Q_i$ into the following Lagrange equation

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\phi}_i} \right) - \frac{\partial T}{\partial \phi_i} + \frac{\partial U}{\partial \dot{\phi}_i} = Q_i.$$  \hspace{1cm} (42)

From (33), (38), and (40), we obtain

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\phi}_i} \right) = \sum_i M_i \frac{\partial^2 \phi_i}{\partial \dot{\theta}_i^2},$$

$$\frac{\partial T}{\partial \phi_i} = 0,$$

$$\frac{\partial U}{\partial \dot{\phi}_i} = \frac{\partial}{\partial \dot{\theta}_i} \left( \frac{1}{2} \sum_i M_i (\frac{\partial \phi_i}{\partial \dot{\theta}_i})^2 \right),$$

$$= \sum_i M_i \lambda_i^2 \phi_i^2.$$ \hspace{1cm} (45)

We obtain a differential equation in terms of the general co-ordinator $\phi_i(t)$:

$$\frac{\partial^2 \phi_i}{\partial t^2} + \lambda_i^2 \phi_i = \frac{Q_i(t)}{M_i} = \frac{q}{M_i \beta_i \lambda_i} F(t)$$ \hspace{1cm} (46)

where

$$F(t) = \left\{ \frac{\sinh \beta_i t \sin \beta_i t}{\sinh \beta_i t \sin \beta_i t} \right\} - \left\{ \frac{\cosh \beta_i L + \cos \beta_i L}{\sinh \beta_i L + \sin \beta_i L} \right\} \times \left\{ \frac{\cos \beta_i L - \cos \beta_i t}{\sinh \beta_i L + \sin \beta_i L} \right\}.$$ \hspace{1cm} (47)

The general solution of (46) for $\phi_i(t)$ can then be obtained as

$$\phi_i(t) = \phi_i(0) \cos \lambda_i t + \frac{1}{\lambda_i} \left( \frac{\partial \phi_i(0)}{\partial t} \right) \sin \lambda_i t$$

$$+ \frac{q}{M_i \beta_i \lambda_i} \int_0^t F(\xi) \sin \lambda_i (t - \xi) d\xi.$$ \hspace{1cm} (48)

The complete solution of (1) can be expressed as

$$w(x,t) = \sum_i \theta_i(x) \Phi_i(t)$$

$$= \sum_i \left\{ \left( \frac{\cosh \beta_i x - \cos \beta_i x}{\sinh \beta_i L + \sin \beta_i L} \right) \times \left( \frac{\cosh \beta_i L + \cos \beta_i L}{\sinh \beta_i L + \sin \beta_i L} \right) \times \left( \frac{\cos \beta_i L - \cos \beta_i t}{\sinh \beta_i L + \sin \beta_i L} \right) \times \left( \frac{\sinh \beta_i t - \sin \beta_i t}{\sinh \beta_i L + \sin \beta_i L} \right) \times \left( \frac{\cos \beta_i L - \cos \beta_i t}{\sinh \beta_i L + \sin \beta_i L} \right) \right\}$$

$$\times \left\{ \phi_i(0) \cos \lambda_i t + \frac{1}{\lambda_i} \left( \frac{\partial \phi_i(0)}{\partial t} \right) \sin \lambda_i t$$

$$+ \frac{q}{M_i \beta_i \lambda_i} \int_0^t F(\xi) \sin \lambda_i (t - \xi) d\xi \right\}.$$ \hspace{1cm} (49)

with $F(t)$ defined by (47).

Assuming that the initial conditions are as

$$At \ t = 0 \left\{ \frac{\phi(0)}{\partial t} = 0 \right\} = 0$$ \hspace{1cm} (50)

equation (48) becomes

$$\phi_i(t) = \frac{q}{M_i \beta_i \lambda_i} \int_0^t \left\{ \frac{\sinh \beta_i t \sin \lambda_i (t - \xi)}{\sinh \beta_i L + \sin \beta_i L} \right\}$$

$$- \left\{ \frac{\cosh \beta_i L + \cos \beta_i L}{\sinh \beta_i L + \sin \beta_i L} \right\} \times \left\{ \frac{\cos \beta_i L - \cos \beta_i t}{\sinh \beta_i L + \sin \beta_i L} \right\} \right\} \times \sin \lambda_i (t - \xi) d\xi.$$ \hspace{1cm} (51)

Considering the integration

$$\phi_i(t) = \frac{q}{M_i \beta_i \lambda_i} \int_0^t \left\{ \frac{\sinh \beta_i t \sin \lambda_i (t - \xi)}{\sinh \beta_i L + \sin \beta_i L} \right\}$$

$$- \left\{ \frac{\cosh \beta_i L + \cos \beta_i L}{\sinh \beta_i L + \sin \beta_i L} \right\} \times \left\{ \frac{\cos \beta_i L - \cos \beta_i t}{\sinh \beta_i L + \sin \beta_i L} \right\} \right\} \times \sin \lambda_i (t - \xi) d\xi.$$ \hspace{1cm} (52)

or

$$\phi_i(t) = \frac{q}{M_i \beta_i \lambda_i} \left\{ (1) + (2) + (3) + (4) + (5) \right\}$$ \hspace{1cm} (53)

where

$$\left\{ (1) = \int_0^t \left\{ \frac{\sinh \beta_i t \sin \lambda_i (t - \xi)}{\sinh \beta_i L + \sin \beta_i L} \right\} \times \sin \lambda_i (t - \xi) d\xi \right\}$$ \hspace{1cm} (54)

$$\left\{ (2) = - \int_0^t \left\{ \frac{\sinh \beta_i t \sin \lambda_i (t - \xi)}{\sinh \beta_i L + \sin \beta_i L} \right\} \times \sin \lambda_i (t - \xi) d\xi \right\}$$ \hspace{1cm} (55)

$$\left\{ (3) = \left\{ \frac{\cosh \beta_i L + \cos \beta_i L}{\sinh \beta_i L + \sin \beta_i L} \right\} \times \int_0^t \cosh \beta_i \sin \lambda_i (t - \xi) d\xi \right\}$$ \hspace{1cm} (56)
Substituting (20), we have

\[
\beta_i^2 v_i^2 - \lambda_i^2 = 0. 
\]

Substituting (20), we have

\[
\beta_i^2 v_i^2 - \lambda_i^2 = \frac{EI}{m} \beta_i^4 + \frac{k}{m} = 0 
\]

which determines the critical velocity as

\[
v = \frac{1}{\beta_i^2} \left( \frac{EI}{m} \beta_i^4 + \frac{k}{m} \right). 
\]

The first term comes from the rail with cantilever support along and the second term is contribution of the rail support (i.e., containment, insulator, and their support). A parametric study was performed based on the properties listed in Table I.

Fig. 4 show the resonance velocity as function the length of rail. There are two competitive terms which affect the resonance velocity. For a cantilevered supported rail along, the resonance velocity decreases as the rail length increase. However, the length of the rail support \(k\) increases simultaneously and
Fig. 5. Resonance velocity of the rail varies with the supporting structure.

results in the increase of the resonance velocity. The resonance velocity resulting from these effects is illustrated in Fig. 4, and the rail support is the dominated term in this case. Accordingly, the rails receive more support as the length of the gun increases. The resonance velocity of a cantilevered rail along is validated by comparing it with a simple supported (supported at both ends) beam subjected to a moving concentrated load derived in [6]. It is the most relevant comparison obtained so far from the literature which provides certain confidence on the model.

Fig. 5 shows the resonance velocity increase with the variation of the elastic support on the rail. As a common laminated steel laboratory gun, the rail is supported by a combined insulator and containment structure at various levels of stiffness. The stiffness of the elastic support is influenced by the insulator and containment material and structural design. The level of stiffness can be quite different by the variation of insulator design and material along. For a tactical gun, the gun assembly is supported differently (i.e., cantilevered supported entirely). The resonance velocity can also be evaluated using the model developed with certain assumptions.

VI. CONCLUSION

The completed solution of a rail gun with finite length and cantilevered support subjected to a moving magnetic pressure has been derived in detail. The solution indeed can be applied to obtain the displacement and stress profiles as well as resonance velocity of the rail gun with various ways of support. The resonance velocity of a laboratory type rail gun with the elastic support of containment and insulator is simulated to demonstrate the developed model. The analysis can be further implemented to predict a more complex geometry and gun support; therefore, it can be applied to guide numerical solution, which in general involves lengthy computation.

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