Slack Bus Modeling for Distributed Generation and Its Impacts on
Distribution System Analysis, Operation and Planning

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Shiqiong Tong
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Abstract
Slack Bus Modeling for Distributed Generation and Its Impacts on Distribution System Analysis, Operation and Planning
Shiqiong Tong
Karen Miu, Ph.D.

Distribution system operating environments are changing rapidly. Proper distributed generation placement and operating will bring benefits for supporting voltage, reducing system loss, enhancing system reliability, releasing T&D capacity and improving energy management flexibility. Distributed generation will play an important role in distribution systems. However, with increasing number of distributed generators (DGs) installed within distribution systems, the traditional methods for distribution system modeling, analysis and planning needs to be revisited, and new tools are required to be developed.

This thesis addresses these challenges through slack bus modeling studies. First, traditional distribution power flow with a single slack bus model is revisited and a distributed slack bus model is proposed for distribution system analysis and planning. Network-based participation factors to distribute slack are developed. These participation factors capture network parameters, load distributions and generator capacities. Then, impacts on distribution operating and planning functions of slack bus modeling are investigated. Two examples, energy cost analysis and switch placement for DG islanding operation are discussed in detail. Last, the problem of distribution system expansion considering DG placement and feeder upgrade is addressed.
Chapter 1. Introduction

Distribution system operating environments are changing rapidly. With large number of distributed generators (DGs) installed within distribution systems, distribution systems are facing great challenges: the traditional methods for distribution system analysis and planning needs to be revisited, and new tools are required to be developed. This thesis addresses these challenges through slack bus modeling study. Distributed slack bus model is proposed for distribution system analysis and planning. Its impacts on distribution applications are also investigated. Then, a distribution system expansion planning problem is addressed.

1.1 Motivations

Distributed generation has been growing rapidly in power systems. Studies by the Electric Power Research Institute (EPRI) and the Natural Gas Foundation indicate that 20% or higher of new generation will be distributed generation by 2010 [24, 25]. As such, distributed generation will play an important role in power systems. Since distributed generation is sited close to load centers, it may bring following benefits:

- voltage support and loss reduction
- system reliability enhancement
- T&D capacity release and infrastructural deferment
- more energy management flexibility.

In order to achieve above benefits, distributed generators must be carefully installed and operated and the behaviors of distribution systems with distributed generation must be
accurately analyzed. However, the inclusion of large numbers of DGs within distribution systems will fundamentally change distribution system analysis, operating and planning.

At the core of these changes, power flow is an essential tool for steady state analysis. Since traditional distribution systems are generally designed without DGs, its power flow computation uses a single slack bus, which generally is the substation. With DGs introduced to distribution systems, the assumption of single slack bus for unbalanced distribution power flow solvers need to be revisited. New models and tools for distribution system analysis need to be developed.

DG placement is an attractive option for distribution system planning. As load levels increase and increased reliability is required for selected customers, the installation of distributed generation may be used to address these challenges. However, traditional distribution system expansion planning rarely includes DG placement. Thus, new strategies and methods for distribution system expansion planning, which include DG placement and operating, also need to be designed.

1.2 Objectives

To achieve the objective of improving distribution utilization with DG placement, the following problems need to be concerned

- Develop analysis tools and models for DGs
- Evaluate analysis tools and models
- Design new strategies for DG placement and operation

This thesis addresses these objectives by making the following contributions.
1.3 Contributions

This thesis contributes to distribution system modeling, analysis and planning for distributed generation as follows:

- Developed the essential steady state analysis tool of three-phase power flow:
  - Proposed a distributed slack bus model for DGs in unbalanced power flow
  - Introduced scalar participation factors to distribute uncertain real power system loss for three-phase power flow calculations
  - Provided two methods to calculate network-based participation factors: sensitivity-based method and generator domain based method
  - Evaluated different slack bus models

- Investigated impacts of slack bus modeling to distribution system applications
  - Designed a cost analysis method, which distinguishes loss and load contributions of individual DGs
  - Demonstrated slack bus modeling effects on switch placement for DG islanding operation

- Formulated problems of distribution system expansion planning, which are readily used by intelligent system methods to search expansion strategies including feeder upgrades and DG placement options under load increases and network faults

- Proposed a GA-based heuristic algorithm to solve the combinational optimization problem of distribution system expansion planning

1.4 Organization of Thesis

The framework of this thesis is shown in Figure 1.1:
Since distribution systems are usually unbalanced, single-phase analysis is not suitable for distribution systems. Three-phase analysis is required for unbalanced distribution systems. Therefore, in Chapter 2, detailed three-phase component models including distributed generators, capacitors, loads, lines, switches and transformers are briefly reviewed.

Three-phase power flow is a vital analysis tool for unbalanced distribution systems. In Chapter 3, the first objective is addressed to study slack bus modeling for three-phase power flow. The traditional distribution power flow with a single slack bus is revisited, and distributed slack bus models are proposed for DG studies. Different methods to compute network-based participation factors are developed to distribute real power system loss to participating sources for three-phase power flow calculations. Numerical
results are studied for different slack bus implementations. Participation factors based on the concept of multi-phase generator domains show their advantages to capture network characteristics and to distinguish loss and load contributions of individual distributed generators.

In Chapter 4, the second objective is addressed by investigating the impacts of slack bus models on distribution application problems. Two examples of distribution applications are reevaluated and reformulated: a cost analysis method applied the distributed slack bus model with generator domain participation factors is designed; the impacts of slack bus modeling to switch placement for DG islanding operation is studied. Simulation results of these two applications are also provided and discussed.

The problem of distribution expansion planning with DG placement to minimize planning cost is studied in Chapter 5 to address the third objective. With increasing complexity, feeder upgrades, DG placement without/with islanding operation and DG placement with feeder upgrades are discussed. These problems are formulated as combinational optimization problems minimization subject to electrical, operational and network constraints. A GA-based algorithm to solve the optimization problem for distribution planning with DG placement is proposed.

In Chapter 6, conclusions are drawn, and the contributions of this thesis are summarized. Then, some possible future research to extend this work is also been discussed.
Chapter 2. Review of Distribution System Modeling

Steady state component models for unbalanced distribution systems will be reviewed in this chapter. The component models presented in this chapter including distributed generator models, shunt capacitor models, load models and branch models, will be used for distribution system analysis, operation and planning in the later chapters of this thesis.

2.1 Distributed Generator Models

Distributed generators are installed within distribution systems and inject power. Their types and control schemes will be given an overview in Subsection 2.1.1. Then, existing DG models will be briefly reviewed and DG models to be used in this thesis will be presented in Subsection 2.1.2.

2.1.1 Distributed Generator Overview

Distributed generators can be categorized into four types [20]: reciprocating piston engine, gas turbine, fuel cell and renewable resource distributed generators based on their electric power generating methods. Each type is briefly described as following:

*Reciprocating Piston Engine Distributed Generators* are the most widely used DG units and the oldest type of DG technology. A very wide choice of fuel types, such as pure hydrogen, propane, methane, gasoline, natural gas, normal fuel oil, diesel oils etc. can be used by reciprocating piston engines. These DGs produce electric power in this way: first, the heat and pressure from combustion moves a piston inside a cylinder; then, this linear motion is converted to rotation by a crankshaft to spin a generator. The greatest advantages of this type of DGs are their low initial investment cost and simple maintenance needs, which overwhelm their disadvantages of exhaust emissions, noise
and vibration. The sizes of this type of DGs range from less than 5 kW to larger than 25,000 kW [20].

*Gas Turbine Powered Distributed Generators* can be used to fit many situations due to their distinctly different size, fuel, efficiency and operating characteristics. This type of generators is using turbines spun by the rapid gases of combustion to rotate electric generators. Gas turbine generators have the advantages of low-cost maintenance, durability, non-vibration and high power-to-weight ratio, but have the disadvantage of low fuel efficiency. Their sizes range from about 15 kW to more than 150,000 kW. [20]

*Fuel Cell Powered Distributed Generators* also have promising application future. These DGs are essentially chemically powered batteries, which produce DC currents through electrochemical processes. They are characterized with very low noise, high fuel efficiency and very low emission, but are currently expensive. Their sizes range from about 5 kW to 1,000 kW. [20]

*Renewable Resource Distributed Generators* are promoted and motivated by environmental considerations. Their power sources are ongoing natural processes such as solar, wind, biomass, geothermal etc. However, their low efficiencies, high initial cost and site requirements limit their applications.

Methods of connecting DG with electric power systems affect DG control schemes. In [45], the IEEE Standard 1547 provides the minimum technical requirements of interconnecting distributed resources with electric power systems. These requirements are functional requirements, and do not specify any particular connection methods or equipments. To achieve some specified planning and operating goals through automatic or manual control, regulating electric power injections from DGs within distribution
systems and their voltages may be required as well as adjusting output frequencies. Using power electronic devices as the connection interfaces between DGs and AC grids [10, 11] has advantages of control flexibility: power electronic devices can handle different types of DGs; frequencies, power injections from DGs as well as voltages on the connecting points with power systems can be regulated. Therefore, this thesis will use power electronic devices as DG interconnection interfaces. In this thesis, DG outputs mean the injections from DGs to power systems. The overview of a DG source connected with AC grid through power electronic devices and its control is shown in Figure 2.1.

![Figure 2.1 Overview of a DG connection and control](image)

In this figure, the DG sources are used to sustain DC voltages, and DC sources are converted to AC sources, which are connected to AC grid. The governors and controllers are responsible for holding the delivered power from the power source and regulated the terminal voltage to desired values. Control schemes of the steady state behavior mainly include the following three modes [11]:

1. Mode A: Governor control
2. Mode B: Inverter control
3. Mode C: Measurement manipulation
1) Control real power injected into the AC grid and regulate voltage magnitudes of the bus connected to the AC grid

2) Control real power and reactive power injected to the AC grid

3) Control real power and reactive power outputs at a fixed ratio; keep constant power factor

2.1.2 Distributed Generator Models for Power Flow Calculation

There exist many distributed generator models. In [18], dynamic models for different types of DGs were developed for transient analysis. In [19], slow dynamic models were provided to investigate load following performance. Some steady-state models were also developed for three-phase power flow calculation: in [20], a three-phase generator model with considering internal voltage behind transient reactance was provided; in [21], fictitious nodes and impedances were used for DG buses to present reactive power injection; in [22], DGs were classified as constant \(PQ\) or \(PV\) nodes.

In this thesis, three-phase voltage-sourced inverters [10] are assumed as the connection interfaces between DGs and distribution networks. These units can control real and reactive power flow and provide balanced three-phase system voltages. Since this thesis will focus on steady state analysis, the DG buses for three-phase power flow calculations are modeled as following:

1) \(P|V|\) bus model

   In this thesis, the \(P|V|\) bus provides balanced three-phase voltage outputs with specified voltage magnitude. In the traditional power flow, which uses a single slack bus model, a \(P|V|\) bus has a fixed real power injection and specified
voltage magnitude. While in the power flow with a distributed slack bus model, the $P|V|$ bus is modeled with a specified voltage magnitude and a real power injection, which is known at each iteration and may change between iterations. Therefore, both real power and reactive power output limits for DGs ($P_{Gi}^{\min} < P_{Gi} < P_{Gi}^{\max}$, $Q_{Gi}^{\min} < Q_{Gi} < Q_{Gi}^{\max}$) are required to be checked during power flow calculation. If a DG’s limit were violated, this $P|V|$ bus would change to a $PQ$ bus. This special $P|V|$ bus model will be described in detail in the Chapter 3.

2) $PQ$ bus model

When DGs are modeled as $PQ$ buses, they have balanced, three-phase voltage outputs. Both their three-phase real and reactive power outputs are specified. The $PQ$ injections from DGs are considered as negative $PQ$ loads.

The other component models including shunt capacitor, load, and branches will be viewed in the following sections. Most are based on standard three-phase models as presented in [14-17].

2.2 Shunt Capacitor Models

Shunt capacitors are often used for reactive power compensation and voltage regulation to reduce system loss in distribution systems. They are modeled as constant capacitance devices. The corresponding injection current of a shunt capacitor to bus $k$, $I_{Ck}$ is a function of voltage at bus $k$:

$$I_{Ck} = y_{Ck} V_k$$ (2.1)
where:

\( y_{ck} \): admittance matrix for the shunt capacitor at bus \( k \)

\( V_k \): complex vector for voltage of bus \( k \)

\( I_{ck} \) is a complex \((p \times 1)\) vector where \( p \) is the number of phases of bus \( k \) for grounded connection. For the shunt capacitor with ungrounded delta connection, \( V_k \) represents line-to-line voltage, and \( I_{ck} \) is a \((2 \times 1)\) complex vector.

### 2.3 Load Models

Three types of loads are considered in this thesis: constant impedance loads, constant current loads and constant power loads. The loads are represented as a current injection into a bus, a linear combination of the previous three types. The injection current of loads at bus \( k \), \( I_{lk} \) is computed as following:

\[
I_{lk} = -y_{lk}V_k + I_{lk} + \left( \frac{S_{lk}}{V_k} \right)^* \tag{2.2}
\]

where:

\( y_{lk} \): admittance matrix for constant impedance load at bus \( k \)

\( I_{lk} \): complex vector for constant current load at bus \( k \)

\( S_{lk} \): complex vector for constant power load at bus \( k \)

\( V_k \): complex vector for voltage of bus \( k \)

Three-phase loads can be balanced and unbalanced. They can be connected in a grounded wye configuration or an ungrounded configuration. Loads in distribution systems may also be single-phase or two-phase loads. Therefore, \( I_{lk} \) is also a complex \((p \times 1)\) vector.
where $p$ is the number of phases of bus $k$ for grounded loads; $I_{lk}$ is a $(2 \times 1)$ complex vector and $V_k$ represents line-to-line voltage for ungrounded delta connected loads.

### 2.4 Branch Models

Distribution branches include lines, switches and transformers. They take the function of electric power delivery. Since distribution systems have unbalanced characteristics, three-phase models for each branch type were developed [14] and reviewed here.

The predominant branch connections in distribution systems are line branches. The standard $\pi$-model is used for distribution line models. Distribution lines have two categories: grounded lines and ungrounded. The grounded line model is represented as:

$$
Y_{ik} = \begin{bmatrix}
-Z_{ik}^{-1} + \frac{1}{2} Y_{ik}^{sh} & -Z_{ik}^{-1} \\
-Z_{ik}^{-1} & Z_{ik}^{-1} + \frac{1}{2} Y_{ik}^{sh}
\end{bmatrix}
$$

(2.3)

where:

- $Y_{ik}$: branch admittance matrix for the grounded line between bus $i$ and bus $k$
- $Z_{ik}$: series impedance between bus $i$ and bus $k$
- $Y_{ik}^{sh}$: charging admittance between bus $i$ and bus $k$

For the ungrounded line model, the charging admittance is ignored, and the phase impedance $Z_{ik}$ in (2.3) is replaced by line impedance $Z_{ik}^{line}$. The following relationship exists between $(3 \times 3) Z_{ik}$ and $(2 \times 2) Z_{ik}^{line}$:

$$
Z_{ik}^{line} = \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
\frac{1}{3} Z_{ik} \\
0 \\
-1
\end{bmatrix}
$$

(2.4)
All switches in this thesis are modeled as zero impedance branches. Therefore, the two end buses for a switch branch have the same voltages and currents flowing in and out this branch.

Three-phase transformer interconnections include grounded wye connections on the four-wire side and ungrounded delta connections on three-wire side. The comprehensive transformer models for different connections derived in [14] are summarized in Table 2.1. In this table, the transformer models between bus $i$ and bus $k$ are presented by the admittance matrix:

$$
Y_{ik} = \begin{bmatrix}
Y_{11}^{ik} & Y_{12}^{ik} \\
Y_{21}^{ik} & Y_{22}^{ik}
\end{bmatrix}
$$

(2.5)

$y$ presents per phase leakage admittance, and $\alpha, \beta$ present the primary side and secondary side transformer settings, respectively.
<table>
<thead>
<tr>
<th>Type</th>
<th>Transformer Connection Type</th>
<th>( Y_{ik}^{11} )</th>
<th>( Y_{ik}^{12} )</th>
<th>( Y_{ik}^{21} )</th>
<th>( Y_{ik}^{22} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Grounded Wye</td>
<td>Grounded Wye</td>
<td>( \frac{y}{\alpha^2} \begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix} )</td>
<td>( -\frac{y}{\alpha \beta} \begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix} )</td>
<td>( \frac{y}{\alpha^2} \begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix} )</td>
</tr>
<tr>
<td>2</td>
<td>Grounded Wye</td>
<td>Ungrounded Wye</td>
<td>( \frac{y}{3\alpha^2} \begin{bmatrix} 2 &amp; -1 &amp; -1 \ -1 &amp; 2 &amp; -1 \ -1 &amp; -1 &amp; 2 \end{bmatrix} )</td>
<td>( -\frac{y}{\sqrt{3\alpha \beta}} \begin{bmatrix} 2 &amp; 1 \ -1 &amp; 1 \ -1 &amp; -2 \end{bmatrix} )</td>
<td>( -\frac{y}{\sqrt{3\alpha \beta}} \begin{bmatrix} 2 &amp; 1 \ -1 &amp; 1 \ -1 &amp; -2 \end{bmatrix} )</td>
</tr>
<tr>
<td>3</td>
<td>Grounded Wye</td>
<td>Delta</td>
<td>( \frac{y}{\alpha^2} \begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix} )</td>
<td>( -\frac{y}{\alpha \beta} \begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \ -1 &amp; -1 \end{bmatrix} )</td>
<td>( -\frac{y}{\alpha \beta} \begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \ -1 &amp; -1 \end{bmatrix} )</td>
</tr>
<tr>
<td>4</td>
<td>Ungrounded Wye</td>
<td>Grounded Wye</td>
<td>Opposite ꞌ of type 2</td>
<td>( \alpha )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>5</td>
<td>Ungrounded Wye</td>
<td>Ungrounded Wye</td>
<td>( \frac{y}{\alpha^2} \begin{bmatrix} 2 &amp; 1 \ -1 &amp; 1 \end{bmatrix} )</td>
<td>( -\frac{y}{\alpha \beta} \begin{bmatrix} 2 &amp; 1 \ -1 &amp; 1 \end{bmatrix} )</td>
<td>( \frac{y}{\alpha^2} \begin{bmatrix} 2 &amp; 1 \ -1 &amp; 1 \end{bmatrix} )</td>
</tr>
<tr>
<td>6</td>
<td>Ungrounded Wye</td>
<td>Delta</td>
<td>( \frac{y}{\alpha^2} \begin{bmatrix} 2 &amp; 1 \ -1 &amp; 1 \end{bmatrix} )</td>
<td>( -\frac{\sqrt{3}y}{\alpha \beta} \begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{bmatrix} )</td>
<td>( -\frac{\sqrt{3}y}{\alpha \beta} \begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{bmatrix} )</td>
</tr>
<tr>
<td>7</td>
<td>Delta</td>
<td>Grounded Wye</td>
<td>Opposite ꞌ of type 3</td>
<td>( \alpha )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>8</td>
<td>Delta</td>
<td>Ungrounded Wye</td>
<td>Opposite ꞌ of type 6</td>
<td>( \alpha )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>9</td>
<td>Delta</td>
<td>Delta</td>
<td>Same as type 5</td>
<td>( \alpha )</td>
<td>( \beta )</td>
</tr>
</tbody>
</table>

\( \dagger \): swap \( Y_{ik}^{11} \) and \( Y_{ik}^{12} \) with \( Y_{ik}^{22} \) and \( Y_{ik}^{21} \), respectively, then swap \( \alpha \) with \( \beta \).
Chapter 3  Three-Phase Power Flow with Distributed Slack Bus Model

In this chapter, to accommodate the anticipated growth of distributed generators (DGs) in unbalanced distribution systems, the single slack bus model will be revisited and a distributed slack bus model for unbalanced power flow studies with DGs will be proposed. The single slack bus is used as the reference bus for voltage phase angles and to absorb system real power loss $P_{Loss}$. In this thesis, the distributed slack bus is the generator buses, who absorb $P_{Loss}$, and one of these buses acts as the reference for voltage phase angles. A participation factor approach will be applied to distribute $P_{Loss}$, which means system loss is shared by several generator buses during power flow calculation based on their assigned participation factors. Different methods to compute network-based participation factors will be developed and the advantage to capture network characteristics using participation factors based on the concept of multi-phase generator domains will be shown. Then, the three-phase power flow equations are extended to incorporate the distributed slack bus model and implemented with a Newton-Raphson solver. Numerical results were obtained for different slack bus implementations. The performance and impact of the models with respect to different number of DGs and different levels of DG penetration for a large distribution system will also be discussed.

3.1 Background of Distributed Slack Bus Models

Power flow analysis is a basic tool for power system studies. In a traditional power flow with a single slack bus model, one generator bus is selected to be the voltage phase
angle reference and balances the power mismatch due to uncertain system loss. However, there is no slack bus in actual power systems. The single slack bus model may significantly distort computed power flows. Therefore, to provide more realistic power flows, the distributed slack bus model has been investigated.

In balanced transmission systems, distributed slack buses were introduced to remedy the inadequacy of a single slack bus. Participation factors have been applied to assign the system loss to multiple generators during power flow calculations. In previous works, these participation factors are constant values and can be determined by different methods. In [1, 2], the participation factors are related to the characteristics of turbines on each generator bus and load allocation. In [3], the authors applied participation factors using combined cost and reliability criteria in power flow for fair pricing. In [4], the author provides a method of choosing participation factors based on the scheduled generator outputs.

These previous works focus on balanced transmission systems and, for varying reasons, they may not be suitable for distribution systems with DGs. For example, the main source of a terrestrial distribution system is the substation; therefore, no turbine characteristics related to the substation are available. In addition, due to the high R/X ratios of distribution systems (e.g. typical R/X ratios of distribution lines range between 0.15 and 0.5, while those of transmission lines ranges between 0.05 and 0.1), the loss allocation during calculation should be considered. As such, the load distribution and the network topology play a critical role in the participation of each generator with respect to servicing loads and loss. Therefore, network-based participation factors for distributed slack bus models will be developed in this chapter.
3.2 Distributed Slack Bus Model

A distributed slack bus is modeled using scalar participation factors to assign the unknown system loss for participating sources. In distribution systems, participating sources include the substation and distributed generators whose real power outputs can be adjusted. The distributed slack bus model for three-phase power flow will now be proposed, and the different methods to calculate its participation factors will be developed and discussed in the later sections.

In the distributed slack bus model, the system real power loss $P_{\text{Loss}}$ is treated as an unknown and distributed to participating sources according to their assigned participation factors, $K_i$. The sum of all participation factors is one.

$$\sum_{i=0}^{m} K_i = 1 \quad (3.1)$$

where:

- $m$: the number of participating distributed generators in the system
- $0$: the substation index

By applying the participation factors, the total real power outputs of participating sources can now be expressed as:

$$P_{G_i} = P_{G_i}^{\text{load}} + K_i P_{\text{Loss}} \quad i = 0,1,2 \cdots m \quad (3.2)$$

where:

- $P_{\text{Loss}}$: total real power loss in the system
- $P_{G_i}^{\text{load}}$: real power load associated with participating source $i$
Not all DGs in distribution systems are allowed to adjust their real power outputs, since many are small machines and may not have the necessary control technologies. Consequently, two types of DG models for power flow are considered:

- non-participating DGs \( (PQ \text{ model}) \)
- participating DGs \( (P|V| \text{ model}) \)

As such, participation factors for distributing slack are only applied to the set of participating sources including the substation and DGs with adjustable outputs.

Since the real power output \( P \) is adjustable for participating sources during a power flow calculation, the \( P \) is specified at each iteration. Assuming voltage-source inverter (VSI) connections [10] for the participating DGs, a new type of \( P|V| \) buses applied to participating sources has following characteristics:

- total injected real power \( P_{Gi} \) is adjustable per-iteration
- voltage magnitudes of each phase \( |V_i^p| \) are equal and specified
- three voltage phase angles \( \theta_i^p \) are unknown, however, the difference between any pairs are 120° (e.g. \( \theta_i^a, \quad \theta_i^b = \theta_i^a - 120^\circ, \quad \theta_i^c = \theta_i^a + 120^\circ \))

These \( P|V| \) buses provide three-phase balanced voltage outputs and adjustable real power inputs. Thus, there is only one unknown, one voltage phase angle, at each participating DG bus. In this thesis, the phase \( a \) voltage angle, \( \theta_i^a \) is selected as the unknown for generator buses. The corresponding equations below are used:

\[
P_{Gi} - \sum_{p=a}^{c} P_{Di}^p - \sum_{p=a}^{c} P_i^p = 0 \quad i = 1, 2, \ldots, m \quad (3.3)
\]
where:

- $P_{Di}^p$: real and reactive demand on bus $i$, phase $p$
- $P_{i}^p$: real power flow equation on bus $i$, phase $p$

\[
    P_i^p = \left| V_i^p \right| \left[ \sum_{k=0}^{n} \left( g_{ik}^p \cos(\theta_i^p - \theta_k^p) + b_{ik}^p \sin(\theta_i^p - \theta_k^p) \right) \right]
\]

There are $m$ variables of phase angle $\alpha$ on participating DG buses corresponding to these $m$ equations (3.3).

Since power flow with a distributed slack bus model identifies $P_{Loss}$ as an additional unknown, an additional equation at the reference bus is required. In this thesis, the substation is treated as one of the participating sources. It is selected as the reference bus with a specified voltage phase angle and voltage magnitude. Therefore, the additional equation at the substation is as follows:

\[
    (P_{G0}^{load} + K_0 P_{Loss}) - \sum_{p=1}^{c} P_{Di}^p = \sum_{p=1}^{c} P_0^p
\]  

(3.4)

where:

- $P_{Di}^p$: real power demand on phase $p$ of the substation bus
- $P_0^p$: real power flow equation on the substation, phase $p$

\[
    P_0^p = \left| V_0^p \right| \left[ \sum_{k=0}^{n} \left( g_{0k}^p \cos(\theta_0^p - \theta_k^p) + b_{0k}^p \sin(\theta_0^p - \theta_k^p) \right) \right]
\]

On the generator buses, limitations on the range of the machine’s total power output and the converter’s total power output must be adhered to:

\[
    P_{Min}^p < P_G^p < P_{Max}^p
\]

(3.5)

\[
    Q_{Min}^p < Q_G^p < Q_{Max}^p
\]

(3.6)
These conditions and constraints can be integrated into existing distribution power flow solvers in order to distribute the slack bus. When the calculated real/reactive power output of a DG violates its limits during power flow calculation, this DG cannot be considered as a participating source and is modeled as a constant $PQ$ injection for the next iteration. With all load buses as $PQ$ buses, the power flow equations are summarized as following:

**Power Flow Equations:**

**Unknowns:**

1. $P_{\text{Loss}}$
2. $\theta_i^p$ \quad $i = 1, \cdots, m$
3. $\theta_i^p, \left| V_i^p \right|$ \quad $i = m+1, \cdots, n$ and $p = a, b, c$

For the substation bus and m generator buses:

$$f_{P_i} = (P_{\text{load}}^{\text{sub}} + K_i P_{\text{Loss}}) - \sum_{p=a}^{c} P_{Di}^p - \sum_{p=a}^{c} P_i^p = 0 \quad i = 0, 1, 2, \cdots m$$  \hspace{1cm} (3.7)

For n-m load buses:

$$f_{P_i}^p = -P_{Di}^p - P_i^p = 0 \quad i = m+1, m+2, \cdots, n$$  \hspace{1cm} (3.8)

where:

$P_{Di}^p, Q_{Di}^p$ : real and reactive demand on bus $i$, phase $p$

$P_i^p, Q_i^p$ : real and reactive power flow equation on bus $i$, phase $p$

$$P_i^p = \left| V_i^p \right| \sum_{k=0}^{n} V_k^p \left[ g_{ik}^p \cos(\theta_i^p - \theta_k^p) + b_{ik}^p \sin(\theta_i^p - \theta_k^p) \right]$$
\[ Q^p = \left| V_i^p \right| \sum_{k=0}^{n} V_k^p \left[ g_{ik}^p \sin(\theta_i^p - \theta_k^p) - b_{ik}^p \cos(\theta_i^p - \theta_k^p) \right] \]

In these equations, the system total real power loss \( P_{\text{loss}} \) is treated as a variable and an additional equation on the substation is included. The system real power loss is distributed to participating sources according to the participation factors. The above equations can be solved with a Newton-Raphson solver, which will be presented in Section 3.4.

An extended Jacobian matrix can be formed for the following update equations (3.9). The relationship of the variables and the participation factors for three-phase power flow with a distributed slack bus can be seen. The right sub-matrix is the Jacobian matrix for three-phase power flow with a single slack bus.

\[ \text{Update Equations:} \]

\[
\begin{bmatrix}
K_n & \frac{\partial f_{p0}}{\partial \theta_i} & \ldots & \frac{\partial f_{p0}}{\partial \theta_{n+1}} & \ldots & \frac{\partial f_{p0}}{\partial \theta_{m+1}} & \ldots & \frac{\partial f_{p0}}{\partial \theta_{m+1}} & \ldots & \frac{\partial f_{p0}}{\partial \theta_{m+1}} \\
& \frac{\partial f_{p1}}{\partial \theta_i} & \ldots & \frac{\partial f_{p1}}{\partial \theta_{n+1}} & \ldots & \frac{\partial f_{p1}}{\partial \theta_{m+1}} & \ldots & \frac{\partial f_{p1}}{\partial \theta_{m+1}} & \ldots & \frac{\partial f_{p1}}{\partial \theta_{m+1}} \\
& \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
& \frac{\partial f_{p_n}}{\partial \theta_i} & \ldots & \frac{\partial f_{p_n}}{\partial \theta_{n+1}} & \ldots & \frac{\partial f_{p_n}}{\partial \theta_{m+1}} & \ldots & \frac{\partial f_{p_n}}{\partial \theta_{m+1}} & \ldots & \frac{\partial f_{p_n}}{\partial \theta_{m+1}} \\
\end{bmatrix} \begin{bmatrix}
\frac{\partial \Delta P_{\text{loss}}}{\partial \theta_i} \\
\frac{\partial \Delta \theta_i}{\partial \theta_i} \\
\vdots \\
\frac{\partial \Delta \theta_{n+1}}{\partial \theta_i} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{\partial \Delta \theta_{m+1}}{\partial \theta_i} \\
\frac{\partial \Delta \theta_{m+1}}{\partial \theta_i} \\
\vdots \\
\frac{\partial \Delta \theta_{m+1}}{\partial \theta_i} \\
\end{bmatrix} \begin{bmatrix}
\frac{\partial \Delta V_{n+1}}{\partial \theta_i} \\
\frac{\partial \Delta V_{m+1}}{\partial \theta_i} \\
\vdots \\
\frac{\partial \Delta V_{m+1}}{\partial \theta_i} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{\partial \Delta V_{m+1}}{\partial \theta_i} \\
\frac{\partial \Delta V_{m+1}}{\partial \theta_i} \\
\vdots \\
\frac{\partial \Delta V_{m+1}}{\partial \theta_i} \\
\end{bmatrix}
\]
Simplified as

\[-F = J_e \cdot \Delta x\]  \hspace{1cm} (3.9)

where:

- \( J_e \): extended Jacobian matrix

3.3 Network-Based Participation Factors

Participation factors for distribution systems should reflect network parameters, load distribution, generator locations and capacities. Two methods to calculate such network-based participation factors will be discussed:

- network sensitivity participation factors
- generator domain participation factors

3.3.1 Network Sensitivity Participation Factors

The network sensitivity participation factors incorporate the concept of network sensitivities and penalty factors to distribute the slack. These participation factors implicitly include effects of network parameters and load distribution through the sensitivities of system real power loss to real power injections. Since the sensitivities can be negative, penalty factors are applied to keep participation factors nonnegative.

First, the sensitivities \( \frac{\partial P_{loss}}{\partial P_i} \), where \( P_{loss} \) represents real power loss and \( P_i \) represents the real power injection to bus \( i \), will be addressed. They were derived and can be computed at each power flow iteration [12]:

\[
\begin{bmatrix}
\frac{\partial P_{\text{loss}}}{\partial P} \\
\frac{\partial P_{\text{loss}}}{\partial Q} \\
\frac{\partial P_{\text{loss}}}{\partial \theta} \\
\frac{\partial P_{\text{loss}}}{\partial |V|}
\end{bmatrix} = \left[J^T\right]^{-1}
\begin{bmatrix}
\frac{\partial P_{\text{loss}}}{\partial P} \\
\frac{\partial \theta}{\partial P} \\
\frac{\partial P_{\text{loss}}}{\partial \theta} \\
\frac{\partial |V|}{\partial \theta}
\end{bmatrix}
\]  
(3.10)

where:

\( J \): Jacobian matrix for three-phase power flow with a single slack bus

Since R, X values of network components, voltage phase angles \( \theta \) and voltage magnitudes |\( V \)| are included in \( J \), the system network parameters, and load distribution are implicitly included in the sensitivities.

Nonnegative participation factors are desired. However, \( \frac{\partial P_{\text{loss}}}{\partial P_i} \) can be negative. It is noted that in economic dispatch [12, 26] with line loss considerations, penalty factors, \( \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \), were derived through the method of Lagrange multipliers. These penalty factors based on sensitivities are nonnegative, and reflect the impact of transmission system loss to real power injections from units, which are dispersed throughout the system. Therefore, these penalty factors are introduced here to obtain nonnegative participation factors.

In addition, since unbalanced systems are considered, phase sensitivities on the same bus could be different. Therefore, the average phase sensitivity or maximum phase sensitivity can be utilized. In addition, for a single slack bus model, the system loss is independent of the power injection of the reference bus, whose penalty factor is set as one. Thus, the penalty factors \( L_i \) are defined as:

\[ L_0 = 1 \]  for the reference bus

A. Based on average phase sensitivity
\[ L_i = \frac{1}{1 - \frac{1}{3} \left( \frac{\partial P_{Loss}}{\partial P_{Gi}^a} + \frac{\partial P_{Loss}}{\partial P_{Gi}^b} + \frac{\partial P_{Loss}}{\partial P_{Gi}^c} \right)} \quad i = 1, 2, \ldots m \]  

(3.11)

**B. Based on maximum phase sensitivity**

\[ L_0 = 1 \quad \text{for the reference bus} \]

\[ L_i = \frac{1}{1 - \text{Max} \left( \frac{\partial P_{Loss}}{\partial P_{Gi}^a}, \frac{\partial P_{Loss}}{\partial P_{Gi}^b}, \frac{\partial P_{Loss}}{\partial P_{Gi}^c} \right)} \quad i = 1, 2, \ldots m \]  

(3.12)

In (3.11) and (3.12), all penalty factors are nonnegative. At first glance, the sensitivity values are not necessarily nonnegative; however, when calculating in per unit with realistic power distribution components, the sensitivity values are less than one, which results in nonnegative \( L_i \).

These penalty factors also capture DGs’ effects to system loss through sensitivities: when a participating source is installed far from load centers, more loss occurs on the path to serve the same amount of load from this source; then, its sensitivity should be larger than the sources, who are installed closer to load centers. In other words, a larger sensitivity value results in a larger penalty factor.

In addition, since sensitivities or these penalty factors only represent the ratios of system real power loss changes, the associated real power load served by each participating source, \( P_{Gi}^{load} \), should also need to be included in its participation factor to scale its associated real power loss. Therefore, network sensitivity participation factors applied penalty factors are determined as following:

\[ K_i = \frac{L_i P_{Gi}^{load}}{\sum_{j=0}^{m} L_j P_{Gi}^{load}} \quad i = 0, 1, 2 \ldots m \]  

(3.13)
where:

\[ P_{Gi}^{\text{load}} : \text{real power load associated with generator } i \]

Since \( J \) changes at each iteration, \( L_i \) and the participation factors are iterative. The real power load associated with generator \( i \), \( P_{Gi}^{\text{load}} \) is a set value before power flow calculations, which can be considered as generator \( i \)'s scheduled output to serve a desired amount of load. In fact, \( L_i P_{Gi}^{\text{load}} \) in (3.13) is used to represent the loss contribution of participating source \( i \) and to determine the loss contribution ratio for (3.2). Another way to find the loss contribution of each participating source will be discussed in the following subsection.

### 3.3.2 Generator Domain Participation Factors

Participation factors based on the concept of generator domains are now discussed. The concept of multi-phase generator domains strives to distinguish the loss and load associated with each participating source. As such, an associated loss with each participating source can be quantified. The effects of network parameters, load distributions and generator capacities are explicitly included in these participation factors.

The generator domain participation factors are defined as follows:

\[
K_i = \frac{P_{Gi}^{\text{loss}}}{P_{\text{Loss}}} \quad i = 0,1,2,\cdots m \tag{3.14}
\]

where:

\[
P_{Gi}^{\text{loss}} = P_{Gi}^{\text{loss,a}} + P_{Gi}^{\text{loss,b}} + P_{Gi}^{\text{loss,c}} \tag{3.15}
\]

and
In the distributed slack bus model, the real power outputs of participating sources are iterative. Generator domains and loss contributions vary with changing source injections. Thus, the participation factors are iterative during power flow calculations. The process for determining three-phase generator domains will be presented in the following subsections. First, the concept of three-phase generator domains will be discussed.

### 3.3.2.1 Three-Phase Generator Domains

The concept of generator domains and commons originates from a transmission system approach in [8]. Each generator’s contribution to loads and losses can be distinguished using generator domains and commons. Generator domains and commons were determined by post processing a power flow solution or from available system measurements.

This thesis will adapt the transmission based concepts of generator domains and commons to distribution systems. Since the loads and network are unbalanced in distribution systems, the buses and branch flows supplied by the same source may be different across phases. Thus, to emphasize and clarify individual phases to capture unbalanced situations encountered in distribution systems, generator domains will be extended to multi-phase generator domains in this thesis. Since there are some regions of the network that cannot be assigned to just one generator, the concept of generator commons will also be revisited. Finally, this thesis will also determine generator domains and commons through an interactive process within power flow analysis.

For each generator, and a given $P_{Gi}$,
\[ P_{Gi} = P_{load}^{Gi} + P_{loss}^{Gi} \quad i = 0, 1, 2, \cdots m \]  

where:

\[ P_{load}^{Gi} = P_{load,a}^{Gi} + P_{load,b}^{Gi} + P_{load,c}^{Gi} \]  

\[ P_{loss}^{Gi} = P_{loss,a}^{Gi} + P_{loss,b}^{Gi} + P_{loss,c}^{Gi} \]

and

\( P_{load}^{Gi} \): real power load associated with generator \( i \)

\( P_{load,p}^{Gi} \): real power load associated with generator \( i \), phase \( p \)

The three-phase domain of a generator is defined as the set of nodes and branches by phase, whose power is supplied by this generator. With each node (bus and phase) of interest, generator domains vary for each phase and are assigned based on:

- positive power flow direction
- proportionality of common areas

Specifically, a positive power flow direction is defined and used to “trace” the power back to a generator or substation and to allocate loads to several sources for common areas.

**A. Positive Power Flow Direction**

The positive power flow direction will be used to assign a directed graph onto the distribution system. It is defined in the following manner: for two directly connected buses, bus \( i \) and bus \( j \),

- If \( \text{Re}(V_i^p I_y^p) - \text{Re}(V_j^p I_y^p) > 0 \), we define that real power flows from bus \( i \) to bus \( j \) over phase \( p \);
• If $\text{Im}(V_i^p I_j^p) - \text{Im}(V_j^p I_i^p) > 0$, we define that reactive power flows from bus $i$ to bus $j$ over phase $p$.

where:

$V_i^p$: the complex voltage on bus $i$ in phase $p$

$I_j^p$: the complex current from bus $i$ to bus $j$ over phase $p$

By using on-line measurements or a base power flow analysis, for example a power flow with a single slack bus, voltages and currents can be estimated. Then, the power flow directions and the power injected to buses can be determined.

The positive real power flows and positive reactive power flows may be different. Since we are interested in the real power supply and real power loss, we use positive real power flow directions to trace sources. Based on positive real power flow directions, the concept of a generator common for unbalanced systems is now discussed.

**B. Generator Commons**

The loss on a branch or the load on a single node may be supplied by different sources. Therefore, if the domains of different generators intersect in phase, they would have the branch or load in common. Therefore, the definition of a generator common is modified to be a set of contiguous nodes and branches by phase, whose power is supplied by the same generators.

A proportionality assumption is applied to distinguish each generator’s loss and load contribution within commons. It assumes that the proportion of loss and loads supplied by different sources to a common is the same as the proportion of the positive real power injected by the sources to this common. By applying this assumption, the proportion of loads and losses of a common are assigned to the corresponding generator domains.
3.3.2.2 An Example for Illustration

A source’s generator domain consists of commons. If a portion of the network were supplied only by one source, this generator common would belong to one source; if a portion of the network were supplied by several sources together, this common would be shared by its supplying sources. Thus, commons will be first distinguished; then, generator domains will be found.

A 6-bus unbalanced system with two participating sources is used as an example to illustrate how to find generator domains/commons and how to distinguish the loss and load contribution for each source. The example system is shown in Fig. 3.1 and phase $a$ is selected for demonstration.

The arrows in Fig. 3.1 represent positive, real power flow directions. From these directions, three commons on phase $a$ can be identified. Common 1, represented by the dashed curve, is assigned to the substation only. Common 2, represented by the dotted curve, is assigned to the DG only. Common 3, represented by the dot-dashed curve, is the
remaining portion of the phase a network and is proportionally assigned to both the substation and the DG. A directed graph for phase \( a \) is shown in Figure 3.2.

![Directed graph for phase \( a \)](image)

Figure 3.2: Directed graph for phase \( a \)

The real power injected into Bus 5, phase \( a \) from Branch 2-5 and from Branch 4-5 are \( P_{2-5}^a \) and \( P_{4-5}^a \). Then, the total real power injected into Common 3 is \( P_{2-5}^a + P_{4-5}^a \). Applying the proportionality assumption, \( \alpha = \frac{P_{2-5}^a}{P_{2-5}^a + P_{4-5}^a} \) and \( \beta = \frac{P_{4-5}^a}{P_{2-5}^a + P_{4-5}^a} \) are the ratios of the total real power supplied by the substation and the DG, respectively, into Common 3. Then,

\[
\begin{align*}
P_{G0}^{loss,a} &= P_{com1}^{loss,a} + \alpha P_{com3}^{loss,a} \\
P_{G0}^{load,a} &= P_{com1}^{load,a} + \alpha P_{com3}^{load,a} \\
P_{G1}^{loss,a} &= P_{com2}^{loss,a} + \beta P_{com3}^{loss,a} \\
P_{G1}^{load,a} &= P_{com2}^{load,a} + \beta P_{com3}^{load,a}
\end{align*}
\]

where:

\( P_{G0}^{loss,a} \): real power loss associated with the substation, phase \( a \)
Then, the generator domains of phase $a$ are then formed as:

- The substation: Common 1 and Common 3
- The DG: Common 2 and Common 3

In the same way, phase $b$ and $c$ can be analyzed. Then, all the real power loads and losses in the network are assigned to individual generators using directed graphs. As such, each $P_{Gi}^{loss}$ can be computed and, subsequently, the proposed participation factors (3.14) can be determined for each source.

With the presented methods for distributed slack bus models, the following section will discuss their solution algorithms for power flow solvers.

### 3.4 Solution Algorithm

A Newton-Raphson solver incorporating the distributed slack model with iterative participation factors is used. This algorithm works for both network sensitivity and generator domain participation factors. The steps of the algorithm are as follows:

**Step 1.** Choose an initial guess $X^{(0)}$

**Step 2.** Set the iteration counter $k = 0$ 

**Step 3.** Set desired $P_{Gi}^{load}$ and initial $K_i$: 

\[ P_{load,a}^{G0} : \text{real power load associated with the substation, phase } a \]

\[ P_{loss,a}^{G1} : \text{real power loss associated with the DG, phase } a \]

\[ P_{load,a}^{G1} : \text{real power load associated with the DG, phase } a \]

\[ P_{loss,a}^{comi} : \text{total real power loss in Common } i, \text{phase } a \]

\[ P_{load,a}^{comi} : \text{total real power load in Common } i, \text{phase } a \]
- For each participating DG \( i \): \( P_{Gi}^{\text{load}} < P_{Gi}^{\text{rated}} \), \( K_i^{(k)} \leq 1 \)

- For the substation: \( P_{G0}^{\text{load}} = \sum_{i=0}^{m} \sum_{p=a}^{n} P_{Di}^p - \sum_{i=1}^{m} P_{Gi}^{\text{load}} \), \( K_0^{(k)} \leq 1 \)

Step 4. Evaluate \( F^{(k)}(x^{(k)}) \)

Step 5. Stop if \( \|F^{(k)}\| \leq \text{tolerance} \)

Step 6. Evaluate \( J_e^{(k)} = \frac{\partial F}{\partial x} \bigg|_{x=x^{(k)}} \)

Step 7. Solve \( J_e^{(k)} \Delta x^{(k)} = -F^{(k)} \)

Step 8. Let \( x^{(k+1)} = x^{(k)} + \Delta x^{(k)} \)

Step 9. Let \( k = k + 1 \)

Step 10. Check real and reactive power limits of participating DGs: If the calculated real/reactive power output of a DG violated its limits, this DG can not be considered as a participating source which accounts for slack and is modeled as a constant PQ injection. Then, go to Step 3

Step 11. Upgrade calculation information

- For sensitivity participation factors: calculate sensitivities
- For generator domain participation factors: find positive power flow directions and distinguish generator domains for the substation and participating DGs

Step 12. Calculate participation factors \( K_i^{(k)} \) and \( K_0^{(k)} \), and go to Step 4

The flow chart of this solution algorithm is shown in Figure 3.3.
Figure 3.3: Flow chart of the solution algorithm
It is also noted for this solution algorithm that initial participation factors are not dependent on a base power flow solution, e.g. they could be set based on generator domain information without considering losses.

The above models and algorithms will be demonstrated in the following simulation sections. In Section 3.5, three-phase power flow results obtained using different slack bus models will be studied using 20-bus test systems. In Section 3.6, different numbers of DGs and different levels of DG penetration within large scale distribution systems will be studied and the post processed method to separate loss and load contribution is included for comparison.

3.5 Numerical Studies of Power Flow with Different Slack Bus Models

In this section, different slack bus models will be applied to unbalanced 20-bus test systems for simulations. Then, these models will be compared and evaluated according to numerical results.

3.5.1 20-Bus Test System Cases

A 20-bus test distribution system with total system loads of 6.0451 MW and 3.2724 Mvar is shown in Figure 3.4 below. If no DG is installed, the total system real power loss, \( P_{\text{Loss}} \), is 226.23 kW or 3.74%. In order to test whether and how participation factors reflect network parameters, the 20-bus network represents a portion of an existing power distribution system with real network parameters. It can be considered as having two different portions. The transformer between Bus 2 and Bus 3 services 1.6669 MW and 0.9626 Mvar high density loads. The transformer between Bus 2 and Bus 4 services
4.3782 MW and 2.30975 Mvar dispersed loads in a commercial and residential area. With no DG installed 3.51 kW (1.56% of total system real power loss) occurs in the high density load area and 204.59 kW (90.43% of the total loss) occurred in the commercial and residential area from its higher network resistances and branch currents.

Figure 3.4: The one-line diagram of the 20-bus test system

Two cases will be investigated. In each case, simulation results from three-phase power flow analysis using different slack bus models will be compared. They are as follows:

- **Case 1**: the DG is installed on Bus 3
- **Case 2**: the DG is installed on Bus 4

In both cases, one DG is assumed to service 1,500kW loads (that is $P_{DG}^{load} = 1,500$ kW, approximately 25% DG penetration.). The DG installed on Bus 4 is expected to have a larger impact on system real power loss and a larger percentage of system loss contribution. Thus, it should be assigned a larger participation factor than the DG installed on Bus 3 to serve the same amount of real power loads.
3.5.2 Different Slack Bus Models

Four different slack bus models for three-phase power flow are summarized as following:

A. *Three-phase power flow with a single slack bus* [14]

This model assumes the substation has participation factor 1, absorbing all system loss; and the DG has participation factor 0.

\[
K_0 = 1 \quad \text{for the reference bus}
\]

\[
K_i = 0 \quad i = 1, 2, \cdots m
\]

In this model, these participation factors do not reflect the fact that each source contributes to the load and loss at the same time.

B. *Three-phase power flow with distributed slack bus based on capacities*

This model considers that all sources absorb part of the loss proportional to their scheduled real power outputs, \(P_{Gi}^{sch}\), and \(P_{Gi}^{sch} = P_{Gi}^{load}\) [4]:

\[
K_i = \frac{P_{Gi}^{sch}}{\sum_{j=0}^{m} P_{Gj}^{sch}} \quad i = 0, 1, 2, \cdots m
\]

In this model, a DG has the same participation factor regardless of its location in the system. This model does not include the network parameters which affect the loss contributions.

C. *Three-phase power flow with distributed slack bus based on average phase sensitivity*

This model was proposed in Section 3.3, and its participation factors were discussed in Section 3.3.1 A.:
\[ K_i = \frac{L_i P_{G_i}^{load}}{\sum_{j=0}^{m} L_j P_{G_j}^{load}} \quad i = 0,1,\cdots m \]

where:

\[ L_0 = 1 \quad \text{for the reference bus} \]

\[ L_i = \frac{1}{1 - \frac{1}{3} \left( \frac{\partial P_{Loss}^a}{\partial P_{G_i}} + \frac{\partial P_{Loss}^b}{\partial P_{G_i}} + \frac{\partial P_{Loss}^c}{\partial P_{G_i}} \right)} \quad i = 1, 2, \cdots m \]

In this model, the participation factors for DGs are different at different locations based on their average phase sensitivity. The participation factors reflect the network parameters and represent loss contributions for each source.

D. Three-phase power flow with distributed slack bus based on maximum phase sensitivity

This model was proposed in this dissertation, and its participation factors were discussed in Section 3.3.1 B.:

\[ K_i = \frac{L_i P_{G_i}^{load}}{\sum_{j=0}^{m} L_j P_{G_j}^{load}} \quad i = 0, 1, \cdots m \]

where:

\[ L_0 = 1 \quad \text{for the reference bus} \]

\[ L_i = \frac{1}{1 - \text{Max} \left( \frac{\partial P_{Loss}^a}{\partial P_{G_i}}, \frac{\partial P_{Loss}^b}{\partial P_{G_i}}, \frac{\partial P_{Loss}^c}{\partial P_{G_i}} \right)} \quad i = 1, 2, \cdots m \]

In this model, the participation factors for DGs are different at different locations based on the maximum phase sensitivity. The participation factors reflect the network parameters and represent loss contributions for each source.

E. Three-phase power flow with distributed slack bus based on generator domains
This model was proposed in Section 3.3, and its participation factors were discussed in Section 3.3.2.

\[
K_i = \frac{P_{Gi}}{P_{Loss}} \quad i = 0,1,2,\ldots,m
\]

In this model, the participation factors for DGs are different at different locations. The participation factors reflect the network parameters and represent loss contributions for each source.

3.5.3 Simulation Results of the 20-Bus Test Systems

The voltage profiles for DG installations are shown in figure 3.5. It is can be observed that DG installed on Bus 4 has bigger impacts on system voltage profile than DG installed on Bus 3. Simulation results including the participation factors and real power outputs obtained using different slack bus models of the 20-bus systems are shown in Table 3.5.1 and Table 3.5.2 for Case 1 and Case 2 respectively.

![Figure 3.5: Voltage Profile](image_url)
Table 3.5.1 Participation factors and real power outputs using different slack bus models

Case 1: 20-bus system, one DG on Bus 3 to service 1,500kW load

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub.Par. $K_s$</td>
<td>1</td>
<td>0.7519</td>
<td>0.7633</td>
<td>0.7557</td>
<td>0.9861</td>
</tr>
<tr>
<td>DG Par. $K_1$</td>
<td>0</td>
<td>0.2481</td>
<td>0.2367</td>
<td>0.2443</td>
<td>0.0139</td>
</tr>
<tr>
<td>$P_{sub}^{gen}$ (kW)</td>
<td>4769.31</td>
<td>4713.66</td>
<td>4713.66</td>
<td>4713.66</td>
<td>4766.20</td>
</tr>
<tr>
<td>$P_{DG}^{gen}$ (kW)</td>
<td>1500.00</td>
<td>1555.64</td>
<td>1555.64</td>
<td>1555.64</td>
<td>1503.12</td>
</tr>
<tr>
<td>$P_{loss}^{gen}$ (kW)</td>
<td>224.233</td>
<td>224.212</td>
<td>224.212</td>
<td>224.212</td>
<td>224.231</td>
</tr>
</tbody>
</table>

Table 3.5.2 Participation factors and real power outputs using different slack bus models

Case 2: 20-bus system, one DG on Bus 4 to service 1,500kW load

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub.Par. $K_s$</td>
<td>1</td>
<td>0.7519</td>
<td>0.7468</td>
<td>0.7497</td>
<td>0.6749</td>
</tr>
<tr>
<td>DG Par. $K_1$</td>
<td>0</td>
<td>0.2481</td>
<td>0.2532</td>
<td>0.2503</td>
<td>0.3251</td>
</tr>
<tr>
<td>$P_{sub}^{gen}$ (kW)</td>
<td>4752.05</td>
<td>4700.55</td>
<td>4700.55</td>
<td>4700.55</td>
<td>4684.59</td>
</tr>
<tr>
<td>$P_{DG}^{gen}$ (kW)</td>
<td>1500.00</td>
<td>1551.31</td>
<td>1551.31</td>
<td>1551.31</td>
<td>1567.21</td>
</tr>
<tr>
<td>$P_{loss}^{gen}$ (kW)</td>
<td>206.971</td>
<td>206.781</td>
<td>206.781</td>
<td>206.781</td>
<td>206.723</td>
</tr>
</tbody>
</table>
Figure 3.6: Participation factor comparisons for the 20-bus systems

Figure 3.7: DG real power output comparisons for 20 bus systems
From these numerical studies show, the impacts of different slack bus models were observed, and the following comments are made:

- For the single slack bus model, both cases keep the DG at the same output $P_{DG}^{out} = 1,500\text{kW}$

- The distributed slack bus model with non-iterative participation factors based on scheduled generator outputs alone has the same participation factor values in both cases. Thus, with the same DG output, the amount of the output attributed to loads versus losses from (3.2) would be the same even though the DG is located at different locations. Since this method does not capture the effects of DG locations on system studies, it is not recommended.

- The distributed slack bus model with sensitivity participation factors were computed in two ways: based on average sensitivities and maximum phase sensitivities. The resulting participation factors were slightly different between these two methods. It is noted that both methods assigned larger participation factors to the DG on Bus 4 than when the DG was placed on Bus 3. Thus the sensitivity and penalty factor approach performed, as expected, with respect to attributing higher losses to the DG at bus 4. However, the difference in
participation factors between the DG at bus 3 vs. bus 4 was small. Thus, concerns arise as to whether sensitivity measures are significant enough to fully capture the effects of DG locations.

- In contrast, the distributed slack bus model with generator domain participation factors has a much larger participation factor for the DG on Bus 4 than the DG on Bus 3 (0.3251 in Case 2 vs. 0.0139 in Case 1). This demonstrates that explicitly relating the participation factors with generator locations, network parameters and load distribution yield more distinct distributed slack bus participation factors.

Therefore, the participation factors determined by generator domains are recommended for the distributed slack bus model. The following section will do further numerical studies using this recommended slack bus model.

### 3.6 Numerical Studies of Power Flow Under Different Levels of DG Penetration

In this section, the distributed slack bus model with generator domain participation factors will be applied to numerical studies for distribution systems with different numbers of DGs and different levels of DG penetration.

#### 3.6.1 394-Bus Test System Cases

The test network is a 394-bus, unbalanced radial network. Its one line diagram is shown in Figure 3.7. The total loads of the system are 26.96MW and 9.61Mvar. All loads are constant $PQ$ loads in these simulations. The total nominal loads on each phase are

- Phase $a$: 8.99MW, 3.30Mvar
- Phase $b$: 8.95MW, 3.29Mvar
- Phase $c$: 9.02MW, 3.02Mvar
Figure 3.8: The one-line diagram of the 394-bus unbalanced test system

The network-based distributed slack bus model is applied to study the effects of different DG penetration levels to distribution power systems. The DG penetration corresponds to the percentage of total system loads supplied by DGs.

Up to four participating DGs will be used in this section. The DG limits are decided by their rated outputs for continuous power application. In the simulation, it assumes:

- DG1, DG2 and DG3 have rated outputs 2.4 MW
- DG4 has rated outputs 3 MW
Convergence tolerance for both power and voltage magnitude mismatches are set to 10e-8. Five simulation cases using different methods to compute each source’s loss and load contributions will be analyzed.

### 3.6.2 Different Methods of Computing Load and Loss Contributions

Based on different calculation methods or models, the loss contribution of a generator would be different. Traditionally, outputs of DGs are treated to supply loads, and all system loss is supplied by the substation in distribution systems. In [8], a method of using generator domains based on a power flow solution was proposed to separate load and loss contributions of each generator source for balanced transmission systems. This method will be referred to as post processed generator domains in this thesis, and will be applied for comparison. Power flow analysis based on slack bus model will result in different generator domains. Therefore, three different methods of computing load and loss contributions of DGs’ outputs will be applied to the 394-bus systems:

- a single slack bus model (traditional method)
- a single slack bus model with post processed generator domains to separate $P_{Gi}^{load}$ and $P_{Gi}^{loss}$ based on [8]
- the recommended distributed slack bus model with participation factors based on generator domains

Figure 3.9 shows three different methods of computing load and loss contributions of DGs’ outputs, called Treatment 1, 2 and 3.
Fig. 3.9: Three different treatments used in comparative simulations

### 3.6.3 Simulation Results

Five simulation cases for 394 bus systems with different numbers of DGs and different levels of DG penetration will be discussed:

- **Case 1**: 5% DG penetration with one DG
- **Case 2**: 10% DG penetration with two DGs
- **Case 3**: 15% DG penetration with two DGs
- **Case 4**: 20% DG penetration with three DGs
- **Case 5**: 30% DG penetration with four DGs

Their simulation results of different treatments to account DGs’ load and loss contributions based are now be presented.
**Case 1: 5% DG penetration with one DG**

One DG (DG1) is sited at Bus 59 with initial real power output, 1.35MW. At the last iteration, the participation factors of the proposed algorithm are $K_0 = 0.9729$ and $K_1 = 0.0271$. As expected, the participation factor for the substation is close to 1, as the substation absorbs most of the slack.

Table 3.6.1 shows simulation results of using the three treatments of losses. The per-phase real power outputs of the DG and the substation in this table are calculated by $\Re\{\sqrt{p_G (i_G^p)^*}\}$ from power flow results. The simulation results of using a traditional power flow and using a single slack bus model with post processed generator domains are the same except for the DG’s power to loads.

From the results calculated with a single slack bus, it can be observed that the total DG output from power flow calculation is the same as its specified output, 1.35 MW. This shows that the DG model with VSI connection is successfully implemented into the three-phase power flow.

In Table 3.6.1, for Treatment 1, the real power output of the DG is modeled to supply load, $P_{G1}^{\text{load}} = P_{G1} = 1.35$ MW, then the substation absorbs all the system loss. For Treatments 2 and 3, the loads supplied by the DG are calculated by $P_{G1}^{\text{load}} = P_{G1}^{\text{load},a} + P_{G1}^{\text{load},b} + P_{G1}^{\text{load},c}$, determined from generator domains. From the power flow with a distributed slack bus, DG1 must produce 1.389 MW in order to service 1.35 MW of loads, therefore, if a single slack bus model is used, DG1 actually services only 1.312 MW of loads, 2.8% less than with the distributed slack bus model.
Table 3.6.1 Simulation results of different treatments to compute DG contributions,

Case 1: 5% DG penetration with one DG

<table>
<thead>
<tr>
<th>Real Power Unit: MW</th>
<th>Single Slack Bus</th>
<th>Dist. Slack Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Treatment 1</td>
<td>Treatment 2</td>
</tr>
<tr>
<td></td>
<td>Traditional</td>
<td>With Gen. Dom</td>
</tr>
<tr>
<td>Substation Outputs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A 8.93460583</td>
<td>8.93460583</td>
<td>8.92102587</td>
</tr>
<tr>
<td>C 9.04599416</td>
<td>9.04599416</td>
<td>9.03244545</td>
</tr>
<tr>
<td>Total</td>
<td>27.04589829</td>
<td>27.04589829</td>
</tr>
<tr>
<td>DG1 Outputs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A 0.44398866</td>
<td>0.44398866</td>
<td>0.4690910</td>
</tr>
<tr>
<td>B 0.45039369</td>
<td>0.45039369</td>
<td>0.46332730</td>
</tr>
<tr>
<td>C 0.45561765</td>
<td>0.45561765</td>
<td>0.46853425</td>
</tr>
<tr>
<td>Total</td>
<td>1.35000000</td>
<td>1.35000000</td>
</tr>
<tr>
<td>Load</td>
<td>1.35000000</td>
<td>1.31231218</td>
</tr>
<tr>
<td>Total Loss</td>
<td>1.43310828</td>
<td>1.43310828</td>
</tr>
<tr>
<td>Iteration No.</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

This case demonstrates that the generator domain concept and the resulting participation factors can effectively distribute real power loss. From Treatment 3, the real power output $1.389\,\text{MW}$ ($P_{G1} = P_{G1}^{\text{load}} + P_{G1}^{\text{loss}}$) and the corresponding generator domain yields $P_{G1}^{\text{load}} = 1.35\,\text{MW}$, the same as the desired set value. This means that $K_1 P_{\text{Loss}}$ does represent the loss contributed by the DG1.

Under restructuring, DG owners could be different and accounting for wheeling costs and/or system losses requires improved models for the slack bus. As such, the distributed slack bus model with network-based generator domain participation factors can do this and possibly help DGs design output control schemes based on the load they wish to supply while compensating for the associated losses. The loss contributions of sources from the different treatments are shown in the Table 3.6.2 below.
Table 3.6.2 Ratios of loss contributions

<table>
<thead>
<tr>
<th>Sources/Models</th>
<th>Treatment 1 Traditional</th>
<th>Treatment 2 With Gen. Dom</th>
<th>Treatment 3 Par. Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substation</td>
<td>1</td>
<td>0.9737</td>
<td>0.9729</td>
</tr>
<tr>
<td>DG1</td>
<td>0</td>
<td>0.0263</td>
<td>0.0271</td>
</tr>
</tbody>
</table>

Case 2: 10% DG penetration with two DGs

There are two participating DGs in this case. DG1 and DG2 are sited at Bus 59 and Bus 120, respectively. Both DGs have the same initial real power outputs, 1.35MW. The participation factors of the last iteration for the distributed slack model are $K_0 = 0.9636$, $K_1 = 0.0295$ and $K_2 = 0.0069$. Simulation results of using different methods to compute DGs’ load and loss contributions are in Table 3.6.3.

Table 3.6.3 Simulation results of different treatments to compute DG contributions,

<table>
<thead>
<tr>
<th>Real Power Unit: MW</th>
<th>Single Slack Bus</th>
<th>Dist. Slack Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Treatment 1</td>
<td>Treatment 2</td>
</tr>
<tr>
<td>Substation Outputs</td>
<td></td>
<td>With Gen. Dom</td>
</tr>
<tr>
<td>A</td>
<td>8.45910506</td>
<td>8.45910506</td>
</tr>
<tr>
<td>B</td>
<td>8.58682137</td>
<td>8.58682137</td>
</tr>
<tr>
<td>C</td>
<td>8.53294415</td>
<td>8.53294415</td>
</tr>
<tr>
<td>Total</td>
<td>25.57887058</td>
<td>25.57887058</td>
</tr>
<tr>
<td>DG1 Outputs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.44434077</td>
<td>0.44434077</td>
</tr>
<tr>
<td>B</td>
<td>0.45181644</td>
<td>0.45181644</td>
</tr>
<tr>
<td>C</td>
<td>0.45384279</td>
<td>0.45384279</td>
</tr>
<tr>
<td>Total</td>
<td>1.35000000</td>
<td>1.35000000</td>
</tr>
<tr>
<td>DG2 Outputs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.45239832</td>
<td>0.45239832</td>
</tr>
<tr>
<td>B</td>
<td>0.42915535</td>
<td>0.42915535</td>
</tr>
<tr>
<td>C</td>
<td>0.46844633</td>
<td>0.46844633</td>
</tr>
<tr>
<td>Total</td>
<td>1.35000000</td>
<td>1.35000000</td>
</tr>
<tr>
<td>Load</td>
<td>1.35000000</td>
<td>1.31231774</td>
</tr>
<tr>
<td>A</td>
<td>0.45239832</td>
<td>0.45239832</td>
</tr>
<tr>
<td>B</td>
<td>0.42915535</td>
<td>0.42915535</td>
</tr>
<tr>
<td>C</td>
<td>0.46844633</td>
<td>0.46844633</td>
</tr>
<tr>
<td>Total</td>
<td>1.35000000</td>
<td>1.31608058</td>
</tr>
<tr>
<td>Total Loss</td>
<td>1.31608058</td>
<td>1.31608058</td>
</tr>
<tr>
<td>Iteration No.</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>
This case shows that the two DGs supplying the same amount of real power load have different participation factors. This denotes that the loss contributed by each source is not only related to the outputs of generators but also related to their locations and the network parameters. This also suggests that it is better to consider more than just the scheduled generator outputs as the participation factors for distribution systems.

**Case 3: 15% DG penetration with two DGs**

There are still two participating DGs in this case. DG1 and DG2 are sited at Bus 59 and Bus 120. Both DGs have initial real power output 2MW. The participation factors at the last iteration are $K_0 = 0.9422$, $K_1 = 0.0469$ and $K_2 = 0.0109$. Table 3.6.4 shows the simulation results of using different methods to compute DGs’ load and loss contributions.

### Table 3.6.4 Simulation results of different treatments to compute DG contributions,

<table>
<thead>
<tr>
<th>Substation Outputs</th>
<th>Real Power Unit: MW</th>
<th>Single Slack Bus</th>
<th>Dist. Slack Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Treatment 1</td>
<td>Treatment 3</td>
<td>Treatment 3</td>
</tr>
<tr>
<td></td>
<td>Traditional</td>
<td>With Gen. Dom</td>
<td>Par. Factors</td>
</tr>
<tr>
<td>A</td>
<td>7.99568636</td>
<td>7.99568636</td>
<td>7.97099402</td>
</tr>
<tr>
<td>B</td>
<td>8.12347222</td>
<td>8.12347222</td>
<td>8.09880418</td>
</tr>
<tr>
<td>C</td>
<td>8.07004414</td>
<td>8.07004414</td>
<td>8.04539564</td>
</tr>
<tr>
<td>Total</td>
<td>24.18920272</td>
<td>24.18920272</td>
<td>24.11519474</td>
</tr>
<tr>
<td>DG1 Outputs</td>
<td>A 0.66098178</td>
<td>0.66098178</td>
<td>0.68012382</td>
</tr>
<tr>
<td></td>
<td>B 0.66863437</td>
<td>0.66863437</td>
<td>0.68779275</td>
</tr>
<tr>
<td></td>
<td>C 0.67038385</td>
<td>0.67038385</td>
<td>0.68951708</td>
</tr>
<tr>
<td>Total</td>
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<td>2.00000000</td>
<td>2.05743365</td>
</tr>
<tr>
<td>Load</td>
<td>2.00000000</td>
<td>1.94417015</td>
<td>2.00000029</td>
</tr>
<tr>
<td>DG2 Outputs</td>
<td>A 0.66898505</td>
<td>0.66898505</td>
<td>0.67342304</td>
</tr>
<tr>
<td></td>
<td>B 0.64592649</td>
<td>0.64592649</td>
<td>0.65036811</td>
</tr>
<tr>
<td></td>
<td>C 0.68508846</td>
<td>0.68508846</td>
<td>0.68952952</td>
</tr>
<tr>
<td>Total</td>
<td>2.00000000</td>
<td>2.00000000</td>
<td>2.01332067</td>
</tr>
<tr>
<td>Load</td>
<td>2.00000000</td>
<td>1.98676749</td>
<td>2.00000000</td>
</tr>
<tr>
<td>Total Loss</td>
<td>1.22641272</td>
<td>1.22641272</td>
<td>1.2235907</td>
</tr>
<tr>
<td>Iteration No.</td>
<td>5</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>
As expected, the DGs’ participation factors are larger according to their higher level of real power outputs compared to Case 2.

**Case 4: 20% DG penetration with three DGs**

There are three participating DGs in this case. DG1 and DG2 have the same locations and initial real power outputs as Case 2. DG3 is sited at Bus 262 with initial real power output 1.35MW. The participation factors at the last iteration are $K_0 = 0.9440$, $K_1 = 0.0345$, $K_2 = 0.0080$ and $K_3 = 0.0135$. Table 3.6.5 shows the simulation results of using different methods to compute DGs’ load and loss contributions.

| Table 3.6.5 Simulation results of different treatments to compute DG contributions, Case 4: 20% DG penetration with three DGs |
|---|---|---|---|
| | Real Power Unit: MW | Single Slack Bus | Dist. Slack Bus |
| | | Treatment 1 Traditional | Treatment 2 With Gen. Dom | Treatment 3 Par. Factors |
| Substation Outputs | A | 7.96371941 | 7.96371941 | 7.94158113 |
| | B | 8.07202108 | 8.07202108 | 8.04989871 |
| | C | 8.00266155 | 8.00266155 | 7.98054886 |
| | Total | 24.03840204 | 24.03840204 | 23.97202870 |
| DG1 Outputs | A | 0.44526952 | 0.44526952 | 0.45818961 |
| | B | 0.45244437 | 0.45244437 | 0.46537383 |
| | C | 0.45228611 | 0.45228611 | 0.46519939 |
| | Total | 1.35000000 | 1.35000000 | 1.3876283 |
| | Load | 1.35000000 | 1.31231957 | 1.35000017 |
| DG2 Outputs | A | 0.45269657 | 0.45269657 | 0.45569417 |
| | B | 0.44265491 | 0.44265491 | 0.44562536 |
| | C | 0.45464852 | 0.45464852 | 0.45764481 |
| | Total | 1.35000000 | 1.35000000 | 1.35899161 |
| | Load | 1.35000000 | 1.34106788 | 1.35000000 |
| DG3 Outputs | A | 0.45389486 | 0.45389486 | 0.45894137 |
| | B | 0.44461934 | 0.44461934 | 0.44966831 |
| | C | 0.45148579 | 0.45148579 | 0.45653178 |
| | Total | 1.35000000 | 1.35000000 | 1.36514146 |
| | Load | 1.35000000 | 1.33496399 | 1.35000001 |
| Total Loss | 1.12561204 | 1.12561204 | 1.12213461 |
| Iteration No. | 5 | 5 | 9 |
As expected, these results show that the participation factor of the substation is decreased as the DG penetration level is increased. The participation factors of DG1 and DG2 are varied, although DG1 and DG2 have the same locations as Case 2 and almost the same total real power outputs as Case 2. Adding DG3 causes these changes.

Also, it can be noted that the system loss has been decreasing from case to case. It is believed this behavior is exhibited because the DG buses were modeled with specified voltage magnitudes set to 1.0 p.u. This inherently provided voltage support, which reduced the system loss. Although many DGs may not have voltage control systems, it is still expected that DG installation helps to reduce system loss because, with the sources closer to the loads, less loss should be experienced across branches.

**Case 5: 30% DG penetration with four DGs**

There are four participating DGs in this case. DG1, DG2 and DG3 have the same locations and the same initial output values as Case 4. DG4 is sited at Bus 309 with 2.65MW initial real power output. The participation factors at the last iteration are $K_0 = 0.8867, K_1 = 0.0661, K_2 = 0.0153, K_3 = 0.0148$ and $K_4 = 0.0171$.

Table 3.6.6 Simulation results of different treatments to compute DG contributions,

**Case 5: 30% DG penetration with Four DGs**

<table>
<thead>
<tr>
<th>Real Power Unit: MW</th>
<th>Single Slack Bus</th>
<th>Dist. Slack Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Treatment 1 Traditional</td>
<td>Treatment 2 With Gen. Dom</td>
</tr>
<tr>
<td>Substation Outputs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>6.58144493</td>
<td>6.58144493</td>
</tr>
<tr>
<td>B</td>
<td>6.66342020</td>
<td>6.66342020</td>
</tr>
<tr>
<td>C</td>
<td>6.59109926</td>
<td>6.59109926</td>
</tr>
<tr>
<td>Total</td>
<td>19.83596439</td>
<td>19.83596439</td>
</tr>
<tr>
<td>DG1 Outputs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.66322073</td>
<td>0.66322073</td>
</tr>
<tr>
<td>B</td>
<td>0.66930185</td>
<td>0.66930185</td>
</tr>
<tr>
<td>C</td>
<td>0.66747742</td>
<td>0.66747742</td>
</tr>
<tr>
<td>Total</td>
<td>2.00000000</td>
<td>2.00000000</td>
</tr>
<tr>
<td>Load</td>
<td>2.00000000</td>
<td>1.94417109</td>
</tr>
</tbody>
</table>
In addition, comprehensive experiments on each case above showed that the power flow solution was invariant to the initial participation factors selected. Specifically, for each case, the power flow solutions obtained using various initial $K$’s were within $10^{-11}$ on both $|V|$ and $\theta$.

From the above simulation results, comments and observations are summarized as follows:

- Generator domain network-based participation factors can be used to distribute slack to participating DGs and the substation.
- Network parameters and the locations of DGs affect the DGs’ loss contribution.
- The substation real power outputs with a distributed slack bus are slightly smaller than the real power outputs with a single slack bus.
- While the results only appear to differ slightly, depending on the DG locations and from the DG viewpoints, the amount supplied to loads can differ (up to 2.8%).

| DG2 Outputs | A  | 0.67142993 | 0.67142993 | 0.67587020 |
| DG3 Outputs | A  | 0.65884665 | 0.65884665 | 0.66328702 |
| DG4 Outputs | A  | 0.66972341 | 0.66972341 | 0.67416376 |
| Total Load | 2.00000000 | 2.00000000 | 2.01332098 |
| DG2 Outputs | B  | 0.65884665 | 0.65884665 | 0.66328702 |
| DG3 Outputs | B  | 0.43379111 | 0.44427911 | 0.44858154 |
| DG4 Outputs | C  | 0.66972341 | 0.66972341 | 0.67416376 |
| Total Load | 1.35000000 | 1.35000000 | 1.36290546 |
| DG2 Outputs | C  | 0.45576113 | 0.45576113 | 0.46006257 |
| DG3 Outputs | C  | 0.45995976 | 0.45995976 | 0.46426135 |
| DG4 Outputs | Total | 1.35000000 | 1.35000000 | 1.36290546 |
| Load | 1.35000000 | 1.35000000 | 1.35000000 |
| DG2 Outputs | Load | 2.00000000 | 1.98676715 | 2.00000000 |
| DG3 Outputs | Load | 1.35000000 | 1.33720034 | 1.35000000 |
| DG4 Outputs | Load | 2.65000000 | 2.63525846 | 2.65000000 |
| Total Load | 2.65000000 | 2.65000000 | 2.66482402 |
| Load | 2.65000000 | 2.65000000 | 2.65000000 |
| Total Loss | 0.87317440 | 0.87317440 | 0.86895484 |
| Iteration No. | 4 | 4 | 9 |
which is significant and could be treated as a distribution wheeling indicator for DGs.

- With the penetration level and number of DGs increasing, reductions in system losses were seen.

### 3.7 Comments

The main contributions of this chapter are included:

- A distributed slack bus model for DGs in unbalanced power flow is proposed.
- Scalar participation factors are introduced to distribute uncertain real power system loss for three-phase power flow calculations.
- Two different methods to calculate network-based participation factors are developed and studied.
- The participation factors calculated by sensitivity-based methods and generator domain based method capture the effects of network parameters, load distribution, generator outputs and locations.
- Scalar participation factors are incorporated into three-phase power flow equations.
- Numerical results for different slack bus models are obtained and investigated.

The participation factors based on generator domains, which are explicitly relative to network parameters and load distributions, demonstrate that their ability to capture network characteristics and to scale loss contributions of sources surpasses other participation factors. Therefore, the distributed slack model with generator domain
participation factors is recommended and the following chapters will apply this recommended model to discuss its impacts and applications on cost analysis, switch placement and distribution system planning.
Chapter 4. Impacts of Slack Bus Modeling on Distribution Applications

Distributed slack bus models have been discussed in the previous chapter and the participation factors based on generator domains were recommended. Thus, the impacts of the proposed slack bus models on distribution application problems will be investigated in this chapter. Two examples of distribution applications, cost analysis and switch placement, will be reevaluated and reformulated. Simulation results of these two applications will be provided and discussed.

4.1 Application Functions for Distribution Systems with DGs

Distribution power flow with a network-based distributed slack bus model for unbalanced distribution systems can be applied to:

- develop advanced economic analysis tools and models, which distinguish load and loss contributions of each electric power provider;
- affect other distribution system application techniques, such as switch placement, capacitor placement, DG placement;
- provide planning and operating guides for distribution systems with DG

With respect to economic issues, this thesis will focus on cost analysis. When distributed generators within a distribution system belong to different owners, distinguishing each source’s load and loss contributions becomes significant for fair pricing. Therefore, cost analysis procedures need to be re-evaluated and developed to accommodate the changed operating distribution environment. The distributed slack bus model can be integrated directly into DG operating cost analysis.
For distribution application techniques, power flow analysis with distributed slack bus models may yield different placement and control actions for distributed generators, capacitors and network switches. For example, capacitor placement and control [50, 51], and network reconfiguration [52, 53] may be revised as their problem formulations typically focus on loss reduction. Service restoration schemes [54, 55] will also be affected as they are often formulated in terms of maximum power delivered to the loads.

In this thesis, switch placement will be used an example to show distributed slack bus modeling impacts on new switch locations for DG island operating, and different amount of load to be serviced during fault condition.

Thus, the two examples of distribution slack bus model applications will be studied. The chapter progresses as follows:

- Section 4.2 will study cost analysis for distribution systems with DGs
- Section 4.3 will discuss the problem of optimal switch placement to coordinate DG islanding operating

It is noted that the problem of distribution system planning needs to account for both economic and technical issues. Thus, due to its complexity, it will be discussed as a whole in Chapter 5.

### 4.2 Cost Analysis for Distribution Systems with DGs

The installation of DGs within distribution systems creates opportunities and challenges for both local utilities and independent power providers (IPPs). Both require tools to assess their own cost and to maximize their profits under competition. In addition, since DGs may belong to different owners and local utilities own and operate
the network, fair pricing of power from DGs is important. Therefore, new cost analysis tools for distribution system planning and operating techniques need to be developed to accommodate the changed distribution operating environment.

Since distinguishing load and loss contributions of each source is very important to correctly account for revenue and cost, the distributed slack bus model using generator domains provides advantages over the single slack bus model. In this section, cost analysis based on economic profit formula will be investigated. Detailed mathematical expressions for computing revenue and cost will be provided. These expressions incorporate loss and load contributions of DGs and the substation. The rates of loss contributions of sources are directly obtained by participation factors of distributed slack bus models. As such, the dollar cost impacts of different slack bus models can be quantified.

4.2.1 Cost Analysis Expressions

In this subsection, operating cost analysis based on the economic profit formula is presented with detailed mathematical expressions for the revenue and cost. For each source \( i \), the economic profit, \( EP_i \) is the difference between its total revenue, \( TR_i \) and its total cost \( TC_i \):

\[
EP_i = TR_i - TC_i
\]  

(4.1)

Proposed methods to account for revenue and cost are varied in electric power markets [23, 27, 28]; and methods proposed for distribution systems are also under development. In this thesis, the goal of this section is to demonstrate the possibility for different cost analysis approaches when a distributed slack bus model is used.
Utilities and DGs supply electricity to customers; their revenues are primarily based on customers’ electricity consumption. Therefore, the total revenue of a source can be expressed as follows:

$$TR_i = \sum_{l=1}^{n_l} \sum_{j \in D_i^l} B^l_j P^l_j T^l_j$$  \hspace{1cm} (4.2)

where:

- $n_l$: the number of load levels
- $D_i^l$: the set of loads served by source $i$ at load level $l$
- $B^l_j$: the price of real power load $j$ at load level $l$ (unit: $/kwh$)
- $P^l_j$: the real power delivered to load $j$ at load level $l$ (unit: kw)
- $T^l_j$: the duration of time of load $j$ in hours at load level $l$ (unit: h)

Here $P^l_j$ can be assigned using the models from (3.2) and (3.16) through a power flow at load level $l$ and:

$$\sum_{j \in D_i^l} P^l_j = P^{\text{load,}i}_{Gi}$$  \hspace{1cm} (4.3)

with $P^{\text{load,}i}_{Gi}$ representing the load associated with generator $i$ at load level $l$.

Closer evaluation shows that (4.2) holds for the substation. However, from a DG standpoint the contribution of a DG to losses, using (4.2) results in the DG absorbing the entire cost for producing these loss contributions. Since these loss contributions enhance the distribution network capability and often improve distribution network efficiency, loss contributions from DGs should be encouraged. As such, network operators could also represent revenue sources to DGs by providing financial compensation for the losses provided. Thus, for DGs, (4.2) is modified as:
TR_i = \sum_{l=1}^{n_i} \sum_{j \in D_j^l} B_j^l P_j^l T_j^l + \sum_{l=1}^{n_i} A_i^l P_{\text{loss,}i}^l T_{\text{Gi}}^l \tag{4.4}

where:

$A_i^l$: the price of utilities’ payment to DG real power loss contribution at load level $l$

(unit: $/kwh)$

$P_{\text{loss,}i}^l$: the loss associated with generator $i$ at load level $l$ (unit: kwh)

$T_{\text{Gi}}^l$: the duration of operating time of DG $k$ at load level $l$ (unit: h)

The payment of loss contributions to DGs is accounted for as a part of total cost of utilities. Thus, the total cost of the local utilities has three parts: payments to generation and transmission systems including energy charges, capacity charges, and payment to DG operators for loss contributions. The energy charge for a distribution utility $C_{en}$ is a function of the price per kilowatthour (kWh) and the kWh of real power injection from the transmission system at the substation:

$C_{en} = \sum_{l=1}^{n_i} A_e^l T_l P_{\text{Sinj}}^l$

$= \sum_{l=1}^{n_i} A_e^l [T_l (P_{\text{Load}}^l + P_{\text{Loss}}^l) - \sum_{k=1}^{m} T_{Gi}^l P_{Gi}^l ] \tag{4.5}$

where:

$A_e^l$: the real power price on the substation at load level $l$ (unit: $/kwh)$

$T_l$: the duration of time of load level $l$ (unit: h)

$P_{\text{Sinj}}^l$: the real power injection on the substation from the transmission system at load level $l$ (unit: kw)

$P_{\text{Load}}^l$: the total system real power load at load level $l$ (unit: kw)
\( P_{Loss}^l \): the total system real power loss at load level \( l \) (unit: kw)

\( T_{Gk}^l \): the duration of operating time of DG \( k \) at load level \( l \) (unit: h)

\( P_{Gk}^l \): the real power injection from DG \( k \) at load level \( l \) (unit: kw)

The capacity charge, \( C_{ca} \), is charged at the maximum apparent power on the substation during a period of time:

\[
C_{ca} = [A_c/(365*24)]S_{inj}^{max} \sum_{i=1}^{m} T_i
\]  

(4.6)

where:

\( A_c \): the price of capacity charge on the substation (unit: $/kVA per year)

\( S_{inj}^{max} \): the maximum apparent power injection on the substation during whole period of time (unit: kVA)

Payments to DG operators for loss contributions, \( C_{lo} \), is the sum of payments to participating DGs.

\[
C_{lo} = \sum_{i=1}^{n_s} \sum_{i=1}^{m} A_i P_{Gk}^{loss,l} T_{Gk}^l
\]  

(4.7)

This payment effectively quantifies and encourages DG placements in locations that could benefit the system with respect to voltage support. However, it also results in reducing payments of the distribution utility to the transmission and generation operators. Thus, from the distribution system standpoint, ideally this payment rate should not be higher than the nodal price on the substation. In summary, the total cost of a distribution utility is the sum of its energy charge, capacity charge and payment to DGs’ loss contributions:

\[
TC_{sub} = C_{en} + C_{ca} + C_{lo}
\]  

(4.8)
For DGs, their operating cost mainly comes from their power generating cost. Thus, the total cost of a DG, without connection tariff, can be expressed as

\[ TC_{Gk} = \sum_{l=A}^{n_l} A_{Gk}^{l} P_{Gk}^{l} T_{Gk}^{l} \]  

(4.9)

where:

- \( A_{Gk}^{l} \): the average real power generating price of DG \( k \) at load level \( l \) (unit: $/kwh)
- \( P_{Gk}^{l} \): the real power injection from DG \( k \) at load level \( l \) (unit: kwh)

From the above revenue and cost expressions, the revenue of each source is related to the associated load it services. In addition, with a distributed slack bus model, cost analysis can be fine-tuned to identify different costs/revenues for load contributions and loss contributions. Thus, the distributed slack bus model allows for more detailed profit and cost evaluations of individual sources. If a single slack model is used where all the system loss is assigned to the substation, this may unfairly punish the network operators. However, using computational tools that can distinguish loads and losses, fairer pricing can be achieved with DGs receiving compensation for network benefits they provide. Assuming the payment rate towards loss contributions is less than the transmission system charge; distribution utility profits can actually increase with appropriate compensation to DG operators.

In addition, the network-based approach to assigning slack and associated cost analysis can capture and identify more attractive locations to install distributed generators. These models and cost analysis approaches have been implemented in Matlab; and, in the next section, detailed simulation results will be presented to show the impacts of cost analysis and placement strategies using different slack bus models.
4.2.2 Numerical Analysis

This section will use different slack bus models to perform cost analysis. In order to clearly show the impacts from the distributed slack bus models for DGs, one system load level is applied for a one year time period. It is assumed that the DG is owned and operated by an entity independent of the distribution utility. Cost evaluations for DG installations at different locations will be studied; in addition, various levels of DG output will be investigated. For each case, the following cost parameters are used for this section’s analysis [23, 26]; it is noted that, other cost schemes can be readily incorporated in the program:

- Flat energy charge for all customers within distribution system: 0.085 $/kWh
- Electricity charge on the substation from transmission for the local utility, including two parts:
  - Electricity energy charge: 0.075 $/kWh
  - Electricity capacity charge: 45 $/kVA per year
- The average electricity generating cost of the DG: 0.07 $/kWh
Figure 4.1: One-line diagram of a 27-bus distribution system

Table 4.2.1  Summary for Cost Analysis of Base Case without DG

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Test Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substation Real Power Output (kW)</td>
<td>6939.49</td>
</tr>
<tr>
<td>Substation Apparent Power (kVA)</td>
<td>7453.70</td>
</tr>
<tr>
<td>Total System Real Power Load (kW)</td>
<td>6659.38</td>
</tr>
<tr>
<td>Real Power Load of Bus 3 Downstream (kW)</td>
<td>3080.24</td>
</tr>
<tr>
<td>Real Power Load of Bus 4 Downstream (kW)</td>
<td>3579.14</td>
</tr>
<tr>
<td>Total System Real Power Loss (kW)</td>
<td>280.11</td>
</tr>
<tr>
<td>Real Power Loss of Bus 3 Downstream (kW)</td>
<td>13.53</td>
</tr>
<tr>
<td>Real Power Loss of Bus 4 Downstream (kW)</td>
<td>255.96</td>
</tr>
<tr>
<td>Total Annual Revenue ($)</td>
<td>4,958,574</td>
</tr>
<tr>
<td>Energy Charge</td>
<td>4,559,247</td>
</tr>
<tr>
<td>Capacity Charge</td>
<td>335,420</td>
</tr>
<tr>
<td>Total</td>
<td>4,894,667</td>
</tr>
<tr>
<td>Total Annual Economic Profit ($)</td>
<td>63,907</td>
</tr>
<tr>
<td>Cost for Serving Loss ($)</td>
<td>184,034</td>
</tr>
</tbody>
</table>
A 27-bus test distribution system will be used and its one-line diagram is shown in Figure 4.1. The network was designed from a portion of an existing system with real network parameters. For the simulations, all loads are treated as constant \( PQ \) loads and the total system load is 6.659 MW and 2.539 Mvar. Using a single slack bus unbalanced distribution power flow solver, the total system real power loss, \( P_{\text{loss}} \), is 280 kW or 4.2% of real power injection at the substation. The transformer between Bus 2 and Bus 3 services 3.08 MW and 0.95 Mvar high density loads. The transformer between Bus 2 and Bus 4 services 3.58 MW and 1.58 Mvar dispersed loads in a commercial and residential area. Without DG installed, 255.96 kW (91.4% of total system real power loss) occurs in the commercial and residential area, and only 13.53 kW (4.8% of the total loss) occurs in the high density load area due to shorter branches and lower network resistances. The local utility acquires electricity from the transmission system to supply loads and loss within the system. Using the cost parameters above results in the distribution utility incurring an annual revenue of approximately $4,958,000 USD, annual cost $4,894,700, and annual profit $63,300. A summary of cost analysis for this base case without DG is shown in Table 4.2.1.

In the following examples, three cases will be investigated where penetration is defined as the percentage of the target real power output to the total system real power load:

- Case 1: the DG is installed at Bus 3 with different DG penetration
- Case 2: the DG is installed at Bus 4 with different DG penetration
- Case 3: the DG is installed at different locations with the same DG penetration
It is expected that the ability of the distributed slack bus model to quantify loss and loads will yield significant differences in cost analysis compared to a traditional single slack bus power flow. In addition, the impact of DG locations will be illustrated; and it is expected that, for this case, installations in areas of the distribution network with more dispersed loads will illustrate larger differences in cost than installations within high density load areas.

**Case 1: DG installed at Bus 3**

One DG is sited at Bus 3 to service 0.5, 1, and 1.5 MW real power load respectively. Bus 3 is located near the high density loads. Load and loss contributions obtained from power flow with a single slack bus model and distributed slack bus model are displayed in Table 4.2.2. The real power system loss is modestly reduced when the DG’s penetration is increased, with the 1500kW set point representing 20% penetration. Using the traditional power flow approach of a single slack bus model, the DG is treated to have no contribution to system real power loss. While using the distributed slack bus model, the DG’s participation factors, $K$, and associated real power loss contribution are displayed.
Table 4.2.2 Loss Contributions for Cost Analysis with a DG on Bus 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Single Slack Bus Model</th>
<th>Distributed Slack Bus Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Power Load Serviced by DG (kW)</td>
<td>500  1,000  1,500</td>
<td>500  1,000  1,500</td>
</tr>
<tr>
<td>DG Real Power Output (kW)</td>
<td>500  1,000  1,500</td>
<td>502.17  1,004.33  1,506.50</td>
</tr>
<tr>
<td>Substation Real Power Output (kW)</td>
<td>6,437.80  5,936.89  5,436.17</td>
<td>6,435.63  5,932.55  5,429.66</td>
</tr>
<tr>
<td>Substation Apparent Power (kVA)</td>
<td>6,618.93  6,139.62  5,664.73</td>
<td>6,616.84  6,135.48  5,658.59</td>
</tr>
<tr>
<td>Total System Real Power Loss (kW)</td>
<td>278.42  277.51  276.79</td>
<td>278.41  277.51  276.78</td>
</tr>
<tr>
<td>Real Power Loss Serviced by DG (kW)</td>
<td>0  0  0</td>
<td>2.17  4.33  6.50</td>
</tr>
<tr>
<td>Real Power Loss Serviced by Sub (kW)</td>
<td>278.42  277.51  276.79</td>
<td>276.25  273.17  270.28</td>
</tr>
<tr>
<td>Participation Factor for DG</td>
<td>0  0  0</td>
<td>0.0078  0.0156  0.0235</td>
</tr>
<tr>
<td>Participation Factor for Substation</td>
<td>1  1  1</td>
<td>0.9922  0.9844  0.9765</td>
</tr>
</tbody>
</table>

Table 4.2.3 Cost Analysis for the Local Utility with a DG on Bus 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Single Slack Bus Model</th>
<th>Distributed Slack Bus Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Power Load Serviced by DG (kW)</td>
<td>500  1,000  1,500</td>
<td>500  1,000  1,500</td>
</tr>
<tr>
<td>Total Annual Revenue ($)</td>
<td>4,586,274  4,213,974  3,841,674</td>
<td>4,586,274  4,213,974  3,841,674</td>
</tr>
<tr>
<td>Annual Cost ($)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy Charge</td>
<td>4,229,634  3,900,538  3,571,563</td>
<td>4,228,207  3,897,686  3,567,287</td>
</tr>
<tr>
<td>Capacity Charge</td>
<td>297,852  276,283  254,913</td>
<td>297,758  276,097  254,636</td>
</tr>
<tr>
<td>Payment to DG Loss Contributions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_i = 0$</td>
<td>0  0  0</td>
<td>0  0  0</td>
</tr>
<tr>
<td>$A_i = 0.03$</td>
<td>0  0  0</td>
<td>570  1,139  1,708</td>
</tr>
<tr>
<td>$A_i = 0.075$</td>
<td>0  0  0</td>
<td>1424  2,848  4,271</td>
</tr>
<tr>
<td>Total Annual Economic Profit ($)</td>
<td>58,788  37,153  15,198</td>
<td>60,309  40,191  19,751</td>
</tr>
<tr>
<td>Total Cost for Serving Loss ($)</td>
<td>182,921  182,326  181,851</td>
<td>181,495  179,474  177,574</td>
</tr>
</tbody>
</table>

### Notes
- $A_i$ represents different scenarios or parameters in the cost analysis.
- The cost analysis includes energy charge, capacity charge, and payment to DG for different load scenarios.
- The participation factor indicates the percentage of total loss that a component (DG or substation) is responsible for.
Table 4.2.4 Cost Analysis for the DG on Bus 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Single Slack Bus Model</th>
<th>Distributed Slack Bus Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Power Load Serviced by DG (kW)</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total Annual Revenue ($)</strong></td>
<td>A_0 = 0</td>
<td>372,300</td>
</tr>
<tr>
<td></td>
<td>A_0 = 0.03</td>
<td>372,300</td>
</tr>
<tr>
<td></td>
<td>A_0 = 0.075</td>
<td>372,300</td>
</tr>
<tr>
<td><strong>Total Annual Cost ($)</strong></td>
<td>A_0 = 0</td>
<td>306,600</td>
</tr>
<tr>
<td></td>
<td>A_0 = 0.03</td>
<td>65,700</td>
</tr>
<tr>
<td></td>
<td>A_0 = 0.075</td>
<td>65,700</td>
</tr>
<tr>
<td><strong>Total Annual Economic Profit ($)</strong></td>
<td>A_0 = 0</td>
<td>65,700</td>
</tr>
<tr>
<td></td>
<td>A_0 = 0.03</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>A_0 = 0.075</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total Cost for Serving Loss ($)</strong></td>
<td>A_0 = 0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>A_0 = 0.03</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>A_0 = 0.075</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 4.2: Annual Economic Profit for the Local Utility with a DG on Bus 3
For the cost analysis based on (4.7), three different rates are applied for the utility’s payment to DG operators for loss contributions $A_u = $0, $0.03$ and $0.075$, respectively. Table 4.2.3 displays the resulting cost analysis from the system/distribution utility standpoint. Figure 4.2 displays the annual economic profit of the distribution utility for the various amounts of DG penetration and $A_u$. It is observed that

- for both slack bus models, the total annual revenue, consumption charge, capacity charge, and economic profit of the utility decrease with increased amounts of DG penetration.

- using the distributed slack bus model and for each $A_u$, the utility has higher profit than using the single slack bus model because the DG supplies a portion of the losses at a cost to the utility less than if the utility purchased the same amount from the substation. The delineation of the amount of loss allows for this accounting to be performed. Thus, in Figure 4.2, the profit curves with $0.03$ and $0.075$ rates are located between the curve of the distributed slack bus model with zero rate and the curve of the single slack bus model.

In Table 4.2.4, cost analysis results with respect to the DG are displayed. The total annual revenue, annual cost and economic profit of the DG increase with the increase in DG penetration for both slack bus models. The loss contribution payments from utilities further increased the DG’s profit using the distributed slack bus model.

**Case 2: DG installed at Bus 4**

One DG is sited at Bus 4 to service 0.5, 1, and 1.5 MW real power load respectively. Bus 4 is located closer to the dispersed loads. The simulation results using different slack
bus models are shown in Table 4.2.5. As in Case 1, the real power system loss of this case is also slightly reduced with DG’s penetration increasing for both slack bus models, and the value of the DG’s participation factor is increasing with DG penetration. However, the participation factor assigned to the DG on Bus 4 is much larger than that of Bus 3 at the same penetration. The larger factors reflect the higher percentage of system real power loss occurring downstream of Bus 4 which is identified as the generator’s domain. Thus, the DG installed on Bus 4 is identified to make larger real power loss contributions than a DG placed at Bus 3.

Table 4.2.5 Loss Contributions for Cost Analysis with DG on Bus 4

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Single Slack Bus Model</th>
<th>Distributed Slack Bus Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Power Load Serviced by DG (kW)</td>
<td>500</td>
<td>1,500</td>
</tr>
<tr>
<td>DG Real Power Output (kW)</td>
<td>500</td>
<td>1,500</td>
</tr>
<tr>
<td>Substation Real Power Output (kW)</td>
<td>6430.83</td>
<td>5292.67</td>
</tr>
<tr>
<td>Substation Apparent Power (kVA)</td>
<td>6482.17</td>
<td>5988.73</td>
</tr>
<tr>
<td>Total System Real Power Loss (kW)</td>
<td>271.45</td>
<td>270.29</td>
</tr>
<tr>
<td>Real Power Loss Serviced by DG (kW)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Real Power Loss Serviced by Sub (kW)</td>
<td>271.45</td>
<td>270.29</td>
</tr>
<tr>
<td>Participation Factor for DG</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Participation Factor for Substation</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.2.6 Cost Analysis for the Local Utility with a DG on Bus 4

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Single Slack Bus Model</th>
<th>Distributed Slack Bus Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Power Load Serviced by DG (kW)</td>
<td>500</td>
<td>1,500</td>
</tr>
<tr>
<td>Total Annual Revenue ($)</td>
<td>4,586,274</td>
<td>4,213,974</td>
</tr>
<tr>
<td>Energy Charge</td>
<td>4,225,057</td>
<td>3,895,792</td>
</tr>
<tr>
<td>Capacity Charge</td>
<td>291,697</td>
<td>269,493</td>
</tr>
<tr>
<td>Payment to DG Loss Contributions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_e = 0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_e = 0.03$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_e = 0.075$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Annual Economic Profit ($)</td>
<td>69,520</td>
<td>48,689</td>
</tr>
<tr>
<td>$A_e = 0$</td>
<td>94,054</td>
<td>97,728</td>
</tr>
<tr>
<td>$A_e = 0.03$</td>
<td>84,885</td>
<td>79,389</td>
</tr>
<tr>
<td>$A_e = 0.075$</td>
<td>71,130</td>
<td>51,880</td>
</tr>
<tr>
<td>Total Cost for Serving Loss($)</td>
<td>178,344</td>
<td>177,579</td>
</tr>
<tr>
<td>$A_e = 0$</td>
<td>155,361</td>
<td>131,634</td>
</tr>
<tr>
<td>$A_e = 0.03$</td>
<td>149,973</td>
<td>125,452</td>
</tr>
<tr>
<td>$A_e = 0.075$</td>
<td>178,287</td>
<td>177,482</td>
</tr>
</tbody>
</table>

69
Table 4.2.7 Cost Analysis for the DG on Bus 4

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Single Slack Bus Model</th>
<th>Distributed Slack Bus Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Power Load Serviced by DG (kW)</td>
<td>0.5 1 1.5</td>
<td>0.5 1 1.5</td>
</tr>
<tr>
<td>Total Annual Revenue ($)</td>
<td>372,300 744,600 1,116,900</td>
<td>372,300 744,600 1,116,900</td>
</tr>
<tr>
<td>$A_i = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_i = 0.03$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_i = 0.075$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Annual Cost ($)</td>
<td>306,600 613,200 919,800</td>
<td>327,997 655,992 983,986</td>
</tr>
<tr>
<td>Total Annual Economic Profit ($)</td>
<td>65,700 131,400 197,100</td>
<td>44,303 88,608 132,914</td>
</tr>
<tr>
<td>$A_i = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_i = 0.03$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_i = 0.075$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost for Serving Loss ($)</td>
<td>0 0 0</td>
<td>-1,528 42,792 64,186</td>
</tr>
<tr>
<td>$A_i = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_i = 0.03$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_i = 0.075$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.3: Annual Economic Profit for the Local Utility with a DG on Bus 4
As a result, the cost analysis from the distribution utility standpoint is quite interesting and displayed in Table 4.2.6. For both slack bus models, the total annual revenue, energy charge, and capacity charge of the utility decrease with increasing DG penetration. As we expected, regardless of the model, the DG installed on Bus 4 has more beneficial impacts on the cost analysis than the same DG on Bus 3. However, different impacts on the total annual economic profits of the utility are observed with increases in DG penetration, please see Figure 4.3:

- Using the single slack bus model, the total annual economic profit of the utility first slightly increases with DG penetration and then decreases with continued increase in DG penetration.

- Using the distributed slack bus model, a similar increase in profit with certain levels of DG penetration is also observed. These initial increases in profit imply that the distribution utility can still economically benefit from proper DG placement and sizing through loss reduction and reduced capacity charges even though they may no longer supply some portions of their original customers.

- Using the distributed slack bus model with, $A_u = 0$ zero payments to DG loss contributions, the total annual economic profit of the utility consistently increases with DG penetration.

These different results in profits may cause different behaviors: for the same location, the utility may encourage higher DG penetration based on the analysis using the distributed slack bus model than using the single slack bus model.
In Table 4.2.7, the total annual revenue, annual cost and economic profit of the DG also increase with an increase in DG penetration for both models. For the single slack bus model, Case 1 and Case 2 yield the same cost analysis. As expected, the annual profit results for the distributed slack bus model are significantly different from those in Case 1. While the total system loss decreases with increases in DG penetration, the DG contribution to total system loss increases with a distributed slack bus model. As a consequence, the rate at which a DG may be compensated for providing loss contributions or providing network voltage support is significant; and its impact on profit can also be seen in Figure 4.3. Therefore, some DG installations may be specifically identified for network support and improved electrical and cost efficiency.

**Case 3: DG at different locations**

To study the effects of different DG placement locations using the different slack bus models, a DG is selected to service 1.5 MW load on Bus 6, Bus 13 and Bus 19 respectively. From Figure 4.4 and Table 4.2.8, it can be observed that the DG’s loss contribution, reflected by its participation factors, is significantly impacted by its location. To service the same amount of load, DG’s participation factors are significantly different 0.3163 at Bus 6 vs. 0.027 at Bus 19. When the DG is installed at Bus 6, the system real power loss is reduced from 280.11 kw to 237.55 kw with a single slack bus model and to 227.23 kw with the distributed slack bus model. When DG is at Bus 19, its participation factor is 0.027 and the system real power loss is reduced however by a much smaller level.
Figure 4.4: Annual Economic Profit for the Local Utility with DG Serving 1500kW Load at Different Locations

Figure 4.5: Participation Factors of DG Serving 1500kW Load at Different Locations
Table 4.2.8. Loss Contributions for Cost Analysis with a DG to Service 1.5 MW Load on Different Locations

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Single Slack Bus Model</th>
<th>Distributed Slack Bus Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bus 6</td>
<td>Bus 13</td>
</tr>
<tr>
<td>DG Locations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DG Real Power Output (kW)</td>
<td>1,500</td>
<td>1,500</td>
</tr>
<tr>
<td>Substation Real Power Output (kW)</td>
<td>5,396.93</td>
<td>5,402.76</td>
</tr>
<tr>
<td>Substation Apparent Power (kVA)</td>
<td>5,422.89</td>
<td>5,412.62</td>
</tr>
<tr>
<td>Total System Real Power Loss (kW)</td>
<td>237.55</td>
<td>243.38</td>
</tr>
<tr>
<td>Real Power Loss Serviced by DG (kW)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Real Power Loss Serviced by Sub (kW)</td>
<td>237.55</td>
<td>243.38</td>
</tr>
<tr>
<td>Participation Factor for DG</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Participation Factor for Substation</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.2.9 Cost Analysis for the Local Utility with a DG to Service 1.5 MW Load on Different Locations

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Single Slack Bus Model</th>
<th>Distributed Slack Bus Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bus 6</td>
<td>Bus 13</td>
</tr>
<tr>
<td>DG Location</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Annual Revenue ($)</td>
<td>3,841,674</td>
<td>3,841,674</td>
</tr>
<tr>
<td>Energy Charge</td>
<td>3,545,782</td>
<td>3,549,615</td>
</tr>
<tr>
<td>Capacity Charge</td>
<td>244,030</td>
<td>243,568</td>
</tr>
<tr>
<td>Annual Cost ($)</td>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>Payment to DG Loss Contributions</td>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>$A_e = 0.03</td>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>$A_e = 0.075</td>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>Total Annual Economic Profit ($)</td>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>Total Cost for Serving Loss ($)</td>
<td>$51,862</td>
<td>$48,491</td>
</tr>
<tr>
<td>$A_e = 0.03</td>
<td>$51,862</td>
<td>$48,491</td>
</tr>
<tr>
<td>$A_e = 0.075</td>
<td>$51,862</td>
<td>$48,491</td>
</tr>
<tr>
<td>Total Cost for Serving Loss ($)</td>
<td>$156,070</td>
<td>$159,903</td>
</tr>
<tr>
<td>$A_e = 0.03</td>
<td>$156,070</td>
<td>$159,903</td>
</tr>
<tr>
<td>$A_e = 0.075</td>
<td>$156,070</td>
<td>$159,903</td>
</tr>
</tbody>
</table>
Table 4.2.10. Cost Analysis for the DG to Service 1.5 MW Load at Different Locations

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Single Slack Bus Model</th>
<th>Distributed Slack Bus Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>DG Location</td>
<td>Bus 6</td>
<td>Bus 13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Annual Revenue ($)</td>
<td>$1,116,900</td>
<td>$1,116,900</td>
</tr>
<tr>
<td>$A_0$</td>
<td>$1,116,900</td>
<td>$1,116,900</td>
</tr>
<tr>
<td>$A_{0.03}$</td>
<td>$1,116,900</td>
<td>$1,116,900</td>
</tr>
<tr>
<td>Total Annual Cost ($)</td>
<td>$919,800</td>
<td>$919,800</td>
</tr>
<tr>
<td>$A_0$</td>
<td>$197,100</td>
<td>$197,100</td>
</tr>
<tr>
<td>$A_{0.03}$</td>
<td>$197,100</td>
<td>$197,100</td>
</tr>
<tr>
<td>$A_{0.075}$</td>
<td>$197,100</td>
<td>$197,100</td>
</tr>
<tr>
<td>Total Cost for Serving Loss ($)</td>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>$A_0$</td>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>$A_{0.03}$</td>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>$A_{0.075}$</td>
<td>$0</td>
<td>$0</td>
</tr>
</tbody>
</table>

Table 4.2.9 and 4.2.10 present the cost analysis from the utility viewpoint and from the DG standpoint, respectively. A plot for the utility profits is displayed in Figure 4.5. Results illustrate significant differences between costs when losses are not distributed and one where a distributed slack bus model is employed. From Table 9 and 10, it is possible to quantify cost differences between locations; therefore flat rate interconnection charges may not be optimal to either the distribution utility or the DG operator. Thus network-based participation factors used in the distributed slack bus model can quantify loss contributions of participating sources and provides an advantage for improved cost analysis.

From the above simulations, the follow summary of comments and observations follow:

- The network-based participation factors of the distributed slack bus model reflect the ratios of participating sources’ real power loss contributions to the system real power loss.
• Consequently, the approach is able to quantify the different impacts on system loss based on different DG locations, with larger participation factors indicating a larger impact on system loss.

• Local loss contributions and the release of system capacity from DGs can improve utilities’ profits. Reasonable payment rates to DG operators for loss contributions can bring benefits for both utilities and DG owners.

• For the local utility:
  – The cost analysis results are significantly different using different slack bus models
  – Penetration levels of DGs will affect the economic profit of the local utility. Increasing and decreasing the profit of the local utility are both possible by increasing DG penetration.
  – Locations of DG installation also impact the profit of the local utility

• For the DG:
  – The cost analysis results are significantly different using slack bus models
  – Penetration of DG will affect the profit of the DG
  – The selected location for the DG greatly affects the profit of the DG when a distributed slack bus model is used; while the profit is lower than if the losses are not distributed this method for determining non-load service benefits to the system would be useful to properly determine interconnection charges.
Through these studies, it can be seen that a distributed slack bus approach in power flow analysis and cost analysis can significantly impact distribution application functions such as distributed generator placement and control problems.

### 4.2.3 Comments

In this section, slack bus modeling for distribution power flow is linked to cost analysis for distribution systems with DGs. Its impacts on DG installations within distribution systems have been analyzed. Detailed mathematical expressions for a method of cost analysis have been developed. Simulation results show that different slack bus models may cause significantly different results of cost analysis. The slack bus model with generator domain participation factors can provide more realistic power flow analysis data; and the ability to quantify loss and load contributions from individual source may help regulators to set fair pricing schemes.

### 4.3 Switch Placement for DG Islanding Operation

This section will discuss the distributed slack bus model impacts on the application technique of switch placement. In order to improve radial distribution system reliability, the switch placement schemes are used to coordinate DGs to form self-supported areas under fault conditions.

### 4.3.1 Review of Switch Placement

Under competitive environments, utilities face the challenge to improve reliability for customers with minimal cost investments. Allowing DGs to support an isolated area by opening switches during upstream faults is an option to increase distribution system reliability [29-31].
In [31], the switch placement problem is formulated as a non-differentiable, multi-objective optimization problem subject to electrical, operational and network constraints. The objectives included:

- minimize the number of new switches to be installed;
- maximize the amount of priority load in the island;
- maximize the number of customers in the island;
- maximize the amount of total load in the island;
- minimize the number of switch operations.

In order to solve this problem, a graph-based solution algorithm was proposed. The essential idea of this solution algorithm is: first, build a graphical isolated area to be supported by DGs; then, expand this area by closing existing switches or adding new switches if capacity allows. A three-phase power flow with a single slack bus model was applied in this solution algorithm.

If an islanded area has multiple DGs, slack bus modeling will affect the results of this switch placement problem. The different methods to assign real power loss to generators will directly change the ability of generators to load supply. As such, the amount of load and the number of customer in islanded areas supported by DGs will be affected. Then, to form the islands, new switch installation and the number of switch operation may also be different.

Moreover, DGs within distribution systems may have similar size, and their operating margins are limited compared to the substation. The algorithm in [31] defined a parameter of quickly DG adjustable power $\alpha$ to represent the adjustable output of a DG
for islanding operation. Thus, if all the power loss of an island was assigned to one DG, DG output constraint of one DG may be violated and other DGs still having spare capacity; while slack shared by multiple DGs can reduce such violation during solution search.

Therefore, distributed slack power flow is applied to the switch placement problem in this thesis. Simulations using power flow with different slack bus models are shown and discussed in the following subsection.

### 4.3.2 Numerical Results

A 20 bus system with 5799 KW and 3192 Kvar load is used here for simulation. Its one line diagram is shown in Figure 4.6. Two DGs are installed in Bus 5 and Bus 12. Both DGs have a 1200 KW rating and have 1050 KW output before the fault.

![Figure 4.6: One-line diagram of a 20-bus distribution system for DG islanding operation](image-url)
In the switch placement algorithm, the percentage of quickly adjustable power of a DG with respect to its rating is used to represent its adjustable output margin for islanding operation; the percentage of losses on the branches respect to total power generation, $\beta$, is used to estimate load limits to reduce power flow computation. In this case, both DGs have the same value of $\beta$. Parameter values for the test cases are summarized in Table 4.3.1.

<table>
<thead>
<tr>
<th>$\alpha$, Percentage of adjustable DG output</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial $\beta$, percentage of system loss</td>
<td>6%</td>
</tr>
<tr>
<td>Fault branch</td>
<td>Bus 4-5</td>
</tr>
<tr>
<td>Non-priority loads</td>
<td>100% uncontrollable</td>
</tr>
<tr>
<td>Total load isolated by fault</td>
<td>2820 KW, 2150 Kvar</td>
</tr>
</tbody>
</table>

When a fault occurs on the branch between Bus 4 and Bus 5, the network downstream of this branch will be isolated from the substation. Applying the algorithm in [31], first, an intentional islanded area can be formed with a new switch installed on Bus 8-9. It is obtained by estimating load and generation limits of portion networks. This intentional islanded area is the area within the dashed line in Figure 4.6. 2160 KW and 1650 Kvar load within this area. After the estimated solution was obtained, three-phase power flows were run to check the feasibility of this solution. The power flow results applying different slack bus model are shown in Table 4.3.2.
Table 4.3.2 Power flow results for the intentional islanded area

<table>
<thead>
<tr>
<th></th>
<th>Single Slack Bus Model</th>
<th>Distr. Slack Bus Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participation factor $K$</td>
<td>DG 1: 1</td>
<td>0.5511</td>
</tr>
<tr>
<td></td>
<td>DG 2: 0</td>
<td>0.4489</td>
</tr>
<tr>
<td>DGs’ real power output</td>
<td>DG1: 1190 KW</td>
<td>1141</td>
</tr>
<tr>
<td></td>
<td>DG2: 1078 KW</td>
<td>1130</td>
</tr>
<tr>
<td>Installed and open new switch</td>
<td>Bus 8- Bus 9</td>
<td>Bus 8- Bus 9</td>
</tr>
<tr>
<td>Load served by DGs in the island</td>
<td>2160 KW 1650 Kvar</td>
<td>2160 KW 1650 Kvar</td>
</tr>
</tbody>
</table>

If the distribution power flow with a single slack bus model was applied, the DG installed at Bus 5 was assigned as the only slack bus, it would have 1190 KW real power output from the power flow calculation. This value violates the constraint of DG real power output for islanding operation, $1170\text{ KW} = 1050 + \alpha \cdot 1200$. This violation would prohibit the islanded area from forming. While applying the distributed slack bus model, the islanded area can be supported by the two DGs without constraint violations.

4.3.3 Comments

Switch placement for DG islanding operation may be affected by applying different slack bus models. The simulation results show that an islanded area can be formed using distributed slack bus model, but when using a single slack bus model this area can not be formed due to operating constraint violations.
4.4 Comments

The distributed slack bus model impacts on distribution applications were discussed in this chapter. With DG penetration increasing in distribution systems, proper modeling slack bus will impact distribution applications. This chapter provided two examples, cost analysis and switch placement to show the impacts of slack bus modeling on economic and technical issues for distribution systems with DGs. It was demonstrated that distributed slack model can bring advantages for cost analysis with distinguishing loss and load contributions of individual sources. Slack bus modeling also affects the results of switch placement for DG islanding operation. The switch placement will be included in distribution expansion planning, which will be addressed in the following chapter.
Chapter 5. Distribution System Expansion Planning

In this chapter, DG indicates distributed generator. The problem of distribution system expansion considering DG placement and feeder upgrade will be addressed. Detailed problem formulations for different expansion options will be discussed. A GA-based solution algorithm will be proposed to solve the optimization problem for distribution planning. In all cases, the distributed slack bus model will be implemented to DG islanding operation in the distribution system expansion planning to increase distributed system reliability and reduce planning cost.

5.1 Introduction

DG placement is an option for expanding generation capacity, releasing transmission and distribution system capacity, and enhancing system reliability. However, distribution system expansion through DG placement is different from traditional distribution system expansion, which typically expands system capacity by substation and feeder upgrades. Although the planning problem becomes much more complex when considering DG placement and feeder upgrades together, it provides a more diverse expansion solution for utilities. Therefore, new strategies and methods for distribution system expansion need to be developed to accommodate this challenge.

Historically, methods for optimal distribution expansion planning have been thoroughly investigated without considering DG placement [32-34]. Recently, in some areas, generation expansion could not keep up with the rapid load growth. The number of societal concerns, the dramatically increased cost of building new generation plants,
transmission lines and distribution lines have hampered the installation of scale generation. On the other hand, DG installation and operating costs have been decreasing, and the reliability of environmentally friendly, alternative energy based DGs has improved.

DG placement becomes an attractive method for distribution expansion. Several approaches about optimal distributed generator placement within distribution systems were proposed. In [37], Griffin et. al. provided a method based on loss sensitivity or load distribution to expand system capacity and reduce loss. In [38], Nara et. al. applied tabu search for optimal placement of distributed generators to minimize interruption cost. In [39], Kim et. al. used a fuzzy-GA method to minimize the distribution loss cost under different load level considering constraints of bus voltages and DG capacities. In [40], Teng et. al. considered installation and operating costs and proposed a GA method to maximize the ratio value of benefits/cost of DG placement.

The above methods of DG placement consistently focus on cost minimization for distribution systems. However, the savings from loss reduction or from service interruption reductions alone may not be sufficient to compensate for the DG installation and operating costs. In fact, to maximize the benefit of DG installation, DGs may be required to be operated within islanded areas when faults occur as well as in parallel with the substation. In addition, DG placement will affect other equipment placement, for example, distribution line and transformer upgrades may be reduced through parallel operation, new switch placement may be required to coordinate DG islanding operation. Therefore, the problem of DG placement for distribution expansion planning needs to be carefully evaluated.
The chapter will present:

- problem formulations of distribution expansion planning accounting for feeder upgrades, DG placement and different allowable DG operating modes
- a GA-based algorithm to solve these combinational optimization problems
- simulation results for feeder upgrades with DG placement

5.2 Problem Formulation

The problem of distribution system expansion planning with DG placement and network upgrades is formulated as a non-differentiable optimization problem subject to electrical, operational and network constraints. The generic constrained optimization problem is:

\[
\min_{x,u} f(x,u) \quad \text{(5.1)}
\]

s.t. \( F(x,u) = 0 \) \quad \text{(5.2)}

\( G(x,u) \leq 0 \) \quad \text{(5.3)}

where:

- \( f(x,u) \): the aggregate objective function
- \( x \): continuous state variables representing distribution system’s bus voltages
- \( u \): discrete and continuous control variables
- \( F(x,u) \): electrical equality constraints
- \( G(x,u) \): operational inequality constraints

The problem formulations for distribution system expansion planning with increasing complexity will be investigated in the following subsections: first, only feeder upgrades will be considered in Section 5.2.1; then, DG placement without islanding operation in
Section 5.2.2 and with islanding operation in Section 5.2.3; last, DG placement with feeder upgrade in Section 5.2.4.

5.2.1 Feeder Upgrades

Feeder upgrades are important options for distribution expansion to increase system capacity. This subsection considers feeder upgrades as the only options for distribution expansion. It assumes that feeder upgrades will maintain the existing network configuration.

Branches of this problem include line branches, switch branches and transformer branches. Transformers and switches within the substation are also considered as branches of the system’s feeder. If the conductor type of each branch were given, the impedance and capacity of each branch would be found. Therefore, the control variables are the branch conductor types represented as discrete variables:

\[ u = \begin{bmatrix} u_1^f, u_2^f, \ldots, u_{n_{\text{branch}}}^f \end{bmatrix} \tag{5.4} \]

where:

- \( u_j^f \): the conductor type of branch \( j \)
- \( n_{\text{branch}} \): the number of branches, and

\[ n_{\text{branch}} = n_{\text{line}} + n_{\text{sw}} + n_{\text{sf}} \tag{5.5} \]

with:

- \( n_{\text{line}} \): the number of line branches
- \( n_{\text{sw}} \): the number of switch branches
- \( n_{\text{sf}} \): the number of transformer branches
The total number of control variables is the number of branches. Available options for line, switch and transformer upgrades are considered as branch candidates. The following expression is used to calculate the size of the search space:

\[
\prod_{i=1}^{n_{\text{can}}} n_{\text{can}}^n \cdot \prod_{i=1}^{n_{\text{can}}} n_{\text{can}}^n \cdot \prod_{i=1}^{n_{\text{can}}} n_{\text{can}}^n = (n_{\text{can}}^n)^n \cdot (n_{\text{can}}^n)^n \cdot (n_{\text{can}}^n)^n
\]  

(5.6)

where:

\( n_{\text{can}}^\text{line} \): the number of line candidates

\( n_{\text{can}}^\text{sw} \): the number of switch candidates

\( n_{\text{can}}^\text{xf} \): the number of transformer candidates

The objective is to minimize the total cost over a planning period. Here, total outage cost, total feeder upgrade cost, and total wheeling cost are considered in the objective function. Thus,

\[
f(x, u) = C_{\text{outage}}(x, u) + C_{\text{feeder}}(x, u) + C_{\text{wheel}}(x, u)
\]  

(5.7)

where:

\( C_{\text{outage}}(x, u) \): total outage cost

\( C_{\text{feeder}}(x, u) \): total feeder upgrade cost

\( C_{\text{wheel}}(x, u) \): total wheeling cost

The total outage cost, total feeder upgrade cost and total wheeling cost are discussed in the following subsections.
A. Total Outage Cost

The total outage cost is the sum of all customer interruption cost for all load levels during the planning period.

\[
C_{\text{outage}}(x,u) = \sum_{l=1}^{n_l} \sum_{i=1}^{n} (r_{\text{fail}}^{i} + r_{\text{fail}}^{i} + m^{i-\text{Sub}} \cdot r_{\text{line}}^{i} \cdot T_{i} \cdot (C_{i}^{l,\text{out}} \cdot \sum_{p=a}^{P_{\text{Load},i}^{p,l}} P_{\text{Load},i}^{p,l}(x,u)))
\]

(5.8)

where:

- \( n_{l} \): the number of load levels
- \( n \): the number of bus
- \( r_{\text{fail}}^{i} \): the average power interruption rate of the substation
- \( r_{\text{fail}}^{i} \): the average power interruption rate of a switch or transformer branch
- \( r_{\text{line}}^{i} \): the average power interruption rate of distribution lines for unit length
- \( U_{i} \): set of switch or transformer branches between bus \( i \) and the substation
- \( m^{i-\text{Sub}} \): the feeder length between bus \( i \) and the substation
- \( C_{i}^{l,\text{out}} \): the average rate of outage cost for the load on bus \( i \) at load level \( l \)
- \( T_{i} \): time duration in hours of load level \( l \)
- \( P_{\text{Load},i}^{p,l}(x,u) \): real power load on bus \( i \), phase \( p \) at load level \( l \)

The interruption duration time of customers and their interruption costs are used to account for the outage costs. The customer interruption costs can be estimated by their average rates of outage costs provided by interruption cost surveys [41]. Only radial structures are considered in this thesis. Thus, system factors causing customer interruptions include:
1) Substation outages

A substation outage means a failure of the substation to supply power to the network. It can be caused by faults occurring within transmission systems or equipment failures on the substation. The average power interruption rate of the substation is used to represent the duration time of power interruption within a unit period of time. If the rate of power availability of the substation \( r_{\text{avail}}^{\text{sub}} \) was given (e.g. 99.995% [23]), the interruption rate caused by the substation without service would be:

\[
\frac{r_{\text{fail}}^{\text{sub}}}{r_{\text{sub}}} = 1 - r_{\text{avail}}^{\text{sub}} \quad (5.9)
\]

2) Power interruptions between the substation and a customer

Power interruptions between the substation and a customer are caused by faults occurring on the branches between the substation and the customer. These interruption rates are related to failure rates of branches and their mean time to repair (MTTR). The failure rate is the average number of failures of a component or unit of the system in a given period of time. MTTR is the average or expected time to repair a failed unit. Thus, the power interruption rate caused by a transformer and switch is:

\[
r_{\text{fail}}^{\text{xfsw}} = f_{\text{xfsw}}^{\text{fail}} \cdot t_{\text{xfsw}}^{\text{fail}} \quad (5.10)
\]

where:

\( f_{\text{xfsw}}^{\text{fail}} \): failure rate of a transformer or a switch

\( t_{\text{xfsw}}^{\text{fail}} \): mean time to repair a failed transformer or switch

An interruption rate caused by distribution lines is relative to the length of line. Thus, for a line with unit length, its interruption rate is:

\[
r_{\text{fail}}^{\text{line}} = f_{\text{line}}^{\text{fail}} \cdot t_{\text{line}}^{\text{fail}} \quad (5.11)
\]

where:
$$f_{\text{line}}^{\text{fail}}: \text{failure rate of a distribution line with unit length}$$

$$t_{\text{line}}^{\text{fail}}: \text{mean time to repair a failed distribution line}$$

B. Total Feeder Upgrade Cost

The total feeder upgrade cost is the sum of all branch upgrade costs mainly including the cost of transformer upgrades, switch upgrades and line upgrades. In this thesis, only the replacement of existing feeders will be considered. The transformers and switchgear of the substation are incorporated as branches of the feeder. The cost for each branch includes the new device costs, the cost for removing old devices and the installation cost for the new devices. Then,

$$C_{\text{feeder}}(x,u) = \sum_{j=1}^{n_{\text{max}}} C_{\text{branch}}^j(x,u)$$

where:

$$C_{\text{branch}}^j(x,u): \text{cost of upgrading branch } j$$

C. Total Wheeling Cost

The wheeling costs are determined by the power costs on the substation. Real power prices change with the time of day, which is related to the changes in load demand. The total wheeling cost is the sum of wheeling costs for different load levels in the planning period. The expression is as follows:

$$C_{\text{wheel}}(x,u) = \sum_{j=1}^{n_j} A_j T_j^i (P_{\text{Load}}^j(x,u) + P_{\text{Loss}}^j(x,u)) + A_i S_{\text{max}} \sum_{l=1}^{n_l} T_l^i / (365 \times 24)]

- \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \sum_{l=1}^{n_l} \left( r_{\text{sub}}^{\text{fail}} + \sum_{k=1}^{m_{\text{sub}}} r_{\text{sub},k}^{\text{fail}} + m_{\text{line}}^{\text{fail}} r_{\text{line}}^{\text{fail}} \right) \cdot T_i \cdot (A_i \sum_{p=1}^{c} P_{\text{Load}}^p(x,u))$$

where:
\( A_l \): real power price on the substation for load level \( l \)

\( P^l_{\text{Load}}(x,u) \): total system real power load at load level \( l \)

\( P^l_{\text{Loss}}(x,u) \): total system real power loss at load level \( l \)

\( A_c \): the price of capacity charge on the substation (unit: $/kVA per year)

\( S_{\text{max}}^{\text{inj}} \): the maximum apparent power injection on the substation during whole period of time (unit: kVA)

This expression includes two terms: the first term represents wheeling costs for different load levels in the planning period; the second term represents the costs of estimated outage load, which should not be accounted in the wheeling cost.

D. Constraints

Constraints include both equality and inequality constraints. Three-phase power flow equations \( F(x,u) = 0 \) are the equality constraints. Network operating constraints are represented as inequality constraints:

- Voltage magnitude constraints:
  \[
  V_k^{\min} \leq |V_k^l| \leq V_k^{\max} \quad \forall \text{ nodes } k, \quad k = 1, 2, \ldots, n
  \tag{5.14}
  \]

- Current magnitude constraints:
  \[
  |I_j^l| \leq I_j^{\max} \quad \forall \text{ nodes } j, \quad j = 1, 2, \ldots, n_{\text{branch}}
  \tag{5.15}
  \]

- Feeder capacity constraints:
  \[
  (P_i^l)^2 + (Q_i^l)^2 \leq (S_i^{\max})^2 \quad \forall \quad i \in \mathbb{F}
  \tag{5.16}
  \]

where:
$V^\text{min}_k, V^\text{max}_k$: voltage magnitude limits at bus $k$

$I^\text{max}_j$: line current rating for branch $j$

$S^\text{max}_i$: feeder capacity limit

$\mathbb{F}$: the set of all branches

Feeder upgrades expand distribution system capacity which increases the electric power delivery capability. DG placement within distribution systems can also expand a distribution system’s capability to service load. When DGs are operated in parallel with the substation, the system margin for load supply is increased, since loads are supplied locally. Another possible reliability benefit can be achieved if DGs also are allowed to support loads within islanded areas when faults occur. Next, distribution system expansion through DG placement without/with islanding operation will be discussed.

5.2.2 DG Placement Without Islanding Operation

In this subsection, the problem formulation of distribution system expansion through DG placement without islanding operation will be discussed. The following assumptions are made:

- DGs are always available for operation
- Different real power outputs of a DG for parallel operation are achieved by operating a DG at discrete set points, represented as percentages of its rated real power output
- DGs will provide service after faults downstream of DGs are isolated
- No DGs are allowed to operate in an island
Then, control variables of this problem include DG location, \(u_{DG}^{loc}\), DG unit, \(u_{DG}^{unit}\), DG real power outputs for parallel operation, \(P_G\). Here, the real power output of a DG is the sum of three-phase power injection; due to the unbalanced characteristics of distribution systems, DGs may provide unbalanced outputs within a level of limited imbalanced. All control variables are discrete. The control variable vector is expressed as:

\[
u = [u_{DG}^{loc}, u_{DG}^{unit}, P_G]'\]

where:

\[
\begin{bmatrix}
u_{DG}^{loc} \\
u_{DG}^{unit}
\end{bmatrix}_{n_{DG} \times 1} = [u_1^{loc}, u_2^{loc}, \ldots, u_{n_{DG}}^{loc}]'
\]

\[
\begin{bmatrix}
u_{DG}^{unit} \\
\end{bmatrix}_{n_{DG} \times 1} = [u_1^{unit}, u_2^{unit}, \ldots, u_{n_{DG}}^{unit}]'
\]

\[
\begin{bmatrix}
\begin{bmatrix}P_G^1, \ldots, P_{G_{i,j}}^1, \ldots, P_{G_{i,j}}^n\end{bmatrix}_{G_{DG}^i \times 1}
\end{bmatrix}_{G_{DG}^i \times 1} = [P_G^1, \ldots, P_{G_{i,j}}^1, \ldots, P_{G_{i,j}}^n]\]

and

\(n_{ loc}^{DG} : \) the maximum number of buses for DG installation

\(n_{ gn}^{unit} : \) the maximum number of DGs installed on one bus

\(u_{i,j}^{unit} : \) DG unit of the \(j\)th DG on the \(i\)th installation bus

\(P_{G_{i,j}}^l : \) DG real power output of the \(j\)th DG on the \(i\)th installation bus at load level \(l\)

Thus, the total number of control variables is \((1 + n_{ gn} + n_i \cdot n_{ gn}) \cdot n_{DG}^{loc}\).

Candidates of DG locations are buses, which are allowed to install DGs. Candidates of DG unit are possible DGs to be purchased or are available for installation. Candidates of DG output are discrete outputs, which are percentages of DG rated outputs. The number of candidates are given values for planning. The size of the search space of this problem depends on the candidate numbers of DG location, DG unit and DG output.
\[
\begin{pmatrix}
\text{n}_{\text{DG loc}}^\text{can} \\
n_{\text{DG loc}}
\end{pmatrix}
\begin{pmatrix}
\text{n}_{\text{DG}}^\text{can} \\
n_{\text{DG}}
\end{pmatrix}
^{n_{\text{DG loc}}^\text{can}} \\
\text{n}_{\text{DG loc}}
\begin{pmatrix}
n_{\text{DG}}^\text{output} \\
n_{\text{DG}}
\end{pmatrix}
^{n_{\text{DG loc}}^\text{can}} \\
1
\end{pmatrix} =
\begin{pmatrix}
n_{\text{DG loc}}^\text{can} \\
n_{\text{DG}}^\text{loc}
\end{pmatrix}
\begin{pmatrix}
n_{\text{DG}}^\text{can} \\
n_{\text{DG}}
\end{pmatrix}
^{n_{\text{DG loc}}^\text{can}} \\
1
\begin{pmatrix}
n_{\text{DG}}^\text{output} \\
n_{\text{DG}}
\end{pmatrix}
^{n_{\text{DG loc}}^\text{can}} \\
1
\end{pmatrix}
\] (5.18)

where:

\(n_{\text{DG loc}}^\text{can}\): the number of bus candidates for DG installation

\(n_{\text{DG}}^\text{can}\): the number of DG candidates

\(n_{\text{DG}}^\text{output}\): the number of discrete real power outputs of a DG

and

\(\begin{pmatrix}
n_{\text{DG loc}}^\text{can} \\
n_{\text{DG loc}}^\text{loc}
\end{pmatrix}\) is the size of search space for DG location

\(\begin{pmatrix}
n_{\text{DG}}^\text{can} \\
n_{\text{DG}}^\text{loc}
\end{pmatrix}\) is the size of search space for DG unit

\(\begin{pmatrix}
n_{\text{DG}}^\text{output} \\
n_{\text{DG}}^\text{loc}
\end{pmatrix}\) is the size of search space for DG output

The objective function of distribution expansion planning through DG placement without islanding operation includes four parts: total outage cost, total DG installation cost, total DG operating cost and total wheeling cost:

\[
f(x,u) = C_{\text{outage}}(x,u) + C_{\text{DGin}}(x,u) + C_{\text{DGop}}(x,u) + C_{\text{wheel}}(x,u) \quad (5.19)
\]

where:

\(C_{\text{DGin}}(x,u)\): total DG installation cost

\(C_{\text{DGop}}(x,u)\): total DG operating cost
A. Total Outage Cost

The total outage cost is the sum of all customer interruption cost for all load levels during the planning period. For the problem formulation of DG placement without islanding operation, the expression for total outage cost is the same as this of feeder upgrades in (5.8).

B. Total DG Installation Cost

The total DG installation cost is the sum of the initial cost of installing distributed generators including the equipment cost, installation cost, cost of DG control and protection devices, etc.

\[
C_{DGin} (x,u) = \sum_{i=1}^{n_g} C_{i}^{in} (x,u) \tag{5.20}
\]

where:

\[C_{i}^{in} (x,u) : \text{installation cost for the DG } i\]

\[n_g : \text{number of DG to be installed}\]

C. Total DG Operating Cost

The DG operating cost is the total DG parallel operating cost. For different DG candidates, their operating costs are different. Thus,

\[
C_{DGop} (x,u) = \sum_{i=1}^{n_l} \sum_{j=1}^{n_g} \sum_{l=1}^{n_x} T_{Gi,j}^{l} B_{i,j} P_{Gi,j}^{l} (x,u) \tag{5.21}
\]

\[= \sum_{i=1}^{n_l} \sum_{j=1}^{n_g} \sum_{l=1}^{n_x} (T_{Gi,j}^{l} - T_{Gi,j}^{l,shut}) B_{i,j} P_{Gi,j}^{l} (x,u)\]

where:

\[T_{Gi,j}^{l} : \text{parallel operation time in hours of the } j\text{th DG on the } i\text{th DG installation}\]
bus at load level $l$

$T_l$: time duration in hours of load level $l$

$T_{Gi,j}^{l,shut}$: disconnection duration time of the $j$th DG on the $i$th DG installation bus at load level $l$

$B_{i,j}$: power generating cost of the $j$th DG on the $i$th DG installation bus

$P_{Gi,j}^l(x,u)$: real power output of the $j$th DG on the $i$th DG installation bus at load level $l$

There are three reasons for a DG to disconnect from the network during parallel operation: the DG is disconnected during normal operation; the DG is disconnected due to faults upstream; the DG is disconnected due to faults downstream. Thus, for the $j$th DG on the $i$th DG location bus at load level $l$, its disconnection duration is,

$$T_{Gi,j}^{l,shut} = T_{Gi,j}^{l,shut1} + T_{Gi,j}^{l,shut2} + T_{Gi,j}^{l,shut3}$$ (5.22)

where:

$T_{Gi,j}^{l,shut1}$: disconnection duration during normal network operation

$T_{Gi,j}^{l,shut2}$: disconnection duration because of faults upstream

$T_{Gi,j}^{l,shut3}$: disconnection duration because of faults downstream

The DG disconnection duration during normal network operation is generally a percentage of the duration time of a load level:

$$T_{Gi,j}^{l,shut1} = x_l \cdot T_l$$ (5.23)

$x_l$: DG disconnection percentage during normal operation at load level $l$

If a fault occurred upstream of a DG, this DG would be disconnected. Thus,

$$T_{Gi,j}^{l,shut2} = (r_{sub} \sum_{k \in U_i} r_{xfsw,k} + m_i \sum_{k \in U_i} r_{line}) \cdot T_l$$ (5.24)
If a fault occurred downstream of a DG, this DG should be disconnected from the network first for DG protection, then reconnected to network after the fault is isolated [45-48]. To compute $T_{Gi,j}^{shut}$, the DG operating procedure with a fault downstream will be discussed. Figure 5.1 shows an example and the operating steps are explained in the following:

- At $T_0$, a fault occurs on the downstream of the DG
- At $T_1$, Switch 2 (SW2) is open, and the DG is disconnected from the network
- At $T_2$, Switch 1 (SW1) is open, and the downstream fault is isolated
- At $T_3$, Switch 2 is close, and the DG reconnects to the network
- At $T_4$, the DG’s real power output reaches to its set output

The duration time between $T_1$ and $T_4$ is considered the DG disconnection duration:

$$t_G^{shut} = T_4 - T_1$$  \hspace{1cm} (5.25)
The number of faults occurring on DG downstream are estimated through transformer, switch and line failure rate $f_x^{\text{fail}}$, and $f_{\text{line}}^{\text{fail}}$. Then, the DG disconnection duration time for a load level $l$ is

$$T_{l,\text{shut}}^{\text{d}} = t_{\text{shut}}^{\text{d}} \cdot \left( \sum_{k \in H_i} f_x^{\text{fail}} k + m_{l,\text{down}}^{\text{d}} f_{\text{line}}^{\text{fail}} \right) \cdot T_l$$

(5.26)

where:

- $t_{l,\text{shut}}^{\text{d}}$: average disconnection duration time of the $j$th DG on the $i$th DG installation bus with a downstream fault
- $H_i$: set of switch and transformer on downstream of the $i$th DG installation bus
- $m_{l,\text{down}}^{\text{d}}$: the total feeder length on downstream of the $i$th DG installation bus

When faults occur downstream of DGs, the disconnection duration time of a DG only lasts minutes at most, while planning units are hours. Thus, DGs are assumed to be always on. That is the average disconnection duration time for a DG, when a fault occurs downstream, is considered zero, $t_{l,\text{shut}}^{\text{d}} = 0$. Therefore, the DG disconnection duration time is accounted as zero, $T_{l,\text{shut}}^{\text{d}} = 0$, due to its downstream faults

D. Total Wheeling Cost

The total wheeling cost is the sum of wheeling costs for different load levels in the planning periods. The expression is as follows:

$$C_{\text{wheel}}(x,u) = \sum_{j=1}^{n} A_j [T_i (P_{\text{Load}}^i (x,u) + P_{\text{Loss}}^i (x,u)) - \sum_{j=1}^{n} \sum_{k=1}^{n_{\text{gen}}} T_{l,\text{sub}}^{ij} P_{\text{Gi},j}^i (x,u)] + A_{\text{c}} S_{\text{m}}^{\text{max}} \sum_{j=1}^{n} [T_j / (365 \times 24)]$$

$$- \sum_{l=1}^{n} \sum_{i=1}^{n} (r_{l,\text{sub}}^{\text{fail}} + \sum_{k \in U_i} r_{l,\text{sfrw}}^{\text{fail}} + m_{l,\text{sub}}^{\text{d}} r_{l,\text{line}}^{\text{fail}}) \cdot T_l \cdot (A_j \sum_{p=1}^{c} P_{\text{Load},j}^p (x,u))$$

(5.27)
Since load supplied by DG should not be accounted for wheeling cost, the expression (5.27) has one more term, \( \sum_{i=1}^{n_{DG}} \sum_{j=1}^{n_{loc}} T_{Gi,j}^{l} P_{Gi,j}^{l} (x,u) \), which represents load supplied by DGs, than that of feeder upgrades (5.13). Thus, the wheeling cost with DG placement will be reduced. Since the power price on the substation may be much higher than that of DGs during some period of time due to transmission congestion, proper DG placement and operating may help utilities to reduce system operating cost.

### E. Constraints

Constraints also include both equality and inequality constraints. The equality constraints are the three-phase power flow equations \( F(x,u) = 0 \). Inequality constraints include (5.14) to (5.16), and two more network operating constraints related to DGs:

- Maximum DG penetration constraints:
  \[ \sum_{i} P_{Gi}^{cap} \leq P_{DG}^{max} \]  
  \( (5.28) \)

- DG capacity constraints:
  \[ P_{Gk}^{min} \leq P_{Gk}^{\Phi} \leq P_{Gk}^{max} \]  
  \[ Q_{Gk}^{min} \leq Q_{Gk}^{\Phi} \leq Q_{Gk}^{max} \]  
  \( (5.29) \) 

### 5.2.3 DG Placement with Islanding Operation

In this subsection, the problem formulation of distribution system expansion considering DG placement with DG islanding operation will be discussed. The following assumptions are made:

- DGs are always available for operation
Different real power outputs of a DG for parallel operation are achieved by operating a DG at discrete set points, represented as percentages of its rated real power output.

DGs will provide service after faults downstream of DGs are isolated.

DGs are allowed for islanding operation coordinated with switch placement.

Then, the control variables of this problem include DG location $u_{DG}^{loc}$, DG unit $u_{DG}^{unit}$, DG real power outputs for parallel operation $P_G$, new switch placement, $u^{sw}$, and DG real power output for islanding operation $P_G^{is}$. Since DG real power outputs for islanding operation depends on the load and loss within islanded areas, they can not be set at specified output points. Thus, DG real power outputs for islanding operation are continuous variables. Other control variables are discrete vector variables. A mathematical expression of the control variables are:

$$u = [u_{DG}^{loc}, u_{DG}^{unit}, P_G, u^{sw}, P_G^{is}]$$

where:

$$[u_{DG}^{loc}]_{nDG,nG_1} = [u_{DG}^{loc,1}, u_{DG}^{loc,2}, \ldots, u_{DG}^{loc,nDG}]$$

$$[u_{DG}^{unit}]_{nDG,nG_1} = [u_{DG}^{unit,1}, u_{DG}^{unit,2}, \ldots, u_{DG}^{unit,nDG}]$$

$$[P_G]_{nDG,nG_1} = [P_{G,j,1}^{1}, P_{G,j,2}^{1}, \ldots, P_{G,j,nDG}^{1}, P_{G,j,1}^{2}, P_{G,j,2}^{2}, \ldots, P_{G,j,nDG}^{2}, \ldots, P_{G,j,1}^{nDG}, P_{G,j,2}^{nDG}, \ldots, P_{G,j,nDG}^{nDG}]$$

$$[u^{sw}]_{n_b} = [u^{sw,1}, u^{sw,2}, \ldots, u^{sw,n_b}]$$

$$[P_G^{is}]_{nDG,nG_1} = [P_{G,j,1}^{is,1}, P_{G,j,2}^{is,1}, \ldots, P_{G,j,nDG}^{is,1}, P_{G,j,1}^{is,2}, P_{G,j,2}^{is,2}, \ldots, P_{G,j,nDG}^{is,2}, \ldots, P_{G,j,1}^{is,nDG}, P_{G,j,2}^{is,nDG}, \ldots, P_{G,j,nDG}^{is,nDG}]$$

with:

$u_i^{sw}$: new switch status on branch $i$ (1: install new switch, 0: no new switch)
\[ P_{\text{DG},i,j}^{l} \]: DG real power output of the \( j \)th DG on the \( i \)th DG installation bus at load level \( l \) for islanding operation

Two more control variable vectors, new switch placement and DG real power output of islanding operation exist compared to the problem without DG islanding operation in Section 5.2.2. The total number of control variables of this problem is

\[(1 + n_{\text{gn}} + 2 \cdot n_{\text{gn}} \cdot n_{\text{j}}) \cdot n_{\text{loc}} + n_{\text{branch}}.\]

The size of the search space of this problem depends on candidate numbers of DG locations, DG units, DG outputs, branches for switch placement:

\[
\begin{pmatrix}
(n_{\text{DGloc}}^{\text{can}}) \\
(n_{\text{DG}}^{\text{can}})
\end{pmatrix}
\cdot
\begin{pmatrix}
\gamma_{\text{DG}}^{\text{loc}} \\
\gamma_{\text{DG}}^{\text{can}}
\end{pmatrix}
\cdot
\begin{pmatrix}
\gamma_{\text{DG}}^{\text{output}} \\
\gamma_{\text{DG}}^{\text{can}}
\end{pmatrix}
\cdot
2^{n_{\text{can}} - n_{\text{d}}}
\cdot
\begin{pmatrix}
\gamma_{\text{DG}}^{\text{branch}} \\
\gamma_{\text{DG}}^{\text{can}}
\end{pmatrix}

= \begin{pmatrix}
(n_{\text{DGloc}}^{\text{can}}) \\
(n_{\text{DG}}^{\text{can}})
\end{pmatrix}
\cdot
\begin{pmatrix}
\gamma_{\text{DG}}^{\text{loc}} \\
\gamma_{\text{DG}}^{\text{can}}
\end{pmatrix}
\cdot
\begin{pmatrix}
\gamma_{\text{DG}}^{\text{output}} \\
\gamma_{\text{DG}}^{\text{can}}
\end{pmatrix}
\cdot
2^{n_{\text{can}} - n_{\text{d}}}
\tag{5.32}
\]

where:

- \( n_{\text{DGloc}}^{\text{can}} \): the number of bus candidates for DG installation
- \( n_{\text{DG}}^{\text{can}} \): the number of DG candidates
- \( n_{\text{DG}}^{\text{output}} \): the number of discrete real power outputs of one DG

with:

\[
\begin{pmatrix}
(n_{\text{DGloc}}^{\text{can}}) \\
(n_{\text{DG}}^{\text{can}})
\end{pmatrix}
\text{ is the size of search space for DG location}
\]

\[
\begin{pmatrix}
\gamma_{\text{DG}}^{\text{loc}} \\
\gamma_{\text{DG}}^{\text{can}}
\end{pmatrix}
\text{ is the size of search space for DG unit}
\]

\[
\begin{pmatrix}
\gamma_{\text{DG}}^{\text{output}} \\
\gamma_{\text{DG}}^{\text{can}}
\end{pmatrix}
\text{ is the size of search space for DG output for parallel operation}
\]

or islanding operating
is the size of search space for switch placement

The objective function of distribution expansion planning with DG placement with islanding operation includes four parts: total outage cost, total DG installation cost, total DG operating cost and total wheeling cost:

\[ f(x, u) = C_{\text{outage}}(x, u) + C_{\text{DGin}}(x, u) + C_{\text{DGop}}(x, u) + C_{\text{wheel}}(x, u) \] (5.33)

In this thesis, DG islanding operation cost and saving are accounted for total outage cost. Thus, the expressions of \( C_{\text{DGin}}(x, u) \), \( C_{\text{DGop}}(x, u) \) and \( C_{\text{wheel}}(x, u) \) are the same as the previous subsection 5.2.3 for DG placement without islanding operation, and will not be repeated. Since the \( C_{\text{outage}}(x, u) \) is changed, its expression is discussed as following:

\[ C_{\text{outage}} = \sum_{l=1}^{n_l} C_{\text{noDG}}^{l}(x, u) - \sum_{l=1}^{n_l} C_{\text{DGave}}^{l}(x, u) + \sum_{i=1}^{n_{\text{switch}}} C_{i}^{\text{switch}} \] (5.34)

where:

\[ C_{\text{DGave}}^{l}(x, u) = \sum_{k=1}^{n_{\text{line}}} (r_{\text{sub}} + \sum_{j \in U_{\text{island}, k}^{l}} r_{\text{line}} + m_{l}^{\text{island}, k-\text{Sub}} \cdot T_{l} \cdot \sum_{i \in D_{l}} (C_{\text{out}}^{l} \sum_{p=1}^{c} P_{\text{Load}, p} \cdot j) \] (5.35)

with:

\( C_{l}^{\text{noDG}}(x, u) \): the total interruption cost without DG islanding operation at load level \( l \) (the same as total outage cost of feeder upgrades)

\( C_{l}^{\text{DGave}} \): the saving cost through DG islanding operation at load level \( l \)

\( m_{l}^{\text{island}, k-\text{Sub}} \): feeder length between the \( k \)th islanded area and the substation at load level \( l \)

\( U_{\text{island}, k}^{l} \): set of switch or transformer branches between the \( k \)th islanded
area and the substation at load level $l$

$D_k^l$: set of buses supplied by DGs in the $k$th islanded area at load level $l$

$n_{island}^l$: the number of islanded areas at load level $l$

$C_{switch}^i$: cost for the $i$th new switch

$n_{new}$: the number of new switches to be installed

The constraints of the problem of DG placement with DG islanding operation are the same as those of DG placement in Section 5.2.2 without DG islanding operation.

5.2.4 DG Placement with Feeder Upgrades

In this subsection, the problem of distribution system expansion with DG placement and feeder upgrades is discussed. The problem becomes much more complex than the previous sub-problems. If DG islanding operation is allowed, this problem formulation has the same assumptions as Section 5.2.3.

The control variables of this problem include DG location $u_{DG}^{loc}$, DG unit $u_{DG}^{unit}$, DG real power outputs of parallel operation $P_G$, new switch placement $u^{sw}$, DG real power output of islanding operation $P_{Gis}$, and branch conductor type $u^f$. All control variables except the DG real power output of islanding operation are discrete vector variables. The mathematical expression of the control variables are:

$$ u = [u_{DG}^{loc}, u_{DG}^{DG}, P_G, u^f, u^{sw}, P_{Gis}] \text{ } \text{ } \text{ } \text{ } (5.36) $$

where:

$$ [u_{DG}^{loc}]_{n_{DG} \times 1} = [u_1^{loc}, u_2^{loc}, \cdots, u_{n_{DG}}^{loc}]^{T} $$
Thus, the total number of control variables is \((1 + n_{gn} + 2 \cdot n_{gs} \cdot n_l) \cdot n_{loc}^{DG} + 2 \cdot n_{branch}^{sw}\). The size of search space is also increased dramatically:

\[
\left[ u_{DG}^{sw} \right]_{n_{DG}^{loc} \cdot n_{gs} \cdot n_l}^{n_{DG}^{loc} \cdot n_{gs} \cdot n_l} = \left[ u_{DG}^{sw} \right]_{n_{DG}^{loc} \cdot n_{gs} \cdot n_l}^{n_{DG}^{loc} \cdot n_{gs} \cdot n_l}
\]

This is a large search space. If the inherent system relationship or constraints of control variables are applied, the size of search space would be reduced. For example, a given 20-bus system has 2 transformer, 6 switch and 11 line branches. It is assumed that upgrade options for a branch of line, switch and transformer are 8, 4 and 8, respectively; 12 different DG units are available; 15 buses are candidates for DG installation; 3 is the maximum number of buses to install DG; at most 2 DGs are installed on one bus; DGs only have two outputs for parallel operation: 90% rated outputs or zeros output; there are 3 load levels. Then, the size of search space would be \(1.0334 \times 10^{40}\). Thus, reducing search space is helpful for solving this comprehensive problem. Specifically, some control variables can be determined by inherent system relationships or constraints, when values of other control variables are specified. These control variables are considered as
dependent control variables. For example, feeder upgrades are depended on locations and sizes of DG installation. If feeder upgrades, new switch placement and DG output for islanding operation are considered as dependent control variables, the size of search space would be reduced to $1.0332 \times 10^{15}$. More details about dependent control variables will be discussed in Section 5.3.

The objective function of this problem includes five parts: total outage cost, total DG installation cost, total DG operating cost, total wheeling cost and total feeder upgrade cost:

$$f(x,u) = C_{\text{outage}}(x,u) + C_{\text{DGin}}(x,u) + C_{\text{DGop}}(x,u) + C_{\text{wheel}}(x,u) + C_{\text{feeder}}(x,u)$$ (5.38)

The first four terms in this expression are the same as the expressions of the problem of DG placement with DG islanding, and fifth term, feeder upgrade $C_{\text{feeder}}(x,u)$ is the same as the expression (5.12) in the feeder upgrade subsection.

Since the constraints of this problem are also the same as those of DG placement, they will not be repeated here. Now, solution algorithms for the distribution expansion problem of DG placement with feeder upgrades based on this subsection formulation are discussed.

### 5.3 Solution Algorithm

A GA-based algorithm is proposed to solve the optimization problem of distribution system expansion planning of DG placement and feeder upgrades. The GA-based algorithm includes a genetic algorithm and heuristic portions to handle dependent control variables based on three-phase power flow analysis. The outline of solution algorithm is shown in Figure. 5.2.
In this algorithm, not all control variables are determined by a genetic algorithm. The control variables are divided into two types based on different methods to generate or change their values:

- **GA control variables**
  - Dependent control variables

GA control variables include DG location, DG unit and DG real power output for parallel operation; dependent control variables are feeder upgrades, new switch placement and DG real power output for DG islanding operation.

- The substrings of GA control variables can be randomly initialized, and participate in crossover and mutation;
- The dependent control variables are determined by a heuristic algorithm based on three-phase power flow studies using network and parameter values provided by the GA.

Since the number of GA control variables is less than the number of total control variables and dependent control variables are determined by inherent system relationships
or constraints, the search space of the GA is greatly reduced. As a result, the computation time may be shortened and solution quality should not be affected.

A. Coding

The two types of control variables are coded differently for the GA-based heuristic algorithm:

1. GA control variables

   DG location, DG unit and DG output for parallel operation substrings are coded as follows:

   • An integer coded substring exists for DG placement location. Its length is a user input \( n_{DG}^{loc} \), the maximum number of installation buses.

   • A binary coded substring of a constant multiple of \( n_{DG}^{loc} \) represents DG units at their corresponding locations.

   • For each load level, an individual has a binary coded substring representing discrete outputs for each DG at each DG installation location. The size of the string of \( P_{G} \) is a constant multiple \( n_{DG}^{loc} \cdot n_{DG}^{loc} \cdot n_{i} \). If the binary value is zero for all load levels, no DG is placed at the corresponding location.

2. Dependent control variables

   Feeder upgrades, new switch placement and DG output for islanding operation substrings are used to record system parameter changes based on power flow studies with specified DG placements and operations:
• An integer coded substring of length $n_{\text{branch}}$ represents the branch upgrade substring. 0 represents no upgrade, $i$ represents upgrade option $i$ on this branch ($i = 1, 2, \ldots$). It is decided by a heuristic algorithm for feeder upgrade.

• An integer coded substring of length $n_{\text{branch}}$ represents locations to install new switch on branches. 0 represents no new switch; 1 represent new switch on this branch. It is decided by a heuristic algorithm for switch placement.

• For each load level, an individual has a decimal coded substring representing outputs for each DG at each DG installation location. Its size is a constant multiple $n_{\text{GA}} \cdot n_{\text{DG}}^{\text{loc}} \cdot n_i$.

Please see Table 5.1 for a summary.

<table>
<thead>
<tr>
<th>Substrings</th>
<th>Coding</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>DG location</td>
<td>Integer</td>
<td>Bus number</td>
</tr>
<tr>
<td>DG unit</td>
<td>Binary</td>
<td>Binary code for DG unit</td>
</tr>
<tr>
<td>DG output for parallel operation</td>
<td>Binary</td>
<td>Binary code for DG real power outputs for parallel operation</td>
</tr>
<tr>
<td>Feeder upgrade</td>
<td>Integer</td>
<td>Record branch upgrades</td>
</tr>
<tr>
<td>New switch placement</td>
<td>Integer</td>
<td>Record new switch</td>
</tr>
<tr>
<td>DG output for islanding operation</td>
<td>Decimal</td>
<td>Record DG real power outputs for islanding operation</td>
</tr>
</tbody>
</table>

B. Initialization

Different methods are applied for different substrings.

1. The substrings for GA control variables:

   • DG locations are biased based on the buses on availability of feeder capacities.

     For example, the buses on the downstream path of overloaded equipments have higher possibility to be chosen.
DG unit and DG output for parallel operation substrings are initialized randomly.

2. The recording substring for dependent variables:

The substrings of feeder upgrade, switch placement and DG output for islanding operation are initialized zeros, which will be changed to record results from heuristic parts based on power flow study.

C. Feeder Upgrades

The network upgrade options are chosen to guarantee current magnitude constraints (5.15) and feeder capacity constraints (5.16) to be satisfied corresponding to each individual DG placement, which is provided by GA strings. If some branches can not meet the constraints at any load level for a DG placement individual, these branches are upgraded and their upgrade options would be recorded in the string of feeder upgrade.

D. Switch Placement and DG Islanding Operation

DG islanding operation can continuously support select customers to reduce outage cost, when a fault occurs. Switch placement and operating are required to coordinate DG islanding operation. Its solution algorithm has been discussed in Chapter 4: a graph-based algorithm of switch placement in [31] is adapted to use a three-phase power flow with the distributed slack bus model using generator participation factors.

E. Fitness Evaluation

In order to evaluate the fitness of the above strings, a scaled fitness function incorporates the objective function as an optimization problem and penalty functions are used. The fitness function \( f \) can be expressed as follows:
where $f_i$ is from (5.38), the aggregate objective function of total cost of individual $i$; $\Phi_1(x,u)$ and $\Phi_2(x,u)$ are penalty function associated with the constraints of voltage magnitudes (5.14) and maximum DG penetration (5.28). The equality constraint, $F(x,u) = 0$, and other inequality constraints (5.15), (5.16), (5.29) and (5.30) are satisfied and includes in $f_i$. Then, the fitness function is scaled by the individual in the current population with maximum fitness:

$$ff_i = f_i - \Phi_1(x,u) - \Phi_2(x,u)$$

(5.39)

The GA uses values from (5.40) for selection.

**F. Selection**

Stochastic sampling without replacement (roulette wheel) is applied for the selection [43].

**G. Crossover and Mutation**

Single crossover for each substring of GA control variables is performed based on the given crossover rate. Finally, a small mutation rate is employed for each bit of these substrings. The procedure of this GA-based algorithm is shown in the flow chart of Figure 5.3.
Figure 5.3: Flow Chart of the GA-Based Algorithm
5.4 Simulation Results

A 20-bus distribution system is used for simulation analysis. The one-line diagram of this system is shown in Figure 5.4. This system has two 4 MW transformers, 12 loads, and 11 line feeders with total length of cables 37,890 feet (about 7.18 miles). In the following cases, total costs using different methods to expand the distribution system will be compared.

Parameter values have been selected as followed:

Cost data

- Wheeling cost:
  - Electricity energy charge: 0.075 $/kWh
  - Electricity capacity charge: 45 $/kVA per year

- Transformer upgrade cost: $432,000 each (from 4MVA to 6MVA)

- Line upgrade cost: $30,000 per 1000 feet (from 300 Amps to 450 Amps)
• DG cost: see Table 5.2

<table>
<thead>
<tr>
<th>DG Units</th>
<th>Size (MW)</th>
<th>Installation Cost (US dollars)</th>
<th>Operation Cost (US dollars/ kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reciprocating</td>
<td>0.5</td>
<td>217,000</td>
<td>0.0735</td>
</tr>
<tr>
<td>Reciprocating</td>
<td>1.0</td>
<td>433,000</td>
<td>0.070</td>
</tr>
<tr>
<td>Reciprocating</td>
<td>1.2</td>
<td>510,000</td>
<td>0.0680</td>
</tr>
<tr>
<td>Reciprocating</td>
<td>1.5</td>
<td>649,000</td>
<td>0.0665</td>
</tr>
<tr>
<td>Mini Gas</td>
<td>0.5</td>
<td>210,000</td>
<td>0.0824</td>
</tr>
<tr>
<td>Mini Gas</td>
<td>1.0</td>
<td>420,000</td>
<td>0.0880</td>
</tr>
<tr>
<td>Mini Gas</td>
<td>1.2</td>
<td>523,000</td>
<td>0.0850</td>
</tr>
<tr>
<td>Mini Gas</td>
<td>1.5</td>
<td>630,000</td>
<td>0.0836</td>
</tr>
<tr>
<td>Fuel Cell</td>
<td>0.5</td>
<td>375,000</td>
<td>0.0788</td>
</tr>
<tr>
<td>Fuel Cell</td>
<td>1.0</td>
<td>750,000</td>
<td>0.0750</td>
</tr>
<tr>
<td>Fuel Cell</td>
<td>1.2</td>
<td>910,000</td>
<td>0.0730</td>
</tr>
<tr>
<td>Fuel Cell</td>
<td>1.5</td>
<td>1,125,000</td>
<td>0.0712</td>
</tr>
<tr>
<td>Wind</td>
<td>0.5</td>
<td>875,000</td>
<td>0.085</td>
</tr>
<tr>
<td>Wind</td>
<td>1.0</td>
<td>1,750,000</td>
<td>0.0765</td>
</tr>
<tr>
<td>Wind</td>
<td>1.2</td>
<td>2,150,000</td>
<td>0.0715</td>
</tr>
<tr>
<td>Wind</td>
<td>1.5</td>
<td>2,625,000</td>
<td>0.065</td>
</tr>
</tbody>
</table>

**Load levels**

• Three load levels:
  - low-load level: 0.7 times of the base load
  - medium-load level: the base load
  - high-load level: 1.1 times of the base load

• Each load level lasts one year

• The total three-phase base load is 6.6343 MW and 2.0964 Mvar:
  - phase a: 2.2144 MW, 0.7149Mvar
  - phase b: 2.2095MW, 0.6782Mvar
  - phase c: 2.2104MW, 0.7033Mvar
Outage information

- Outage costs:
  - high priority load: 45.82 $/kWh
  - medium priority load: 7.61 $/kWh
  - low priority load: 2.07 $/kWh

- Interruption rates:
  - the substation: about 4.38 hours/years (power availability: 99.995%)
  - transformers and switches: 0 (assumed always available)
  - distribution lines: 0.0152 hours/year per 1000 ft

Case 1. Original System

If the original system is used, the capacity constraint of 4 MVA for a transformer will be violated under High-load level: power through the transformer between Bus 2 and Bus 4 is 4.2008 MVA. This transformer overload cannot be released by transferring loads within this system. Thus, the system must be expanded for safe operation.

Case 2. Feeder upgrade without DG placement

In this case, the transformer between Bus 2 and Bus 4 was upgraded, which is the minimum upgrade cost to satisfy the operation constraints. Then, the total cost for three-year operation:

Total costs = Outage costs + Wheeling costs + Upgrade costs

= $2,828,570 + $12,601,690 + $432,000

= $15,862,260
Case 3. Feeder upgrade with DG placement

In this case, feeder upgrade and DG placement are considered together using the GA-based algorithm and a pure GA algorithm, which is used for comparison. All control variables are GA variables in the pure GA algorithm. The crossover and mutation rates for both algorithms are 0.5 and 0.3, respectively to obtain diversity populations. The result planning strategies is shown in the following:

Planning Strategy 1, obtained from the proposed GA-based heuristic algorithm:

- No feeder upgrade;
- One 1 MW reciprocating DG is installed on Bus 5; its operation outputs are 0, for low-load levels, and 90% of rated output for medium-, and high-load levels;
- No new switch is installed; an islanded area including Bus 5, 6, 12 is formed for DG islanding operation at low-load level.

Planning Strategy 2, obtained from a pure GA algorithm:

- The line branches 4-5,8-9,3-15 are upgrades;
- One 1 MW reciprocating DGs is installed on Bus 12; its operating outputs parallel to the substation are 0 for low-load levels, and 90% of rated output for medium-, and high-load levels;
- New switches are installed on branches between Bus 5-6,7-8,13-14,16-17; no islanded area can be formed for DG islanding operation.

Costs of these planning strategies are shown in Table 5.3.
Table 5.3 Planning Costs for a 20-Bus System Expansion

<table>
<thead>
<tr>
<th>Costs ($ USD)</th>
<th>Planning Strategy 1 GA-based heuristic algorithm</th>
<th>Planning Strategy 2 Pure GA algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outage Costs</td>
<td>2,655,460</td>
<td>2,834,002</td>
</tr>
<tr>
<td>DG Installation Costs</td>
<td>433,000</td>
<td>433,000</td>
</tr>
<tr>
<td>DG Operating Costs</td>
<td>1,839,600</td>
<td>1,226,400</td>
</tr>
<tr>
<td>Wheeling Costs</td>
<td>10,554,095</td>
<td>11,162,935</td>
</tr>
<tr>
<td>Feeder Upgrade Costs</td>
<td>0</td>
<td>186,000</td>
</tr>
<tr>
<td>Total Costs</td>
<td>15,482,155</td>
<td>15,842,337</td>
</tr>
</tbody>
</table>

Figure: 5.5 Performances of Various Algorithm
In this case, the proposed GA-Based algorithm provides a high quality solution, which is $380,105 cheaper than the solution provided by pure feeder upgrade in Case 2. Compared to pure GA algorithm, cost of the solution from GA-based algorithm is also $360,182 less. Figure 5.5 shows performances of the algorithms. Through DG islanding operation, the total outage cost can be reduced by $173,110. Although the solution from the GA-based algorithm is DG placement only and no feeder upgrade, it does not mean that DG placement always can beat feeder upgrade. Considering feeder upgrade and DG placement together could provide a more diverse expansion solution for utilities. Here, the simulation demonstrates that proposed GA-based algorithm can find high quality optimal solution for distribution system expansions.

From above simulation, comments and observations are summarized as follows:

- The proposed GA-based algorithm successfully found high quality solutions;
- DG placement with feeder upgrade can provide more diverse expansion solutions;
- DG placement can avoid or delay equipment upgrades;
- DG islanding operation coordinated with switch placement can improve reliability and reduce outage costs.

5.5 Comments

In this chapter, a problem formulation of distribution system expansion planning with DG placement is proposed. The impacts of distributed generation under load expansion were considered. A cost-based objective function considering outage costs, wheeling costs, feeder upgrade costs, DG installation costs and DG operating costs were discussed. The problem is also subject to electrical, operational and network constraints.
A GA-based algorithm was proposed to solve the distribution system expansion planning optimization problem. In the proposed algorithm, DGs’ initial locations are biased using available feeder capacities. A heuristic based on three-phase power flow studies determines dependent control variables within the genetic algorithm. By these ways, the algorithm’s computation time is reduced and performance is improved. The simulation results show that the proposed algorithm can provide high quality solutions.
CHAPTER 6. Conclusions

With distributed generation introduced to distribution systems, traditional methods for distribution system analysis and planning need to be revisited. The objective of this work was to develop modeling, analysis and planning tools for distribution systems with distributed generation. Toward this objective, the contributions made in this thesis will be summarized in this chapter. In addition, extensions and future work will be discussed.

6.1 Contributions

This thesis provided work toward developing modeling, analysis and planning tools for distribution systems with DGs. Slack bus modeling for distribution power flow analysis has been investigated, and the following work has been contributed:

- distribution power flow with a distributed slack bus model for DGs
- scalar participation factors to distribute uncertain real power system loss for three-phase power flow calculations
- two methods to calculate network-based participation factors
  - sensitivity-based method
  - generator domain based method
- a Newton-Raphson solver implemented the distributed slack model with iterative participation factors
- numerical studies on a 20-bus distribution system for different slack bus models
- detailed simulation results on a 394 bus system with different numbers and different levels of DG penetration
The distribution power flow with a single slack bus model was revisited, and a slack bus model was developed for distribution systems with DGs. The participation factors based on generator domains, which are explicitly relative to network parameters and load distributions, demonstrate their ability to capture network characteristics and to scale loss contributions of sources surpasses other participation factors. Therefore, the distributed slack model with generator domain participation factors was recommended.

Then, impacts of the recommended distributed slack bus model have been investigated with the following contributions made:

- application functions for economic and technical issues of distribution systems with DGs
- a cost analysis method with distinguishing loss and load contributions of individual DGs
  - detailed mathematical expressions
  - extensive simulation analysis on a 27-bus distribution system with different DG locations and penetrations
- switch placement for DG islanding operation

As an example for economic applications, a cost analysis method was discussed. Slack bus modeling for distribution power flow was linked to cost analysis for distribution systems with DGs. The distributed slack bus model with generator domain participation factors demonstrated their advantages for the proposed cost analysis method through quantifying loss and load contributions from individual sources. The proposed method for cost analysis may help regulators to set fair pricing schemes. For a technical issue, switch
placement for DG islanding operation demonstrated that different slack bus models can result in different results of switch placement and operation.

Finally, in order to design strategies for DG placement and operation, the following work has been presented:

- problem formulations for distribution system expansion planning
  - feeder upgrades
  - DG placement without islanding operation
  - DG placement with islanding operation
  - DG placement with feeder upgrades
- a GA-based algorithm to solve the combinatorial optimization problem of distribution system expansion planning with DG placement and feeder upgrades
- simulation results on a 20-bus, unbalanced, radial distribution system for expansion planning with DG placement

The objective of expansion planning was to minimize operation and planning cost subject to constraints. Applications of the recommended distributed slack bus model are included in cases of planning. Feeder upgrades, DG placement and different allowable DG operating modes have been considered in the planning. To shorten computation time for these complex problems, a GA-based algorithm has been proposed. Simulations demonstrated that the GA-based algorithm can provide high quality solutions for distribution expansion planning within a shorter time than GA algorithm. These solutions provided a guide for DG placement and operating planning.
6.2 Extensions and Future Work

In this thesis, the distributed slack bus applied participation factors to distribute real power loss to participating sources. With increasing interest on reactive power dispatch and control in distribution systems, reactive power control for DGs also becomes possible [10, 56]. Thus, the following problems related to slack bus modeling also deserve to be explored:

- how to apply reactive power loss distribution to a distributed slack bus model in power flow study:
  - what are differences between participation factors of reactive power and real power loss contributions;
  - how to combine these different kinds of participation factors;

- what application functions and what will be the impacts, when distributed slack model for reactive power loss is introduced.

The distribution power flow with a distributed slack bus model presented in this thesis can be applied in many areas. As discussed in Chapter 4, slack bus modeling may widely affect economic and technical issues of distribution systems. Only two examples were studied in detail in Chapter 4. Moreover, if reactive power loss were considered for distributing slack, some applications such as capacitor placement could be highly affected. Thus, distribution applications, such as capacitor placement, network reconfiguration, service restoration, using distribution power flow with a distributed slack bus model need to be further investigated.

In Chapter 5, the problem formulation of distribution expansion planning was to minimize the cost for utilities and assuming they owned all the DGs. All cost of
equipments operating and installation were accounted by one owner. If DGs are owned by customers or different owners, the planning methods for utilities are subject to change. Moreover, customers or other DG owners would view the problems for DG planning and operating differently and may require planning strategies different from utilities. Thus, the proposed cost analysis method in Chapter 4, and distributed slack bus model may need to be further studied. Therefore, further research on analysis, operation and planning for distribution systems with distributed generation can be investigated from the work presented in this thesis.
List of References


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