Participation Factor Studies for Distributed Slack Bus Models in Three-Phase Distribution Power Flow Analysis

Shiqiong Tong, *Student Member, IEEE*, and Karen Nan Miu, *Member, IEEE*

***Abstract* – With the increase of distributed generators (DGs) installed within power networks, new analysis tools for planning and operating are required to develop for distribution systems with multiple sources. Power flow with a distributed slack bus model is considered more realistic than a single slack bus model, since there is no slack bus in real power systems. Participation factors can be applied to assign slack for distributed slack bus models. Previous works about participation factors for distributed slack bus models focused on transmission systems. Due to special characteristics of distribution systems, such as high R/X ratios and network imbalance, these previous works cannot be applied directly. This paper presents a three-phase distribution power flow with a distributed slack bus model and studies of two different methods to calculate participation factors. The methods are based on network sensitivity factors and on the concepts of generator domains.**

***Index Terms* -- Power flow, Distributed slack bus, Generator domain, Sensitivity**

I. INTRODUCTION

DISTRIBUTED generation has been growing rapidly in power systems. Previously, one dominant source, the substation, existed in distribution systems. Now, the consistent supply of energy from DGs, the number of sources and the percentage of real power injections from DGs have significantly increased. Therefore, this work will re-evaluate distribution power flow analysis with a single slack bus. Improvements in distribution power flow tools are expected to impact power system planning, operations and economics [15-17].

In a traditional power flow with a single slack bus model, one bus is selected to absorb all system loss. However, there is no slack bus in real power systems. The single slack bus model may significantly distort computed power flows. Therefore, to provide more realistic power flows, power flow analysis with a distributed slack bus model has been investigated.

Previously, participation factors have been applied to assign distributed slack bus during power flow calculations. In [1, 2], the participation factors are related to the characteristics of turbines on each generator bus and load allocation. In [3], the authors applied participation factors using combined cost and reliability criteria in power flow for fair pricing. In [4], the author provides a method of choosing participation factors based on the scheduled generator outputs. While these models are novel in transmission systems for various reasons, they could not be directly applied in distribution systems with DGs. For example, the main source of a terrestrial distribution system is the substation; therefore, no turbine characteristics related to the substation are available. In addition, due to the high R/X ratios of distribution systems, the loss allocation should be considered. As such, the load distribution and the network topology play a critical role in the participation of each generator with respect to servicing loads and loss. Thus, participation factors for distribution power flow should reflect network parameters, load distribution, generator locations and capacities.

In [5,6], iterative participation factors based on the expanded concept of generator domains [8, 9] were proposed for three-phase power flow with a distributed slack bus model. These participation factors capture the effects of unbalanced network parameters, loads and generator locations. Another approach is to apply sensitivity of system loss to power injections. Therefore, in this paper, we will study participation factors calculated by these two different ways for three-phase distribution power flow with a distributed slack bus model.

A summary of a distributed slack bus model for three-phase power flow is presented in Section II. Two methods to calculate participation factors are discussed in Section III. Simulation results for distribution systems with DGs are reported in Section IV. Summary and conclusions are made in Section IV.

II. DISTRIBUTED SLACK BUS MODEL

In this work, a distributed slack bus is modeled using scalar participation factors to assign the unknown system loss for participating sources. Participating sources are ones, whose real power outputs can be adjusted. Participation factors can be calculated by different methods. Two methods will be discussed in Section III.

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S. Tong is with the Electrical and Computer Engineering Department, Drexel University, Philadelphia, PA 19104 USA (e-mail: st58@drexel.edu)

K. N. Miu is with the Electrical and Computer Engineering Department, Drexel University, Philadelphia, PA 19104 USA (e-mail: miu@ece.drexel.edu)

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In a distributed slack bus model, the system real power loss P_{Loss} is treated as an unknown and distributed to participating sources according to their assigned participation factors. Thus, the sum of participation factors K_i is one.

$$\sum_{i=1}^{m} K_i = 1 \qquad (1)$$

where

m : the number of participating sources in the system

In distribution systems, we do not expect all DGs to be allowed to adjust their real power outputs. Only the set of participating generators with adjustable real power outputs will be modeled using participation factors. In distribution systems, the participating sources including the substation and participating DGs.

$$P_{Gi} = P_{Gi}^{load} + K_i P_{Loss} \qquad i = 1, 2 \cdots m \qquad (2)$$

where

P_{Loss} : total real power loss in the system

P_{Gi}^{load} : load associated with participating source i

These equations show that the participation factors K_i represent participating sources' contributions ratio of power system real power loss P_{Loss}, which is related to network topology, load distribution and source capacities.

Since the total system real power loss P_{Loss} is unknown and varies according to the slack distribution, an additional equation at the reference bus is required for solving equations compared to a single slack bus model. Here, the substation is the reference bus, assigned as Bus 1. Also, distributed generators are assumed to be connected to their buses through inverters [5, 10]. These buses are treated as $P|V|$ buses with balanced three-phase voltages having specified voltage magnitudes. Their three-phase real power outputs are adjustable. In such a case, there is only one unknown at each participating source, the voltage phase angle θ_i^a and the real power balance equation is required. Therefore, the three-phase power flow equations are summarized as following:

For m generator buses

$$f_{Pi} = (P_{Gi}^{load} + K_i P_{Loss}) - \sum_{p=a}^{c} P_{Di}^{p} - \sum_{p=a}^{c} P_i^p = 0 \qquad (3)$$

$$i = 1, 2, \cdots m$$

For n-m load buses

$$\begin{aligned} f_{Pi}^{p} &= -P_{Di}^{p} - P_i^p = 0 \\ f_{Qi}^{p} &= -Q_{Di}^{p} - Q_i^p = 0 \end{aligned} \qquad i = m+1, m+2, \cdots, n \qquad (4)$$

where

$$P_i^p = \left|V_i^p\right| \sum_{k=0}^{n} \left|V_k^p\right| \left[g_{ik}^p \cos(\theta_i^p - \theta_k^p) + b_{ik}^p \sin(\theta_i^p - \theta_k^p) \right]$$

$$Q_i^p = \left|V_i^p\right| \sum_{k=0}^{n} \left|V_k^p\right| \left[g_{ik}^p \sin(\theta_i^p - \theta_k^p) - b_{ik}^p \cos(\theta_i^p - \theta_k^p) \right]$$

P_{Di}^p, Q_{Di}^p : the real and reactive load on bus i, phase p

n : the number of buses

Now, methods to calculate participation factors for this distributed slack bus model are discussed.

III. PARTICIPATION FACTORS

Participation factors for distribution systems should reflect network parameters, load distribution, generator locations and capacities. Two methods to calculate such participation factors will be discussed:

- network sensitivity participation factors
- generator domain participation factors

A. Network Sensitivity Participation Factors

The network sensitivity participation factors incorporate the concept of network sensitivities and penalty factors to distribute the slack. These participation factors implicitly include effects of network parameters and load distribution through the sensitivities of system real power loss to real power injections. These sensitivities are then applied to penalty factors which capture characteristics linked to distributed generator locations.

In [12], the sensitivities $\partial P_{Loss} / \partial P_i$ were delivered and can be computed at each power flow iteration:

$$\begin{bmatrix} \dfrac{\partial P_{Loss}}{\partial P} \\ \dfrac{\partial P_{Loss}}{\partial Q} \end{bmatrix} = [J^T]^{-1} \begin{bmatrix} \dfrac{\partial P_{Loss}}{\partial \theta} \\ \dfrac{\partial P_{Loss}}{\partial |V|} \end{bmatrix} \qquad (5)$$

where

J : the power flow Jacobian matrix

Since J depends on the system R and X values and $J\Delta x = -f(x)$, where f is the mismatch power, network parameters and load distribution are represented.

To more directly capture the effects of DG location, the concept of penalty factors is used. If a participating source is installed far from load centers, it should have a larger penalty factor to indicate that more loss occurred on the path to serve the same amount of load from this source. In addition, we desire nonnegative penalty factors. However, $\partial P_{Loss} / \partial P_i$ can be negative. With these desired characteristics in mind, it is noted that penalty factors used in economic dispatch [12] can be applied to reflect these properties.

Since unbalanced systems are considered, phase sensitivities on the same bus could be different. Therefore, average phase sensitivity or maximum phase sensitivity can be utilized. The penalty factors L_i are defined as:

i) Based on average phase sensitivity

$$L_1 = 1 \qquad \textit{for the reference bus}$$

$$L_i = \frac{1}{1-\frac{1}{3}(\frac{\partial P_{Loss}}{\partial P_{Gi}^{a}}+\frac{\partial P_{Loss}}{\partial P_{Gi}^{b}}+\frac{\partial P_{Loss}}{\partial P_{Gi}^{c}})} \tag{6}$$

$$i = 2,\cdots m$$

ii) Based on maximum phase sensitivity

$$L_1 = 1 \qquad \textit{for the reference bus}$$

$$L_i = \frac{1}{1-\mathrm{Max}(\frac{\partial P_{Loss}}{\partial P_{Gi}^{a}},\frac{\partial P_{Loss}}{\partial P_{Gi}^{b}},\frac{\partial P_{Loss}}{\partial P_{Gi}^{c}})} \tag{7}$$

$$i = 2,\cdots m$$

At first glance, these penalty factors are not necessarily nonnegative; however, when calculating in per unit with realistic power distribution components, the sensitivity values are less than one. Therefore, these penalty factors can be applied to determine network sensitivity participation factors:

$$K_i = \frac{L_i P_{Gi}^{load}}{\sum_{j=1}^{m} L_i P_{Gi}^{load}} \qquad i = 1,2\cdots m \tag{8}$$

Since J changes at each iteration, L_i and the participation factors are iterative.

B. Generator Domain Participation Factors

Participation factors based on the concept of generator domains are now discussed. Generator domains strive to associate a set of buses and branches and their power flows to specific participating generators. As such, an associated loss with each participating source can be quantified. The effects of network parameters, load distributions and generator capacities are explicitly included in these participation factors.

A given real power injection P_{Gi} is to associate portions of the electrical network, their loads and losses:

$$P_{Gi} = P_{Gi}^{load} + P_{Gi}^{loss} \qquad i = 1,2,\cdots m \tag{9}$$

where:

$$P_{Gi}^{load} = P_{Gi}^{load,a} + P_{Gi}^{load,b} + P_{Gi}^{load,c} \tag{10}$$

$$P_{Gi}^{loss} = P_{Gi}^{loss,a} + P_{Gi}^{loss,b} + P_{Gi}^{loss,c} \tag{11}$$

and

P_{Gi}^{load} : load associated with participating source i

P_{Gi}^{loss} : loss associated with participating source i

$P_{Gi}^{load,p}$: load associated with participating source i, phase p

$P_{Gi}^{loss,p}$: loss associated with participating source i, phase p

The process for determining unbalanced generator domains are reviewed from [5] in the following subsections. The generator domain participation factors are as follows:

$$K_i = \frac{P_{Gi}^{loss}}{P_{Loss}} \qquad i = 1,2,\cdots m \tag{12}$$

In the distributed slack bus model, the real power outputs of participating sources are iterative. Generator domains and loss contributions vary with changing source injections. Thus, the participation factors are iterative during power flow calculations. Next, we review the concept of generator domains which were used.

Generator Domains

The concept of generator domains originated from [8] and was extended to unbalanced systems in [5,6]. In distribution systems, since the loads are unbalanced, the buses and branch flows supplied by the same generator may be different across phases. Thus, the domain of a generator is defined as the set of buses and branches whose power is supplied by the generator.

Domains vary for each phase and are assigned based on:

- positive power flow direction per phase
- proportionality of commons (areas assigned to more than one source)

Each are now discussed.

The positive power flow direction is used to assign a directed graph onto the distribution system. For two directly connected buses, bus i, phase p and bus j, phase p

- If $\mathrm{Re}(V_i^{p} I_{ij}^{p*}) - \mathrm{Re}(V_j^{p} I_{ij}^{p*}) > 0$, we define that positive real power flows from bus i to bus j over phase p;
- If $\mathrm{Im}(V_i^{p} I_{ij}^{p*}) - \mathrm{Im}(V_j^{p} I_{ij}^{p*}) > 0$, we define that positive reactive power flows from bus i to bus j over phase p

where

V_i^{p} : voltage on bus i in phase p

I_{ij}^{p} : current from bus i to bus j over phase p

The positive real power flows and positive reactive power flows may be different on the same branch. In this work, we are interested in the real power slack, so the positive real power flow directions are used to trace each load's power supplying sources.

Based on these directions, the concept of generator commons for unbalanced systems can be described. The loss on a branch or the load on a single node may be supplied by many sources; therefore, the domains of different generators intersect on this phase. As such, they have the branch or load in common. The definition of a generator common is taken to be a set of contiguous nodes and branches by phase, whose power is supplied by the same generators. The proportion of loss and loads supplied by different sources to a common is assumed to be the same as the proportion of the positive real power injected by the sources to this common.

By applying proportionality of commons, the proportions of loads and losses of each common are then assigned to the corresponding sources, and the load and loss contributed by each source can be found [8]. As such, iterative participation factors can be developed within an unbalanced power flow solver.

IV. SIMULATIONS

The unbalanced power flow with a distributed slack bus model is now applied for studies of different participation factor models. The substation and DGs with adjustable real power outputs are participating sources.

A 20-bus test distribution system with total system loads of 6.0451MW and 3.2724 Mvar is shown in Figure 1 below. If no DG is installed, total system real power loss, P_{Loss}^{sys}, is 226.23kW or 3.74%. To test the participation factor models which reflect network parameters, the network was designed from a portion of an existing system with real network parameters. The transformer between Bus 2 and Bus 3 services 1.6669MW and 0.9626Mvar high density loads. The transformer between Bus 2 and Bus 4 services 4.3782 MW and 2.30975 Mvar dispersed loads in a commercial and residential area. With no DG installed 3.51kW (1.56% of total system real power loss) occurs in the high density load area and 204.59kW (90.43% of the total loss) occurred in the commercial and residential area from its higher network resistances and branch currents.

In the following examples, two cases will be investigated:

- Case 1: the DG is installed on Bus 3
- Case 2: the DG is installed on Bus 4

We expect a DG installed on Bus 4 should have a larger impact on system real power loss and a larger percentage of system loss contribution. Thus, it should be assigned a larger participation factor than a DG installed on Bus 3 to serve the same amount of real power loads.

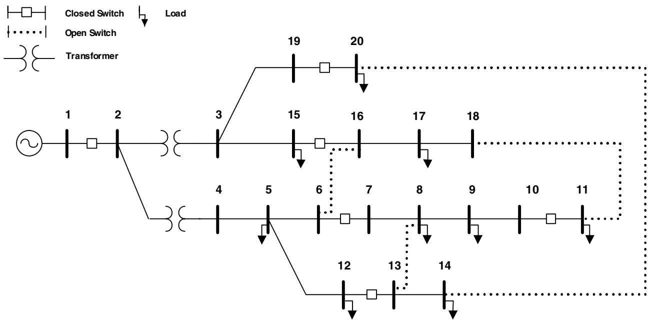

Figure 1. A one-line diagram of the 20-bus distribution system

In both cases, one DG is assumed to service 1,500kW loads (that is P_{DG}^{load} = 1,500kW, approximately 25% DG penetration.) In each case, different power flow models are compared:

- A single slack bus model
- A distributed slack bus model using different participation factors
  - based on scheduled generator outputs [4] yielding constant factors:

$$K_i = \frac{P_{Gi}^{load}}{\sum_{j=1}^{m} P_{Gi}^{load}} \qquad (13)$$

  - based on average phase sensitivities
  - based on maximum phase sensitivities
  - based on generator domains

Simulation results for Case 1 and Case 2 studies are shown in Table 1 and Table 2 respectively.

Table 1. Case1, one DG on Bus 3 to service 1,500kW load

	Single Slack	Distr. Slack Gen Cap.	Distr. Slack Avg Sen.	Distr. Slack Max Sen.	Distr. Slack Gen Dom.
Sub.Par. K_1	1	0.7519	0.7633	0.7557	0.9861
DG Par. K_2	0	0.2481	0.2367	0.2443	0.0139
P_{sub}^{out} (kW)	4769.31	4713.66	4713.66	4713.66	4766.20
P_{DG}^{out} (kW)	1500.00	1555.64	1555.64	1555.64	1503.12
P_{Loss}^{sys} (kW)	224.233	224.212	224.212	224.212	224.231

Table 2. Case 2, one DG on Bus 4 to service 1,500kW load

	Single Slack	Distr. Slack Gen Cap.	Distr. Slack Avg Sen.	Distr. Slack Max Sen.	Distr. Slack Gen Dom.
Sub.Par. K_1	1	0.7519	0.7468	0.7497	0.6749
DG Par. K_2	0	0.2481	0.2532	0.2503	0.3251
P_{sub}^{out} (kW)	4752.05	4700.55	4700.55	4700.55	4684.59
P_{DG}^{out} (kW)	1500.00	1551.31	1551.31	1551.31	1567.21
P_{Loss}^{sys} (kW)	206.971	206.781	206.781	206.781	206.723

From the numerical studies, we observe that:

- For the single slack bus model, both cases keep the DG at the same output P_{DG}^{out} =1,500kW.
- The distributed slack bus model with non-iterative participation factors based on scheduled generator outputs alone has the same participation factor values in both cases. Thus, with the same DG output, the amount of the output attributed to loads versus losses from (2) would be the same even though the DG is located at different locations. Since this method does not capture the effects of DG locations on system studies, it is not recommended.
- The distributed slack bus model with sensitivity participation factors were computed in two ways: based on average sensitivities and maximum phase sensitivities. The resulting participation factors were slightly different between these two methods. It is noted that both methods assigned larger participation factors to the DG on Bus 4 than when the DG was placed on Bus 3. Thus the sensitivity and penalty factor approach performed, as expected, with respect to attributing higher losses to the DG at bus 4. However, the difference in participation factors between the DG at bus 3 vs. bus 4 was small. Thus, concerns arise as to whether sensitivity measures are significant enough to fully capture the effects of DG locations.
- In contrast, the distributed slack bus model with

generator domain participation factors has a much larger participation factor for the DG on Bus 4 than the DG on Bus 3 (0.3251 in Case 2 vs. 0.0139 in Case 1). This demonstrates that explicitly relating the participation factors with generator locations, network parameters and load distribution yield more distinct distributed slack bus participation factors.

Therefore, the participation factors determined using generator domains is recommended for the distributed slack bus model.

V. SUMMARY & CONCLUSIONS

In this paper, two different methods to calculate participation factors for three-phase distribution power flow with a distributed slack bus model were described in detail and studied. Participation factors calculated by sensitivity-based methods and generator domain based methods capture the effects of network parameters, load distribution, generator outputs and locations. Studies comparing a single slack bus power flow model with the two different distributed slack bus models described and one based solely on generator capacities or outputs were presented. A summary of our observation are as follows:

- participation factors based on network sensitivity factors versus generator domains may vary significantly,
- sensitivity participation factors change modestly, because the effects of network parameters, load distribution are implicitly embedded,
- generator domain participation factors change significantly, because the effects of network parameters, load distribution, generator outputs and locations are explicitly included. These participation factors are recommended for the distributed slack bus model.

Thus, for systems with distributed generation, the distributed slack bus model has been incorporated into a general, unbalanced power flow solver. These models will then impact distribution applications, such as DG placement, capacitor placement, network reconfiguration and economic analyses.

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VII. BIOGRAPHIES

Shiqiong Tong (S’02) was born in Chongqiong, China in April 1975. She received her B.S. and M.S. in Automation Technology from Chongqing University in People Republic of China in 1997 and 2000 respectively. She is currently working on her Ph.D. degree in the Power Electronics Laboratory at Drexel University.

Her research interests include power distribution automation, distribution planning and distributed generation.

Karen Nan Miu (M’98) received the Ph.D. degree in electrical engineering from Cornell University. She is currently an associate professor at Drexel University in the department of Electrical and Computer Engineering.

Her research interests include distribution system analysis, distribution automation and optimization techniques applied to power system. She is a recipient of the 2000 NSF Career Award and the 2001 ONR Young Investigator Award.